What do we study in steady state analysis in single phase ac circuit & Transformers.

In state analysis for AC circuits, we delve into analyzing circuits using differential equations to understand how voltages and currents change over time. This includes studying the circuit's behavior under various conditions, such as steady-state AC signals, and applying techniques like phasor analysis and Laplace transforms to simplify and solve the circuit equations.

The unit consist of transformer includes: Magnetic circuits, Working principle of transformer, Types of transformer, E.M.F. equation of transformer, Ideal and practical transformer, Equivalent circuit of transformer, Losses in transformers, Voltage regulation and Efficiency of transformer, Numerical problems.

Why do we need to steady state analysis in single phase ac circuit & Transformers.

Studying steady-state analysis in single-phase AC circuits helps us understand the circuit's behavior when it reaches equilibrium after initial transients. It's crucial for designing and analyzing circuits that operate under continuous AC supply, ensuring accurate predictions of voltage, current, and power distribution, and optimizing performance for real-world applications.

Transformers play a crucial role in electrical distribution by efficiently changing voltage levels, enabling the transmission of electricity over long distances with minimal losses. Understanding their operation, types, and working principle is essential for engineers to ensure reliable and safe electrical systems. This structured approach not only enhances the understanding but also provides practical insights into the vital role that transformers plays in modern electrical infrastructure, making them indispensable in power generation, transmission, and distribution networks worldwide.

Where do we use steady state analysis of single phase ac circuit & Transformers.

Steady-state analysis of single-phase AC circuits is widely used in various practical applications, including:

Power Distribution Systems: To analyze and design electrical systems for efficient energy distribution and load management.

Home Appliances: For designing and troubleshooting appliances like refrigerators, air conditioners, and heaters that operate on single-phase AC power.

Lighting Systems: To ensure proper operation and efficiency of residential and commercial lighting systems.

Transformers are used in a variety of applications, including power generation, transmission and distribution, lighting, audio systems, and electronic equipment.

Power generation: Transformers are used in power plants to increase the voltage of the electricity generated by the plant before it is sent to the grid.

Transmission and distribution: Transformers are used in the transmission and distribution of electricity to increase or decrease the voltage of electricity as it is sent from power plants to homes and businesses.

Lighting: Transformers are used in lighting systems to decrease the voltage of electricity before it is sent to light bulbs.

Audio systems: Transformers are used in audio systems to increase or decrease the voltage of electricity before it is sent to speakers.

Electronic equipment: Transformers are used in a variety of electronic devices, including computers, TVs, radios, and cell phones.

Transformers are a vital part of the electrical grid and are used in a variety of applications to ensure that electricity is delivered safely and efficiently.

Electrical Engineering Design: In designing and optimizing circuits for various electronic devices and systems.

Maintenance and Troubleshooting: To diagnose and fix issues in existing AC circuits by understanding their steady-state behavior.

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2.0 ALTERNATING QUANTITY (AC)

An alternating quantity is that which acts in alternate direction & whose magnitude undergoes a definite cycle of changes in definite intervals of time.
In India, the frequency of AC is 50 Hz.

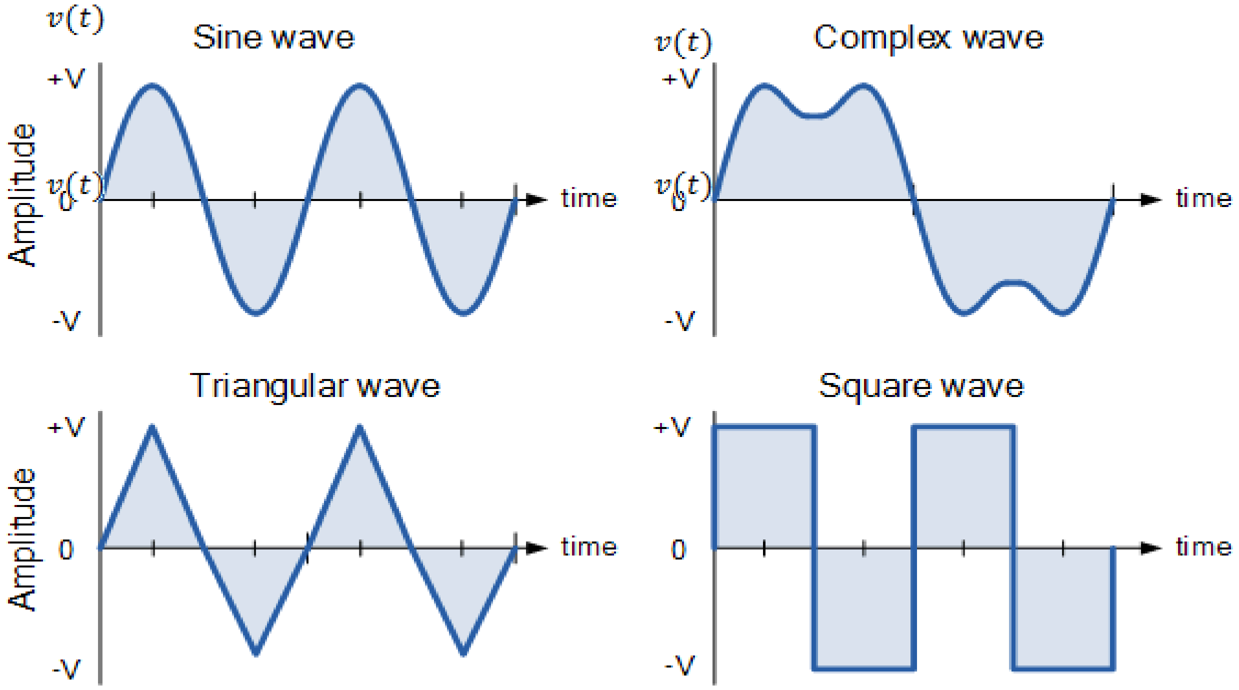


Fig .2.0

2.1. DIRECT CURRENT (DC)

When the electric charge inside the conductor flows in one direction, then such type of current is called direct current. The magnitude of direct current always remains constant and the frequency of current is Zero (0).
It is used in cell phones, electric vehicles, welding electronic equipment etc.,

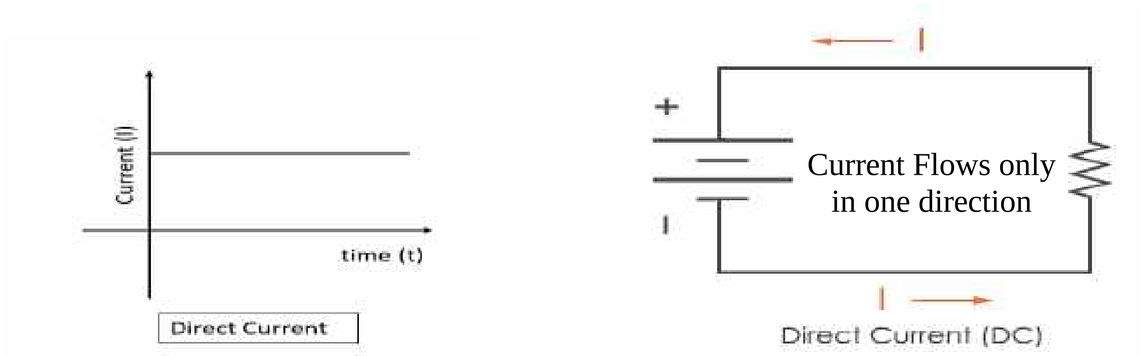


Fig.2.1

2.2. IMPORTANT DEFINITIONS

✶ **Waveform: -** The graph of instantaneous values of an alternating quantity ploted against time is called its waveform.

✶ **Instantaneous: -** The value of an alternating quantity at a perticular instant is called its Instataneus value.

✶ **Cycle: -** One complet set of positive and nagative values and Zero values of an alternating quantity is called a Cycle.

Sometimes, a cycle is specified in terms of angular measure. In that case, one complete cycle is said to spread over $360°$ or 2π radians.

✶ **Time Period: -** Time taken by an alternating quantity to complete one cycle is called its time period 'T'.

$$T = \frac{1}{F}\ seconds$$

✶ **Frequency: -** One of cycles / seconds is called frequency of an alternating quantity. Its unit is Hertz (Hz).

$$F = \frac{1}{T} Sec^{-1}\ or\ Hertz$$

✶ **Amplitude: -** Maximum positive and nagative value, of an alternating quantity is known as its Amplitude.

$$i = 100 \sin \omega t$$

Amplitute is $i_0 = 100\ Amp$

✶ **Angular Frequency: -** It is the frequency expressed in electrical radians per second.As one cycle is of alternating quantity corresponde 2π radians, angular frequency is

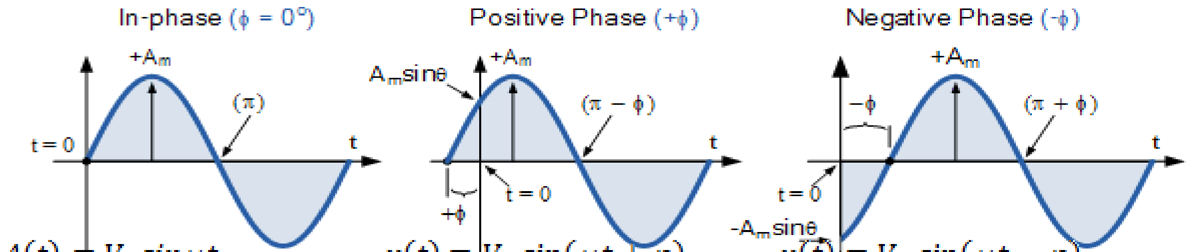
$$\omega = 2\pi F \frac{rad}{sec},\ \omega = \frac{2\pi}{T}\frac{rad}{sec}.$$

✶ **Phase: -** Phase of alternating quantity is the fractional part of time period which shows that ac quantity is how much displased from time zero -axis.

$$e = E_m \sin(\omega t + 90)$$

$I.e + ve\ phase\ 90\ degree\ (leading)$

✶ **Phase Defference: -** It is define as angular displasedment between two zero values or two maximum value of the two alternating having same frequency.



$A(t) = V_m sin\omega t$ $\quad v(t) = V_m \sin(\omega t + \varphi)$ $\quad v(t) = V_m \sin(\omega t - \varphi)$

fig 2.2

✶ **Ading Phase difference**

A quantity which attains its zero or positive maximum value before the compared to other quantity.

✶ **Laging phase difference**

A quantity which attains its zero or positive maximum values after the other quantity.

2.3. AVERAGE VALUE :-

The average value of an alternating current is expressed by that study current which transfers across any circit the same charge as is transferd by that alternting current during the same time.

In case of symmertrical alternating current (i.e. one whose two half cycle are exaclty simler whether sinusoidal or non-sinusoidal). Average value over a complete cycle is zero.But in case of unsymmeterical alternating current (like half-wave rectified current), average value must always to taken over the whole cycle.

$$V_{avg} = \frac{1}{T}\int_0^T f(t).dt$$

2.4. R.M.S VALUE:-

It is defined as that steady current which when flows through a resistor of known resistance for a give period of time as a result the same quantity of heat is produced by an alternating current when flows through the same resistor for the same period of time is called R.M.S or effective value of alternating current.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t).dt}$$

2.5. FORM FACTOR: -It is defined as the ratio of R.M.S value to the average value of an alternating waveform.

$$Form\ Factor\ (K_F) = \frac{R.M.S\ Value}{Average\ Value}$$

$$= \frac{\frac{V_{max}}{\sqrt{2}}}{\frac{2V_{max}}{\pi}}$$

$K_F = 1.11\ For Sinusoidal waveform$

2.6. PEAK FACTOR OR CREST FACTOR OR AMPLITUDE FACTOR: -

It is the ratio of maximum value to the R.M.S value of an alternating wave. For sinusoidal wave of voltage, Peak factor is 1.4142.

$$Peak\ Factor = \frac{Maximum\ value}{R.M.S\ value}$$

For Sinusoidal waveform

$$Peak\ Factor\ (K_p) = \frac{V_{max}}{\frac{V_{max}}{\sqrt{2}}}$$

$$K_p = \sqrt{2} = 1.414$$

Long Questions

Question 1: The equation of alternating current is $i = 141.4 \sin 314t$. What is r.m.s value of current and frequency?

(AKTU2015-2016)

Soultion: R.M.S value will be,

$$I_{r.m.s} = \frac{i_0}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 99.99\ A$$

And frequency will be,

$$\omega = 2\pi f, \qquad f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 49.98 \cong 50\ Hz$$

Question 2 The equation of an alternating current $i = 42.42 \sin 628\ t$.Determine (i) maximum value (ii) frequency (iii) r.m.s value (iv) average value (v) form factor.

(AKTU2017-2018)

Solution: (i) Maximum value i.e

$$i_0 = 42.42\ A$$

(ii) angular frequency i.e

$$\omega = 628\ \ rad/sec.$$

Frequency in Hz would be

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} = \frac{628}{2\pi} = 99.95 \cong 100\ Hz$$

(iii) R.M.S value,

$$i_{r.m.s} = \frac{i_0}{\sqrt{2}} = \frac{42.42}{\sqrt{2}} = 30\ A$$

(iv) Average value,

$$i_{ave} = \frac{2i_0}{\pi} = \frac{2 \times 42.42}{\pi} = 27\ A$$

(v) Form factor i.e,

$$K_f = \frac{i_{r.m.s}}{i_{a.v.e}} = \frac{30}{27} = 1.11$$

Short Questions

Question 3 Explain the form factor and peak factor. (AKTU2017-2018, 2018-2019, 2020-2021)
Question 4 Define the expression for form factor and peak factor. (AKTU2021-2022**)**
Question 5 Define power factor. (AKTU2021-2022)

2.7.1.R.M.S AND AVERAGE VALUES OF SINUSOIDAL WAVEFORMS

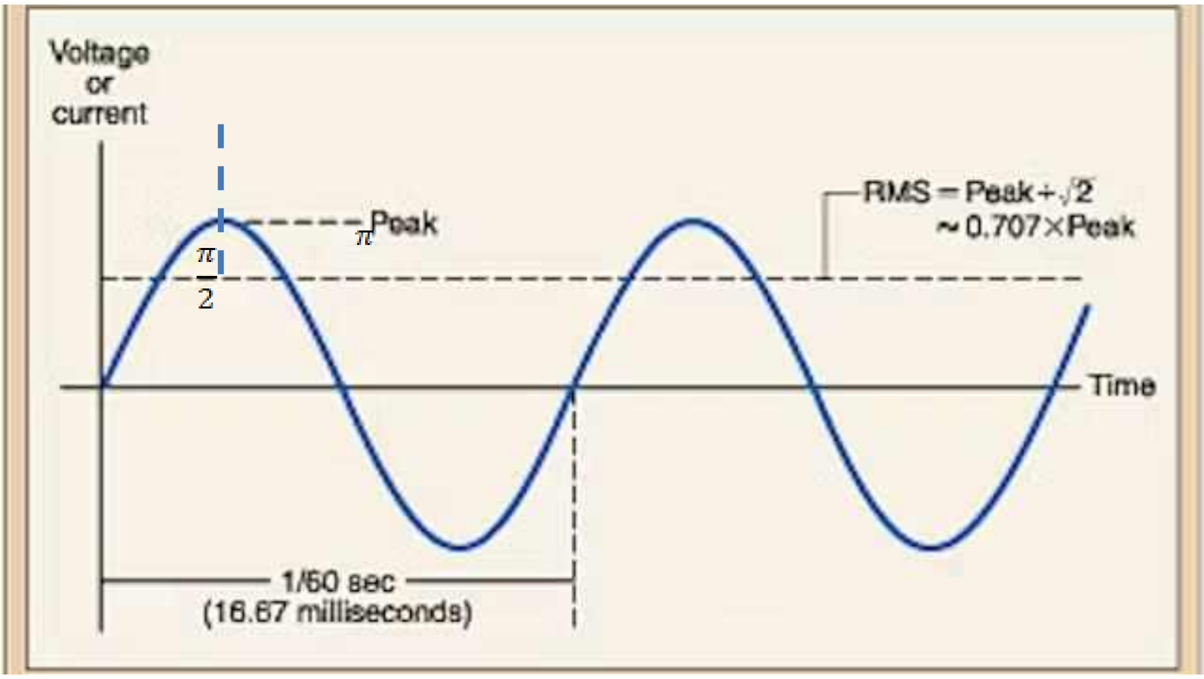


Fig 2.3 SINUSOIDAL WAVEFORMS

$t = 2\pi \quad (length\ of\ the\ base)$

Wave equation $i = I_m sin\theta$

$$v = V_m\, sin\theta$$

R.M.S Value

$$v_{rms}^2 = \frac{1}{2\pi}\int_0^{2\pi} v^2(\theta).d\theta$$
$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{2V_m^2 sin^2\theta}{2}.d\theta$$
$$= \frac{1}{4\pi}V_m^2\left[\int_0^{2\pi}(1-cos2\theta).d\theta\right]$$
$$= \frac{V_m^2}{4\pi}\left[\theta - \frac{sin2\theta}{2}\right]_0^{2\pi}$$
$$V_{rms}^2 = \frac{V_m^2}{4\pi}[2\pi - 0]$$
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$
$$V_{rms} = 0.707V_m$$

Average value

Average value for complete cycle is zero.
So, for half cycle

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta).d\theta$$
$$= \frac{1}{\pi}\int_0^{\pi} V_m sin\theta.d\theta$$
$$= \frac{V_m}{\pi}[-cos\theta]_0^{\pi}$$
$$= \frac{V_m}{\pi}[-cos\pi - cos0]$$
$$= \frac{V_m}{\pi}[-(-1)+1]$$
$$V_{avg} = \frac{2V_m}{\pi}$$

Or

$$V_{avg} = 0.637V_m$$

Form Factor

$$K_F = \frac{V_{rms}}{V_{avg}} = \frac{0.707V_m}{0.637V_m} = 1.11$$

Peak Factor

$$K_p = \frac{V_{max}}{V_{rms}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

2.7.2 FULL WAVE RECTIFIED SINUSOIDAL WAVE FORM

$\pi \rightarrow Length\ of\ base$

Wave equation $v = V_m sin\theta$

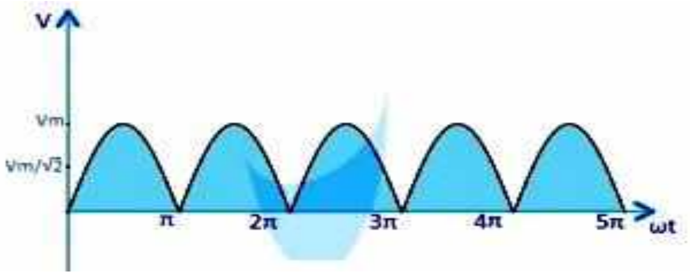


Fig.2.4 Full Wave

R.M.S Value

$$V_{rms}^2 = \frac{1}{\pi}\int_0^{\pi} v^2(\theta).d\theta$$
$$V_{rms}^2 = \frac{1}{\pi}\int_0^{\pi} \frac{2v_m^2 sin^2\theta}{2} d\theta$$
$$V_{rms}^2 = \frac{v_m^2}{2\pi}\int_0^{\pi} 2sin^2\theta d\theta$$
$$V_{rms}^2 = \frac{v_m^2}{2\pi}\int_0^{\pi} (1-cos2\theta).d\theta$$
$$V_{rms}^2 = \frac{v_m^2}{2\pi}\left[\theta - \frac{sin2\theta}{2}\right]_0^{\pi}$$
$$V_{rms}^2 = \frac{v_m^2}{2\pi}[\pi - 0]$$
$$V_{rms}^2 = \frac{v_m^2}{2}$$
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Average Value

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta).d\theta$$
$$= \frac{1}{\pi}\int_0^{\pi} V_m sin\theta d\theta$$
$$= \frac{V_m}{\pi}[-cos\theta]_0^{\pi}$$
$$= \frac{V_m}{\pi}[-cos\pi - cos0]$$
$$= \frac{V_m}{\pi}[-(-1)+1]$$
$$= \frac{2V_m}{\pi}$$
$$V_{avg} = 0.637\ V_m$$

Form Factor

$$K_F = \frac{V_{rms}}{V_{avg}} = 1.11$$

Peak Factor

$$K_p = \frac{V_m}{V_{rms}} = \sqrt{2} = 1.414$$

2.7.3. HALF WAVE RECTIFIED SINUSOIDAL WAVE FORM

$2\pi \rightarrow Length\ of\ base$

$$wave\ equation: \begin{cases} v = V\ sin\theta\ 0\ to\ \pi \\ v = 0, \pi\ to\ 2\pi \end{cases}$$

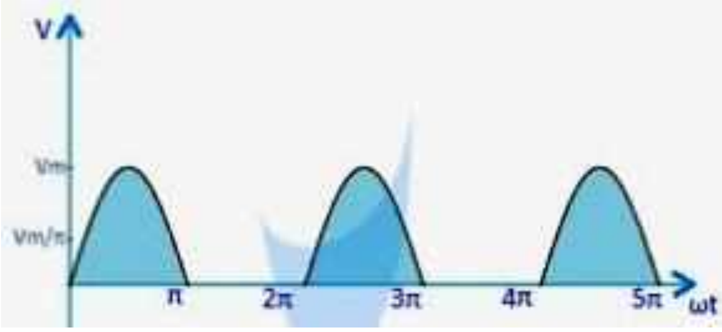


Fig. 2.5 Half Wave

R.M.S Value

$$V_{rms}^2 = \frac{1}{2\pi}\left[\int_0^{2\pi} v^2(\theta).d\theta\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_{rms}^2\, sin\theta\,.d\theta + \int_{\pi}^{2\pi} 0.\,d\theta\right]$$

$$= \frac{V_{rms}^2}{4\pi} - \left[\int_0^{\pi}(1 - cos2\theta)d\theta\right]$$

$$= \frac{V_{rms}^2}{4\pi}\left[\theta - \frac{sin2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_{rms}^2}{4\pi}(\pi - 0)$$

$$= \frac{V_{rms}^2}{4\pi}$$

$$or \qquad V_{rms} = \frac{V_{rms}}{2}$$

Average Value

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta).d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_{rms} sin\theta d\theta + \int_{\pi}^{2\pi} 0 d\theta\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_{rms} sin\theta d\theta\right]$$

$$= \frac{V_{rms}}{2\pi}[-cos\theta]_0^{\pi}$$

$$= \frac{V_{rms}}{2\pi}[-(-1) + 1]$$

$$V_{avg} = \frac{V_{rms}}{\pi} = 0.318\, V_m$$

$$Form\ Factor\ \ (KF) = \frac{\frac{V_m}{2}}{\frac{V_m}{\pi}} = 1.57$$

$$Peak\ Factor\ \ \ (KP) = \frac{V_m}{V_{rms}} = \frac{V_m}{\frac{V_m}{2}} = 2$$

2.7. CONCEPT OF PHASE AND PHASE DIFFERENCE

A.C :

$a.c \rightarrow A < \emptyset \rightarrow angle\ (phase) \begin{cases} Polor\ Form \quad A < \emptyset \rightarrow Phase \\ Rectangular = A + iB \end{cases}$

Phase: Phase of an alternating quantity at any instant is the angle ($\emptyset$) travelled by the phaser representing that alternating quentity upto the instant of consideration, measured form the reference.

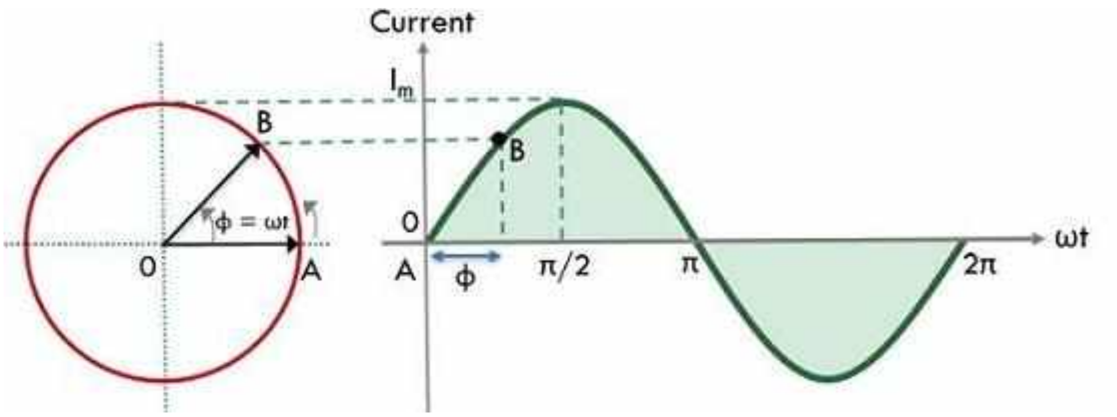


Fig.2.6 Sinusoidal Wave

Equation of ac is given by $e = E_m \sin(\omega t \pm \emptyset)$
There cases are:-
Case I $(\emptyset = 0°) \rightarrow Zero\ phase\ means\ at\ t = 0, instantanous\ value\ of\ a.c\ qualtiy\ is\ zero.$

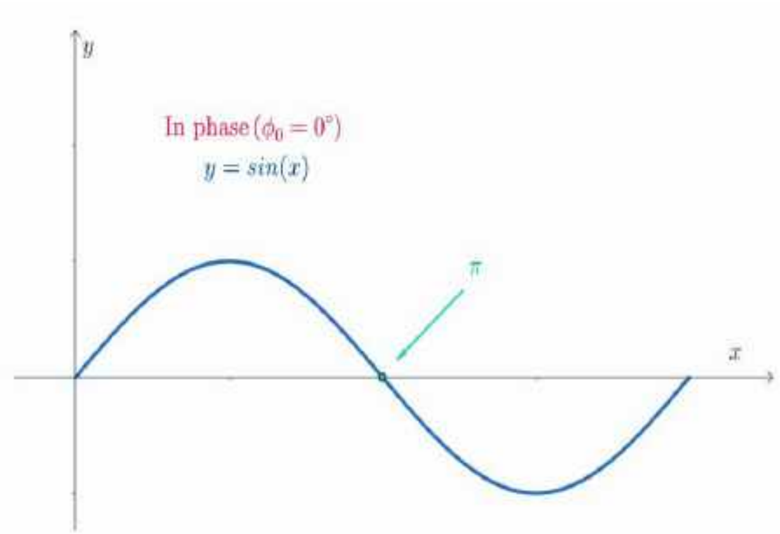


Fig 2.7 Zero phase wave form

Case 2 Positive Phase ; —If at t=0 ,alternating quantity has positive instantaneous value

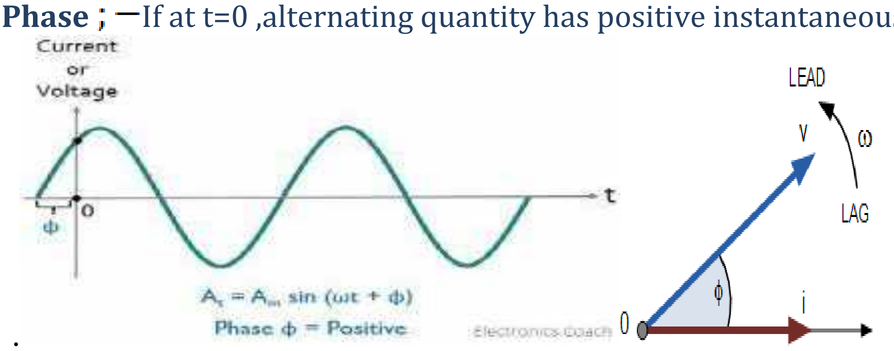


Fig.2.8 positive phase wave form

Case 3 Negative Phase

if at t=0 ,alternating quantity has negative instantaneous value.

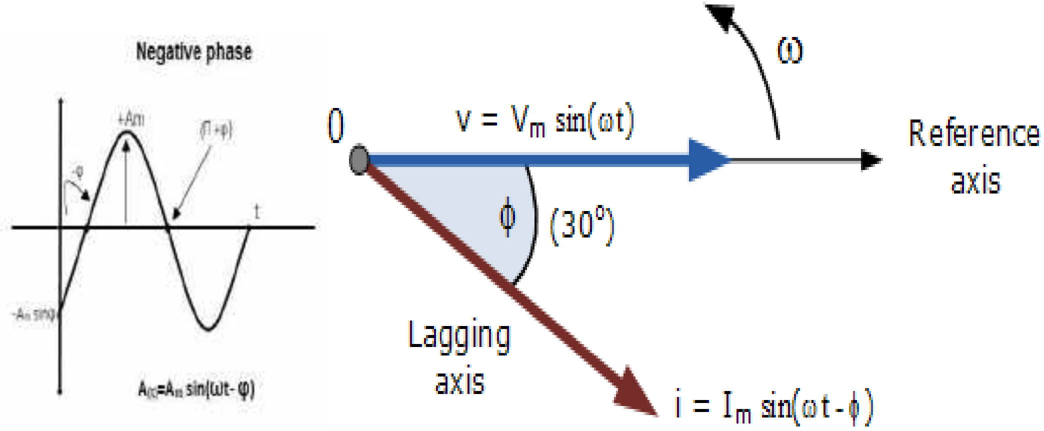


fig.2.9 Negative phase wave form

→Phase is measured w.r .t reference direction i.e +ve x-axis direction.

→Phase measured in anticlockwise direction is +ve while measured in clockwise direction is negative.

→The difference between the phase of the two alternating quantity is called the phase diffrence .

1)- **Zero Phase Difference** ;-

$e = E_m \sin \omega t$

$i = I_m \sin \omega t$

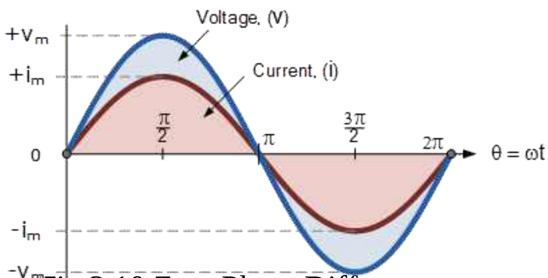


Fig.2.10 Zero Phase Difference

2)- **Lagging Phase Difference** ;-

$e = E_m \sin \omega t$

$i = I_m \sin(\omega t - \varphi)$

e leads i by φ, I lags e by φ.

Phase difference = φ

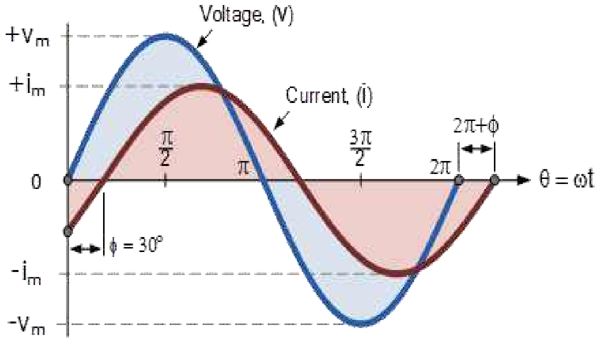


Fig 2.11 Lagging Phase Difference

3)- **Leading Phase Difference ;-**

$e = E_m \sin \omega t$

$i = I_m \sin(\omega t + \varphi)$

e lags i by φ

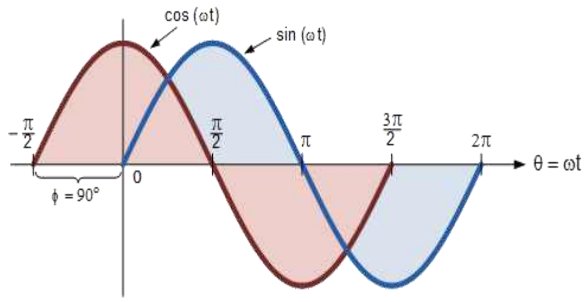


Fig 2.12 Leading Phase Difference

2.8. PURELY RESISTIVE CIRCUIT (R only):-

Let 'Z' be the impedance of the circuit.
Current flowing through the circuit-

$i = \frac{V}{Z} = \frac{V_m sin\omega t}{Z} = \frac{V_m sin\omega t}{R}$

$i = I_m sin\omega t$

Or

$I_m = \frac{V_m}{Z}$

$Z = R$

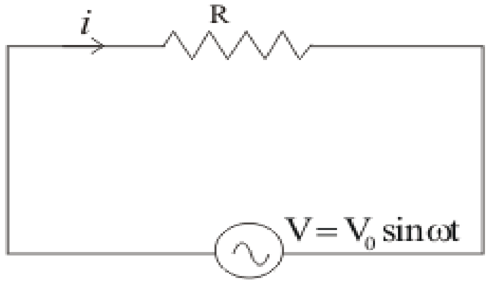


Fig. 2.13 Purely Resistive Circuit

I.) Impedance $Z = R$

II.) $\emptyset$ angle between v & i.$\emptyset = vi = 0°$ f

III.) Power factor $(\cos\emptyset = \cos 0 = 1)$ i.e unity

IV.) v & i are in same phase.

$v = V\angle 0°$ $(v = V_m sin\omega t)$

i=I ∠0$(i = I_m sin\omega t)$

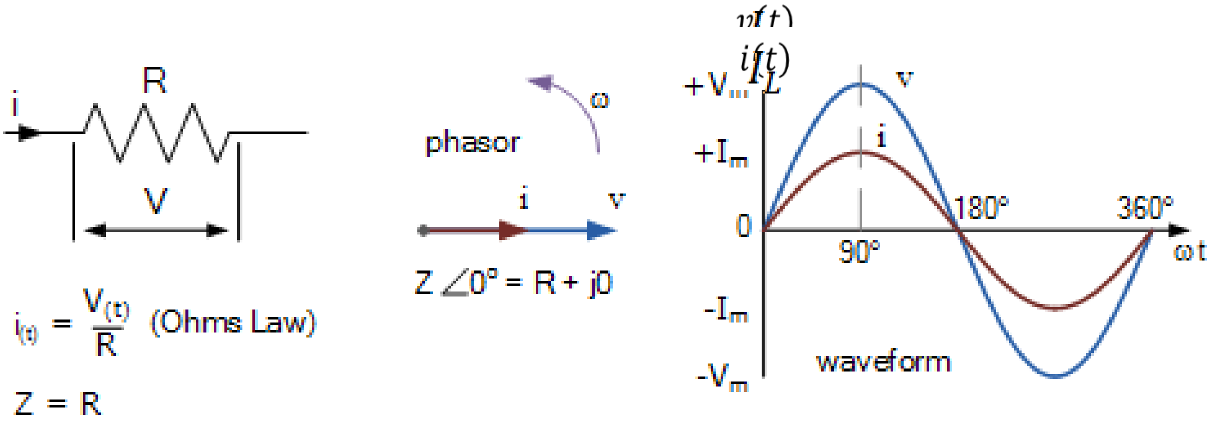


Fig 2.14 Waveform

IMPEDANCE:-

The opposition offered by an a.c circuit in the path of current is called impedance (Z).

$$z = \frac{v}{i} = \frac{V\angle 0°}{I\angle 0°} = R\angle 0°$$

$$z = R\angle 0° (\text{Polar form})$$

$$z = R + j0\ (Rectangular\ form)$$

$$z = R\ \Omega, \angle\emptyset = 0°$$

[Power Factor $(\cos\emptyset) = Unity$]

i.e $\cos\emptyset = \cos 0°$

$$\cos\emptyset = 1\ (unity)$$

POWER:-

$$p = vi$$

$$p = V_m sin\omega t . I_m sin\omega t$$

$$p = \frac{V_m I_m}{2} 2sin^2 \omega t$$

$$p = \frac{V_m I_m}{2}(1 - cos2\omega t)$$

$$p = \frac{V_m I_m}{2} - \frac{V_m I_m cos2\omega t}{2}$$

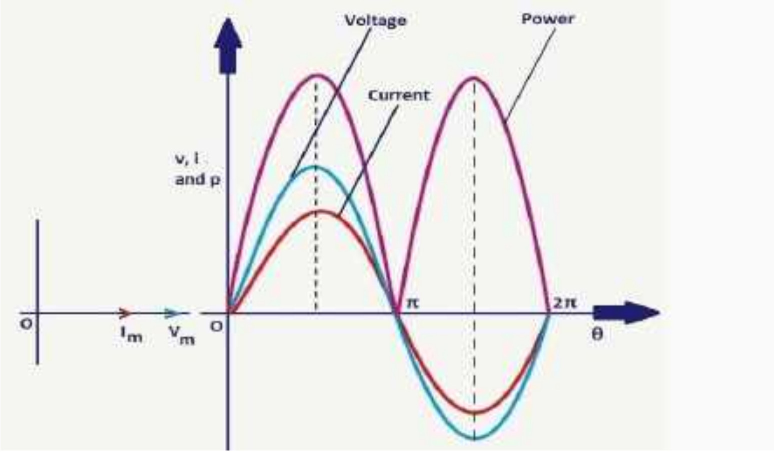


Fig 2.15 Waveform Of Power

Over a complete cycle,

$$\int_0^{\pi} \frac{-V_m I_m}{2} cos2\omega t(d\omega t) = 0$$

Average power over a complete cycle is,

$$p_{avg} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}.\frac{I_m}{\sqrt{2}}$$

$$p_{avg} = v_{rms}.i_{rms}\ (watt)$$

$$p_{avg} = i^2{}_{rms}.R \quad \{v = iR\}$$

$$p_{avg} = \frac{v^2{}_{rms}}{R} \quad \{i = \frac{v}{R}\}$$

So a pure resistance dissipates energy is in the form of heat.

2.9. PURELY INDUCTIVE CIRCUIT (L only);-

When a alternating current ''i'' flows through an inductor L (H)having ''N'' turns it sets up an alternating magnetic field (Ø), $MMF = NI\ (AT)$ around the inductor.

This changing flux links the coil & due to self inductance, e.m.f gets induced in the coil which opposes the supply voltage.

Self induced e.m.f ($e_L = -L\frac{di}{dt}$)

Applied voltage ''v'' is opposite in polarity to e_L

$$\therefore v = -e_L = -(-L\frac{di}{dt})$$

Or

V= $L\frac{di}{dt}$ (1)

$$V_m sin\omega t = L\frac{di}{dt}$$

$$di = \frac{V_m}{L}\ sin\omega t.dt$$

$$i = \int di = \int \frac{V_m}{L}\ sin\omega t.dt$$

$$i = \frac{V_m}{L}(-\frac{\cos\omega t}{\omega})$$

$$i = \frac{V_m}{\omega L}\{-\sin(\frac{\pi}{2} - \omega t)\}$$

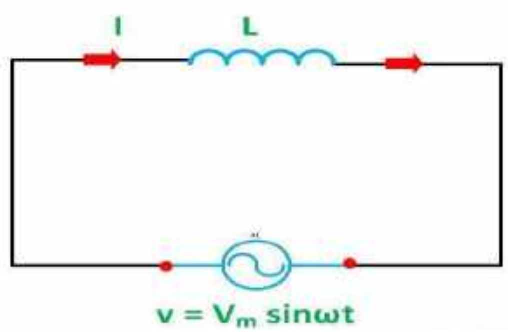


Fig.2.16 Purely Inductive Circuit

$$i = \frac{V_m}{\omega L}\sin(\omega t - \frac{\pi}{2})$$
$$i = I_m \sin(\omega t - \frac{\pi}{2})$$
$$i_m = \frac{V_m}{Z} = \frac{V_m}{X_L}$$

Or Z $= X_L = \omega L = 2\pi f L$ (Ω)

I.) Impendence (Z $= X_L$) Ω

II.) ‘i’ lags ‘v’ by $\frac{\pi}{2}$ or 90°.($\emptyset = 90^\circ$)

III.) Power factor $\cos\emptyset = \cos\frac{\pi}{2} = 0$

IV.) Power (P)=VI $\cos\emptyset$

P = 0 ($\cos\emptyset = 0$)

Impedance (Z) :

$$v = V_m \sin\omega t = V\angle 0^\circ$$
$$i = I_m \sin(\omega t - \frac{\pi}{2}) = I\angle -90^\circ$$
$$z = \frac{V\angle 0^\circ}{I\angle -90^\circ} = X_L\angle +90^\circ = (0 - jX_L)\ \Omega$$

+ ve sign shows that P.F is lagging.

X_L= Inductive reactance (X_L) **is** the opposition offered by the inductance of the ckt to the flow of alternating sinusoidal current.

POWER

p = $v.i$

p = $V_m \sin\omega t . I_m \sin(\omega t - \frac{\pi}{2})$

p $= -\frac{V_m I_m}{2} * 2(\sin\omega t . \cos\omega t)$

p $= -\frac{V_m I_m}{2} \sin 2\omega t *$

$P_{avg} = -\frac{1}{\pi}\int_0^{\pi} \frac{V_m I_m}{2} * \sin 2\omega t\ (d\omega t) = 0$

{over a complete cycle} of power.

$P_{avg} = 0$ →average power is a purely Inductive ckt is zero.

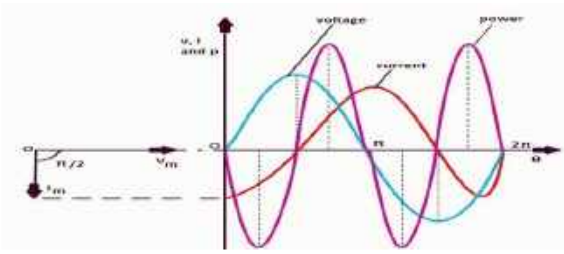


Fig 2.17 Waveform Of Power

.**<u>Previous Years Questions</u> (2 marks)**

Question 1 Derive that average power consumed by a pure inductor is zero .(AKTU 2022-23,2020-2021)

Question 2 What is the real power consumed by a pure inductor? Discuss with suitable diagram.
(AKTU 2022-23,2020-2021)

2.10. PURELY CAPACITIVE CIRCUIT (C ONLY):

Let a pure capacitance (C) is connected to an A.C sinusoidal voltage $v = V_m \sin\omega t$ instaneous charge “q” on plate of the capacitor is

q = CV

current $i = \frac{dq}{dt} = \frac{d(CV)}{dt}$

i$= \frac{Cd(V)}{dt} = C\frac{d(V_m \sin\omega t)}{dt}$

I $= C\omega \frac{\pi}{2}\cos\omega t$

I $= \frac{V_m}{\frac{1}{\omega C}}\sin(\omega t + \frac{\pi}{2})$

i$= I_m \sin(\omega t + \frac{\pi}{2})$

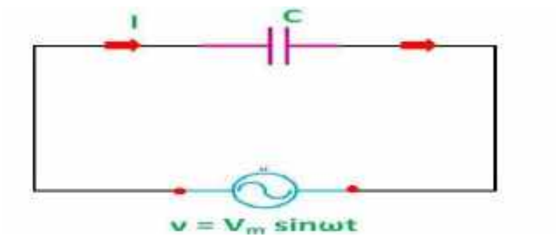


Fig 2.19. Purely Capacitive Circuit

i leads v by $\frac{\pi}{2}$ or 90°

i=$\frac{V_m}{z}$=$\frac{V_m}{\frac{1}{\omega C}}$ = $\frac{V_m}{X_c}$

1. Z=X_c=$\frac{1}{2\pi f C}$ (Ω)

X_c is capacitive reactance.

2. $\phi = 90^\circ$, i leads v by $\frac{\pi}{2}$ or 90°
3. $\cos\phi = \cos 90^\circ$= 0
4. $power\ (P) = 0$ as $\cos\phi = 0$

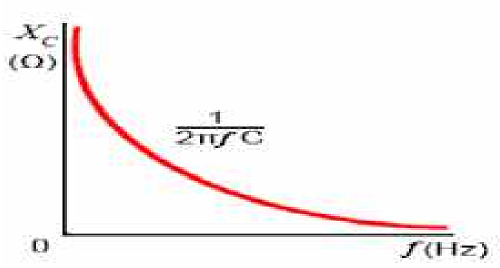


Fig 2.20. .Impedance

IMPEDANCE (Z) :

v = $V_m \sin\omega t$ = V∠0°

i=$I_m \sin(\omega t+\frac{\pi}{2})$= $I\angle 90^\circ$

z = $\frac{V\angle 0^\circ}{I\angle 90^\circ}$ = $X_c\angle -90^\circ$= (0 - jX_c)Ω

-ve leading P.F

Capacitive reactance (**X_c**) **is** the opposition offered by the capacitance of the ckt to the flow of alternating sinusoidal current.

POWER

$$p = v.i$$
$$p = V_m \sin\omega t . I_m \sin(\omega t+\frac{\pi}{2})$$
$$p = \frac{V_m I_m}{2} * 2(\sin\omega t . \cos\omega t)$$
$$p = \frac{V_m I_m}{2} * \sin 2\omega t$$
$$P_{avg} = \frac{1}{\pi}\int_0^{\pi} \frac{V_m I_m}{2} * \sin 2\omega t\,(d\omega t) = 0$$

{over a complete cycle} of power.

$P_{avg} = 0$ →average power is a purely capacitive ckt is zero

∵ $X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$s

∴ $X_c \propto \frac{1}{f}$

∴ graph is rectangular hyperbola.

Short Questions

Question 1 Derive that average power consumed by a pure capacitor is zero.

2.11. SERIES R-L CIRCUIT: -

A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) is known as **RL Series Circuit**. When an AC supply voltage V is applied, the current, I flows in the circuit.

So, I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other. The circuit diagram of RL Series Circuit is shown below:

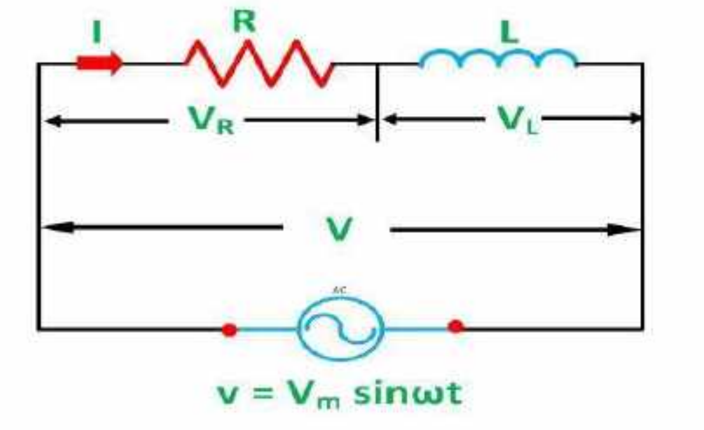


Fig.2.21 Series R-L Circuit

Where,
V_R – voltage across the resistor R
V_L – voltage across the inductor L
V – Total voltage of the circuit

$$\vec{V} = \vec{V_R} + \vec{V_L}$$

$V_R \rightarrow$ It is in phase with " i " for R
$V_L \rightarrow$ It leads " i " by 90^0 for L

2.12.1 Phasor Diagram of the RL Series Circuit:-
The phasor diagram of the RL Series circuit is shown below:
Steps to draw the Phasor Diagram of RL Series Circuit:-
The following steps are given below which are followed to draw the phasor diagram step by step:
1.Current I is taken as a reference.
2.The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.
3.The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
4.The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

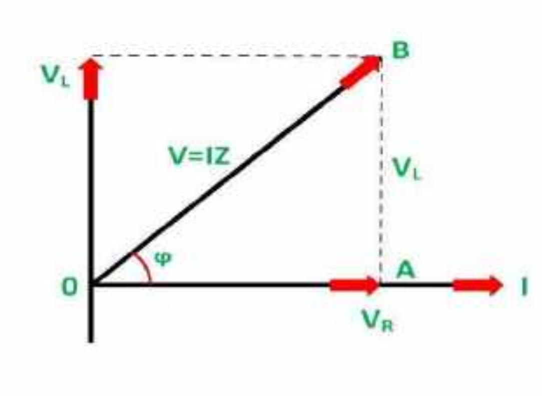


Fig.2.22 Phasor Diagram

Now,
In right-angle triangle OAB
$V_R = I_R$ and $V_L = IX_L$ where $X_L = 2\pi fL$
I lags v by Φ

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t - \phi)$$

From the phasor diagram we get

In right-angle triangle OAB

$$V^2 = V_R^2 + V_L^2$$

$$(iz)^2 = (iR)^2 + (iX_L)^2$$

$$Z = \sqrt{R^2 + X_L^2}$$

$V=V_R=V_L$

$$\tan\varphi = \frac{V_L}{V_R} = \frac{iX_L}{iR}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$\sin\varphi = \frac{V_L}{V} = \frac{IX_L}{IZ} = \frac{X_L}{Z}$$

$$\sin\varphi = \frac{X_L}{Z}$$

$$X_L = Z\sin\varphi$$

Impedance:-

$$v = V\angle 0^o$$

$$i = I\angle -\varphi^o$$

$$Z = \frac{V\angle 0^o}{I\angle -\varphi^o} = |z|\angle +\varphi^o$$

Cos φ is lagging and $0^o < \varphi < 90^o$

$$Z = \sqrt{R^2 + X_L^2}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

2.12.2 Power in RL Series Circuit

Alternating Voltage across the circuit is given as

$$v = V_m \sin\omega t$$

Current can be given as

$$i = I_m \sin(\omega t - \phi)$$

so ,the instantaneous power is given by the Equation

$$p = v\,i$$

now Substituting the value of v and i from the above equation,

$$p = (V_m \sin\omega t) * I_m \sin(\omega t - \phi)$$

$$p = \frac{V_m I_m}{2} * 2\sin(\omega t - \phi)\sin\omega t$$

$$p = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t - \varphi)]$$

$$p = \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)]$$

so the average power consumed in one cycle in the circuit can be given by

$$p = \text{average of } \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} * \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)]$$

so p can be calculated as

$$p = V_{(rms)} * I_{(rms)} \cos\phi = V\,I\cos\phi$$

where cosϕ is the power factor
cosϕ can be calculated as

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Now substituting this Value we will get

$$P = IZ * I * R/Z = I^2 * R$$

Power Consume in the Circuit is $I^2 * R$

2.12.3 Waveform and Power Curve of the RL Series Circuit:-

The various points on the power curve are obtained by the product of voltage and current. If you analyze the curve carefully, it is seen that the power is negative between angle 0 and ϕ and between 180 degrees and (180 + ϕ) and during the rest of the cycle the power is positive. The current lags the voltage and thus they are not in phase with each other.

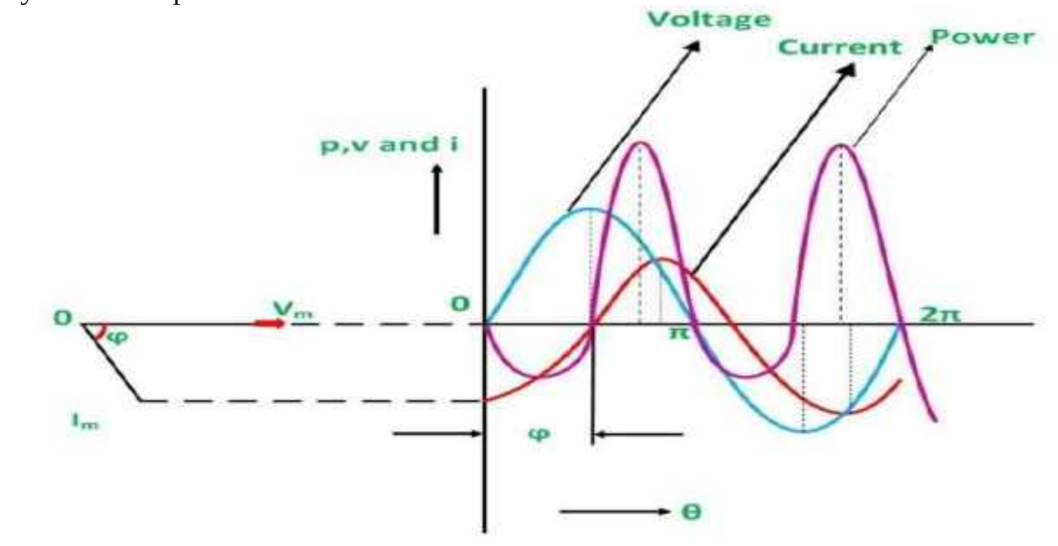


Fig.2.23 Waveform & power curve

Long Questions

Q 1. A resistance and inductance are connected in series across a voltage $v = 283\sin 314t$. An expression of current is found to be $i = 4\sin(314t - 45^0)$. Find the value of resistance, inductance and power factor. (AKTU 2013-2014)

Solution: The value of R.M.S voltage,

$$V_{r.m.s} = \frac{V_m}{\sqrt{2}} = \frac{283}{\sqrt{2}} = 200.11\ V$$

Angular frequency in rad/sec,

$$\omega = 314\ \ rad/sec.$$

The value of R.M.S current,

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828\ A$$

Phase difference voltage and current i.e Φ. Current lags by 45^0 with reference of voltage.

$$\varphi = 45^0$$

Impedance of the circuit would be,

$$Z = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{200.11}{2.828} = 70.75\ \Omega$$

Resistance of the circuit,

$$R = Z\cos\varphi = 70.75 \times \cos 45 = 50\ \Omega$$

Inductive reactance of the circuit,

$$X_L = Z\sin\varphi = 70.75 \times \sin 45^0 = 50\ \Omega$$

Inductance of the circuit,

$$L = \frac{X_L}{\omega} = \frac{50}{314} = 0.159\ H$$

Power factor would be,

$$p.f = \cos\emptyset = \cos 45 = 0.707\ (Laggging)$$

2.12. SERIES R-C CIRCUIT: -

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit.

The RC Series circuit is shown in the figure below:

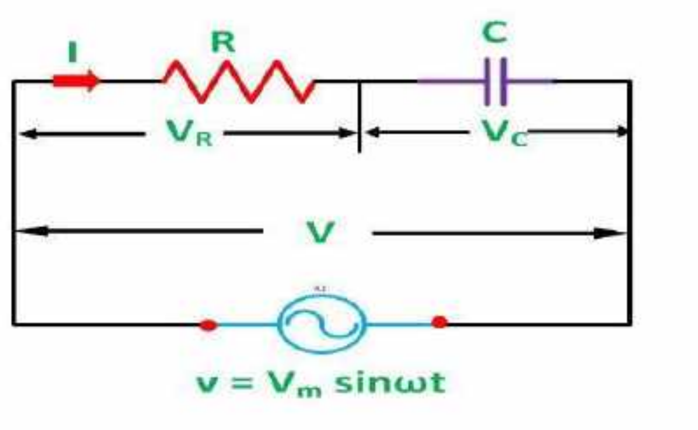


Fig 2.24 Series R-C Circuit

Where,

V_R – voltage across the resistance R
V_C – voltage across capacitor C
V – total voltage across the RC Series circuit

$$\vec{V} = \vec{V_R} + \vec{V_C}$$

$V_R \rightarrow$ It is in phase with " i " for R

$V_C \rightarrow$ It lags " i " by 90^0 for C

V_R= iR [voltage drop across R]
V_C= iX_C [voltage drop across C]

2.13.1 Phasor Diagram of the RC Series Circuit:-

The phasor diagram of the RC Series circuit is shown below:

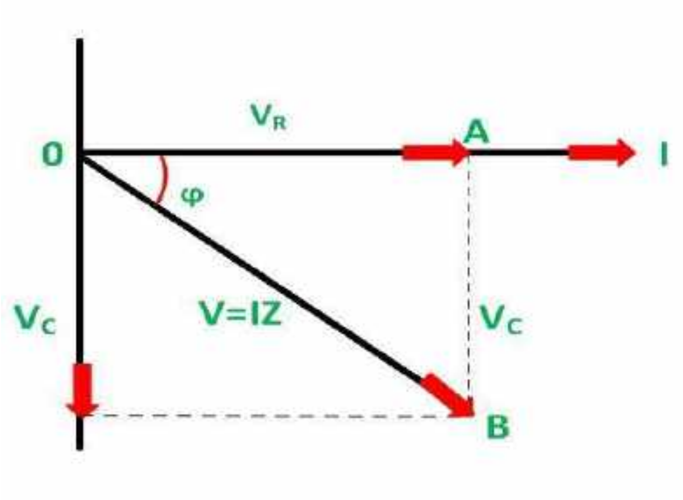


Fig 2..25 Phasor Diagram Series R-C Circuit

Steps to draw the Phasor Diagram of RC Series Circuit:-
The following steps are used to draw the phasor diagram of RC Series circuit
1. Take the current I (r.m.s value) as a reference vector
2. Voltage drop in resistance V_R = IR is taken in phase with the current vector
3. Voltage drop in capacitive reactance VC = IX_C is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
4. The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).
Now,
5. $V_R = IR$
6. $V_C = IX_C$
7. Where $X_C = I/2\pi fC$

In right-angle triangle OAB
$V_R = I.R$ and $V_C = IX_C$ where $X_C = 1/2\pi fC$
I leads v by 90^o

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t+\phi)$$

From the phasor diagram we get
In right-angle triangle OAB

$$V^2 = V_R^2 + V_C^2$$

$$(iz)^2 = (iR)^2 + (iX_C)^2$$
$$Z = \sqrt{R^2 + X_C^2}$$ unit is ohm(Ω)

$V=V_R=V_C$

$$\tan\varphi = \frac{-V_C}{V_R} = \frac{-iX_C}{iR}$$

$$\varphi = \tan^{-1}\frac{-X_C}{R}$$

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

$$\sin\varphi = \frac{V_C}{V} = \frac{IX_C}{IZ} = \frac{X_C}{Z}$$

$$\sin\varphi = \frac{X_C}{Z}$$
$$X_C = Z\sin\varphi$$

Impedance:-

$$v = V\angle 0^o$$

$$i = I\angle +\varphi^o$$

$$Z = \frac{V\angle 0^o}{I\angle\varphi^o} = |z|\angle -\varphi^o$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

2.13.2 Power in RC Series Circuit: -

Alternating Voltage across the circuit is given as

$$v = V_m \sin\omega t$$

Current can be given as

$$i = I_m \sin(\omega t+\phi)$$

so ,the instantaneous power is given by the Equation

$$p = v\, i$$

now Substituting the value of v and I from the above equation,

$$p = (V_m\sin\omega t)*I_m\sin(\omega t+\phi)$$

$$p = \frac{V_m I_m}{2}*2\sin(\omega t+\phi)\sin\omega t$$

$$p = \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}[\cos\varphi - \cos(2\omega t+\varphi)]$$

$$p = \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}\cos(2\omega t+\varphi)]$$

So, the average power consumed in one cycle in the circuit can be given by:-

$$P = \text{average of } \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}\cos(2\omega t+\varphi)]$$

so P can be calculated as

$$P = \frac{V_m}{\sqrt{2}}*\frac{I_m}{\sqrt{2}}\cos\varphi - 0$$

$$P = V_{(rms)}*I_{(rms)}\cos\phi = V.I.\cos\phi$$

$$P = V.I.\cos\phi$$

where $\cos\phi$ is the power factor

$\cos\phi$ can be calculated as

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$

Now substituting this Value we will get

$$P = IZ * I * R/Z = I^2 * R$$

From the above equation it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.

2.13.3 Waveform and Power Curve of the RC Series Circuit: -

The waveform and power curve of the RC series circuit is shown below:

The various points on the power curve are obtained from the product of the instantaneous value of voltage and current. The power is negative between the angle (180° – ϕ) and 180° and between (360° - ϕ) and 360° and in the rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is **positive**.

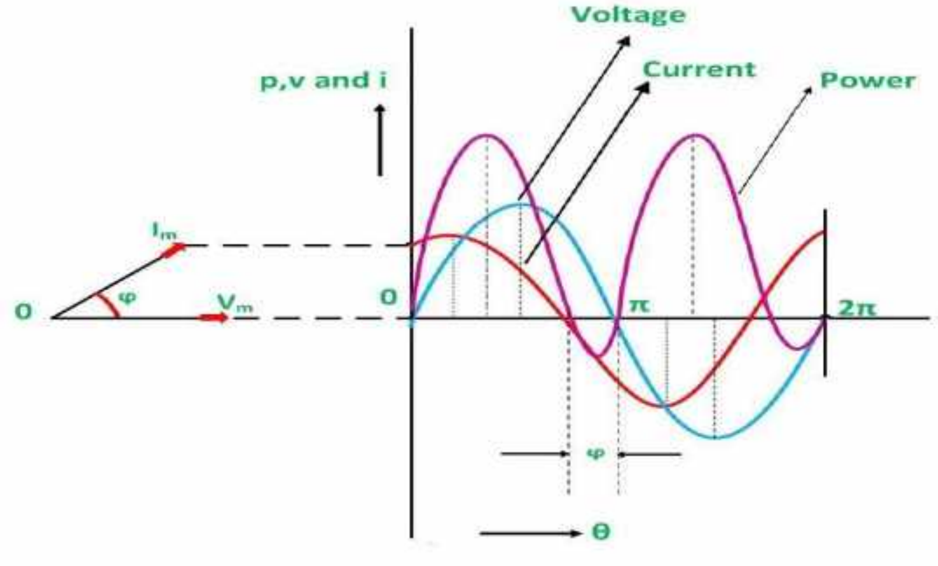


Fig 2.26 Power Waveform of Series R-C Circuit

Long Questions

Q1. A 46 mH inductive coil has a resistance of 10 ohm. How much current will it draw, if connected across 100 V, 50 Hz source? Also determine the value of capacitance that must be connected across the coil to make the power factor of the circuit to be unity. (AKTU 2016-2017)

Solution: Inductive reactance of the coil,

$$X_L = \omega L = 2\pi f.L = 2\pi \times 50 \times 46 \times 10^{-3} = 14.45\ \Omega$$

Now the impedance of the inductive coil,

$$Z_L = R + jX_L = 10 + j14.45 = 17.57 \angle 55.32^0 \Omega$$

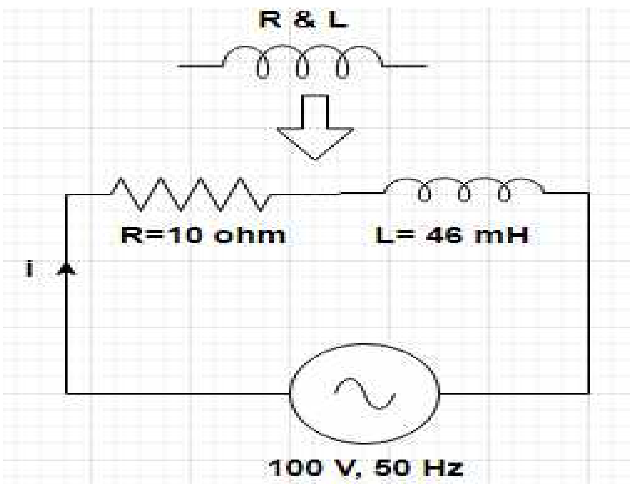


Fig.2.27

Current drawn by the source,

$$i = \frac{V}{Z_L} = \frac{100}{17.57} = 5.69\ A$$

Phase angle,

$$\emptyset = tan^{-1}\left(\frac{X_L}{R}\right) = tan^{-1}\left(\frac{14.45}{10}\right) = 55.52^0$$
$$i = 5.69\ \llcorner - 55.52\ A$$

Power factor can be improved by connecting a capacitor across the coil.

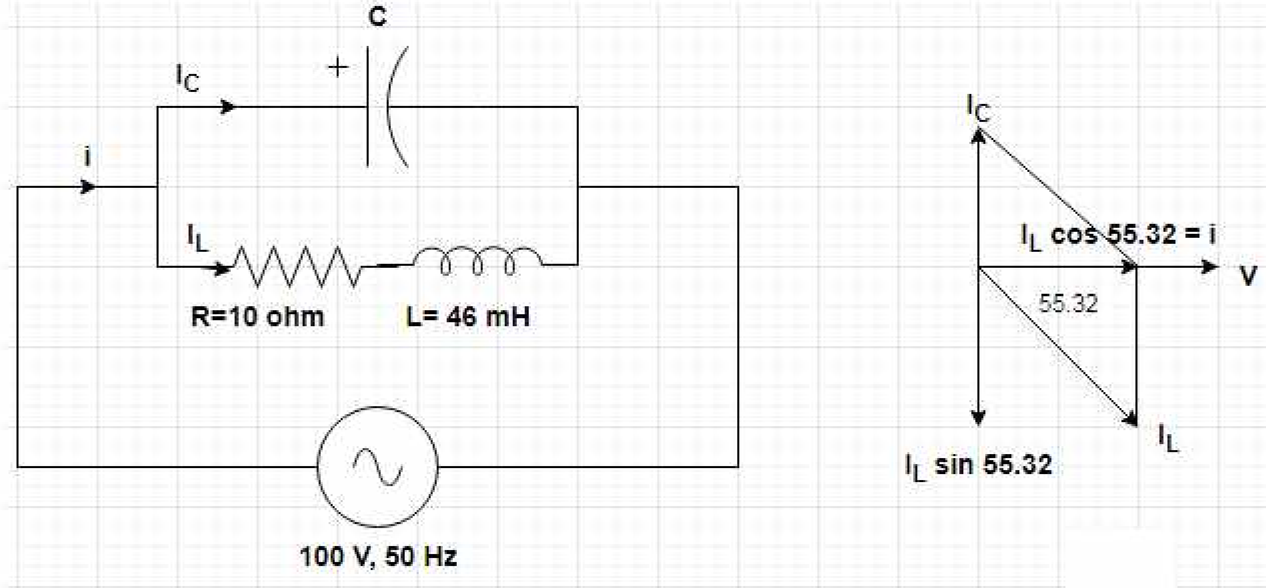


Fig. 2.28

$$I_C = I_L \sin(55.32)$$
$$\frac{V}{X_C} = \frac{V}{Z_L}.\sin(55.32)$$
$$V.2\pi f.C = 5.69 \times 0.822 = 4.679$$
$$C = \frac{4.679}{2\pi \times 50 \times 100} = 148.94\ \mu F$$

Q2. A resistance and inductance are connected in series across a voltage $v = 283 \sin 314t$. An expression of current is found to be $i = 4\sin(314t - 45^0)$. Find the value of resistance, inductance and power factor. (AKTU 2013-2014)

Solution: The value of R.M.S voltage,

$$V_{r.m.s} = \frac{V_m}{\sqrt{2}} = \frac{283}{\sqrt{2}} = 200.11\ V$$

Angular frequency in rad/sec,

$$\omega = 314\ \ rad/sec.$$

The value of R.M.S current,

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2.828\ A$$

Phase difference voltage and current i.e Φ. Current lags by 45^0 with reference of voltage.

$$\varphi = 45^0$$

Impedance of the circuit would be,

$$Z = \frac{V_{r.m.s}}{I_{r.m.s}} = \frac{200.11}{2.828} = 70.75\ \Omega$$

Resistance of the circuit,

$$R = Z\cos\varphi = 70.75 \times \cos 45 = 50\ \Omega$$

Inductive reactance of the circuit,

$$X_L = Z\sin\varphi = 70.75 \times \sin 45^0 = 50\ \Omega$$

Inductance of the circuit,

$$L = \frac{X_L}{\omega} = \frac{50}{314} = 0.159\ H$$

Power factor would be,

$$p.f = \cos \emptyset = \cos 45 = 0.707\ (Laggging)$$

2.13. SERIES R-L-C CIRCUIT:-

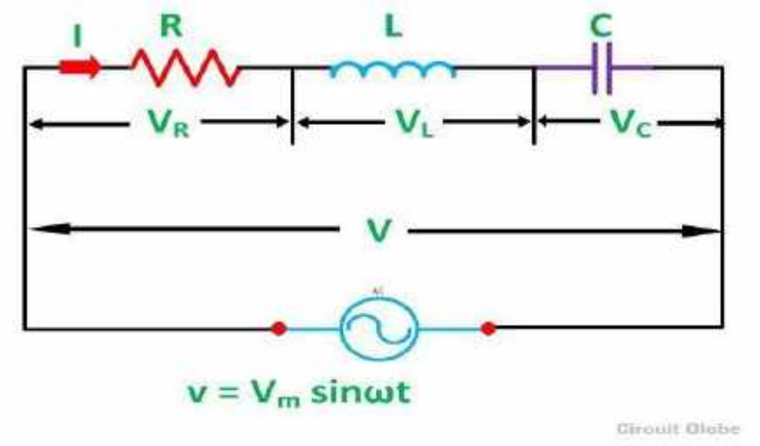


Fig 2.29 Series RLC circuit

Let V and I be the rms values of the applied voltage and current. Potential difference across the resistor = V_R = IR
Potential difference across the inductor = $V_L = I\ X_L$
Potential difference across the capacitor = $V_C = I\ X_C$
The voltage R is in phase with the current I the voltage V leads the current I by 90° and the voltage V_C lags behind the current I by 90°.

$$V = V_R + jV_L - jV_C$$

2.14.1 Phasor Diagram: Since the same current flows through R, L and C, the current I is taken as a reference phasor.
Case (i) $X_L > X_C$

$$Z=R+ j(X_L - X_C)$$

As $X_L > X_C$

$$Z=R+ jX_L$$
$$Z=\sqrt{R^2 + XL^2}$$

Now circuit acts an inductive circuit.
Here, current lags voltage by 90^0.

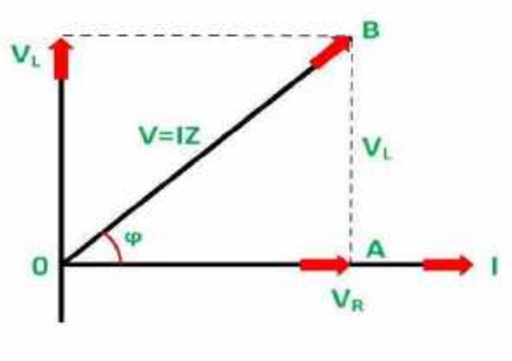


Fig 2.30 Phasor Diagram of Leading P.F

The reactance X will be inductive in nature and the circuit will behave like a series RL circuit.

Case(ii) $X_C > X_L$

$$Z = R + j(X_L - X_C)$$

As $X_L < X_C$

$$Z = R - jX_C$$

$$Z = \sqrt{R^2 + XC^2}$$

Now circuit acts as a capacitive circuit.
Here, the current leads voltage by 90^0.

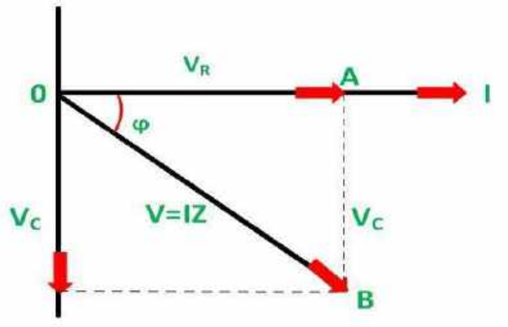


Fig 2.31 Phasor Diagram of Lagging P.F

The reactance X will be capacitive in nature and the circuit will behave like a series RC circuit.

2.14.2 Impedance

$V = V_R + V_L + V_C = RI + jX_L I - jX_C I$
$V/I = R + j(X_L - X_C) = Z$
$Z = Z\angle\phi$

$$Z = \sqrt{R^2 + (XL - XC)^2}$$

2.14.3 Current Equation If the applied voltage is given by $v = V_m \sin \omega t$ then current equation will be

$$i = I_m \sin(\omega t - \phi)$$

'−' sign is used when $X_L > X_C$.
'+' sign is used when $X_C > X_L$.

2.14.4 Power

Average Power	$P = VI \cos \phi = I^2R$
Reactive Power	$Q = VI \sin \phi = I^2X$
Apparent Power	$S = VI = I^2Z$

2.15. AC POWER

In AC circuits, however, the situation is more complicated. There are three categories of abilities here:

1. Active power
2. Reactive power
3. Apparent Power

2.15.1 Active Power

Active or genuine or real power refers to the quantity of power that is dissipated or that does productive work in the circuit.In power systems, it is calculated in watts but is more commonly expressed in (kilowatts) and (megawatts). It is equivalent to an average amount of $P = VI \cos \phi$ and is indicated by P (capital).The circuit or load is driven by the desired output of an electrical system.

$$P = VI \cos \phi$$

2.15.2 Reactive Power

Reactive power is the power that comes back and forth between the source and the load. Reactive power, indicated by the letter Q, is the component that is proportionate to VIsinϕ. This is a power, but it is not calculated in watts because it is a non-active power, so it is calculated in Volt-Amperes-Reactive (VAR). The load power factor determines whether this reactive power is negative or positive. This is due to the fact that inductive loads absorb reactive power, whereas capacitive loads create it.

$$Q=VI \sin \phi$$

2.15.3 Apparent Power

The term apparent power refers to the complicated combination of true power and reactive power. The perceived power is equal to the multiplication of voltage and current, regardless of phase angle.

The perceived power is helpful in determining the rating of power equipment. It can alternatively be written as the square of current times the impedance of the circuit.It is indicated by the symbol S and is calculated in volt-amperes, with (kilovolt-amperes) and (megavolt-amperes) as practical units.

$$S=P+jQ$$

Or we can write as, $S = I^2Z$

2.15.4 Power Triangle

A power triangle is the relationship between active power, reactive power and apparent power that can be explained by describing numbers like the vector in the geometrical form.

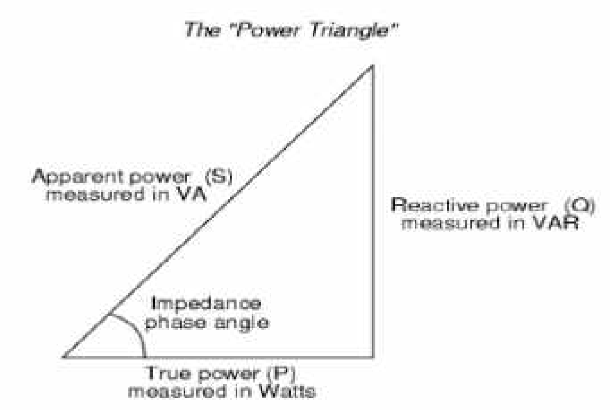


Fig.2.32 Power Traingle

The total of the squares of two sides (active power and reactive power) equals the squares of the diagonal, according to the Pythagoras theorem (apparent power). i.e.,

$$S^2=P^2+Q^2$$

That is,

$$S = \sqrt{(P^2+Q^2)}$$

<u>Short Questions</u>

Q1. What do you mean by apparent power, active power and reactive power?
(AKTU 2015-2016)

<u>Long Questions</u>

Q2. A series circuit consists of a resistance of 10 Ω and inductance of 50mH and a variable capacitance across a 100 V, 50 Hz supply, calculate:- (AKTU2018-2019)
(i)The value of capacitance to produce resonance.
(ii) Voltage across the capacitance.
(iii) Q-factor

Solution: The circuit diagram is shown in fig.

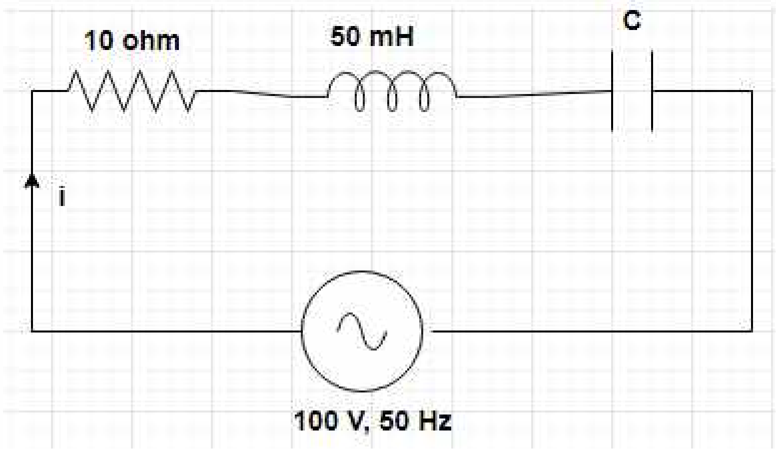


Fig. 2.30

(i) At resonance, source frequency is equal to the resonance frequency. Hence

$$f_s = f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

So, the value of capacitance would be,

$$C = \frac{1}{4\pi^2 f_s^2 L}$$

$$C = \frac{1}{4\pi^2 \times 50^2 \times 50 \times 10^{-3}}$$

$$C = 0.00020264\ F$$

$$C = 202.64\ \mu F$$

(ii) Capacitive reactance i.e X_C

$$X_C = \frac{1}{2 \times \pi \times f \times C} = \frac{1}{2 \times 3.14 \times 50 \times 0.00020264} = 15.72\ ohms$$

$$i = \frac{V}{Z} = \frac{V}{R} = \frac{100}{10} = 10\ A$$

Now the voltage across the capacitance,

$$V_C = i \times X_C = 10 \times 15.72 = 157.2\ V$$

(iii) Q-factor of the series circuit at resonance is,

$$Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{50 \times 10^{-3}}{202.64 \times 10^{-6}}}$$

$$Q - factor = \frac{1}{10}\sqrt{\frac{50 \times 10^{-3}}{202.64 \times 10^{-6}}}$$

$$Q - factor = 1.57$$

Q3. If load draws a current of 10 A at 0.8 p.f lagging, when connected to 100 V supply. Calculate the value of real, reactive and apparent power and also find the resistance of the load.

(AKTU 2020-2021)

Solution

$$Z = \frac{V}{i} = \frac{100}{10} = 10\ \Omega$$

Power factor is given

$$p.f = \cos\varphi$$

$$\varphi = \cos^{-1}(0.8) = 36.86^0$$

Real power,

$$P = V_{r.m.s} \times i_{r.m.s} \times \cos\varphi$$
$$P = 100 \times 10 \times 0.8 = 800\ W$$

Reactive power can be calculate by the following expression,

$$Q = V_{r.m.s} \times i_{r.m.s} \times \sin\varphi$$
$$Q = 100 \times 10 \times 0.6 = 600\ VAR$$

Apparent power can be calculate by the following expression,

$$S = V_{r.m.s} \times i_{rm.s}$$
$$S = 100 \times 10 = 1000\ VA$$

Value of resistance of the load would,

$$R = Z \times \cos\varphi$$
$$R = 10 \times 0.8 = 8\ \Omega$$

Q4. If load draws a current of 10 A at 0.8 p.f lagging, when connected to 100 V supply. Calculate the value of real, reactive and apparent power and also find the resistance of the load.

(AKTU 2020-2021)

Solution

$$Z = \frac{V}{i} = \frac{100}{10} = 10\ \Omega$$

Power factor is given

$$p.f = \cos\varphi$$
$$\varphi = \cos^{-1}(0.8) = 36.86^0$$

Real power,

$$P = V_{r.m.s} \times i_{r.m.s} \times \cos\varphi$$
$$P = 100 \times 10 \times 0.8 = 800\ W$$

Reactive power can be calculate by the following expression,

$$Q = V_{r.m.s} \times i_{r.m.s} \times \sin\varphi$$
$$Q = 100 \times 10 \times 0.6 = 600\ VAR$$

Apparent power can be calculate by the following expression,

$$S = V_{r.m.s} \times i_{rm.s}$$
$$S = 100 \times 10 = 1000\ VA$$

The value of resistance of the load would,

$$R = Z \times \cos\varphi$$
$$R = 10 \times 0.8 = 8\ \Omega$$

2.16. RESONANCE IN SERIES RLC CIRCUIT

Resonance is the condition in a series RLC circuit that -When frequency of supply is varied then at a particular frequency called resonant frequency f_r, the current is maximum & net reactance of the circuit is zero. Circuit behaves as a purely resistive circuit & power factor is unity.

At resonant frequency f_r –

$V_L=V_C$

$X_L-X_C=0$

$X_L=X_C$

$IX_L=IX_C$

$V_L=V_C$

Z=R

$V=V_R$

Current is maximum $I_{MAX}=\frac{V}{R}$

Voltage & current are in same phase.

Φ=VˆI=0

Cosϕ=1 ;unity

2.16.1 RESONANT FREQUENCY

At the resonant frequency, $\mathbf{f_r}$ the inductive reactance (X_L) and capacitive reactance (X_C) are equal, resulting in the circuit's impedance being purely resistive (Z = R).
The current in the circuit reaches its maximum value at $\mathbf{f_r}$ because the impedance is at its minimum.
At f_r-

$$X_L - X_C = 0$$
$$X_L = X_C$$
$$\omega_r L = 1/\omega_r C$$
$$\omega_r = 1/\sqrt{LC}$$

$\omega_r = 2\pi f_r$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Summary: For a series RLC circuit at certain frequency called resonant frequency, the following points must be remembered. So at resonance:

1. Inductive reactance X_L is equal to capacitive reactance X_C.
2. Total impedance of circuit becomes minimum which is equal to Ri.e Z = R.
3. Circuit current becomes maximum as impedance reduces, I = V / R.
4. Voltage across inductor and capacitor cancels each other, so voltage across resistor V_r = V, supply voltage.
5. Since net reactance is zero, circuit becomes purely resistive circuit and hence the voltage and the current are in same phase, so the phase angle between them is zero.
6. Power factor is unity.
7. Frequency at which resonance in series RLC circuit occurs is given.

2.16.2 RESONANCE CURVE:

The resonance curve for a series RLC circuit, also known as a **frequency response curve** illustrates how the circuit's current amplitude varies with the frequency of the applied AC voltage. This curve helps visualize the behavior of the circuit as it approaches and reaches resonance.

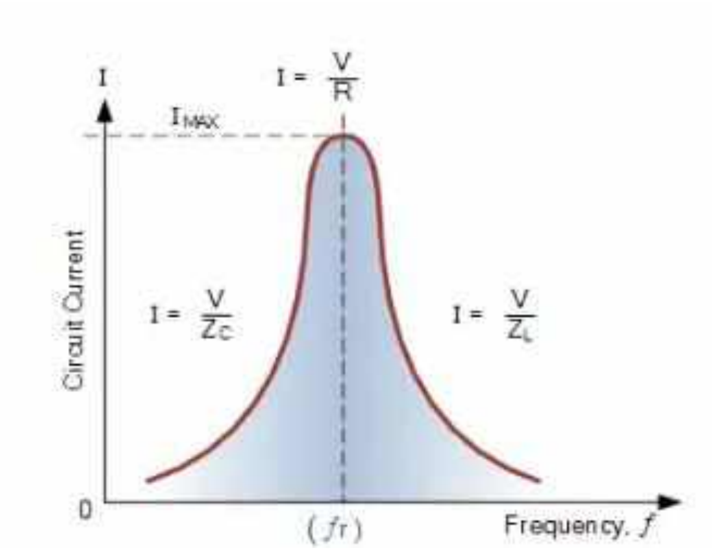


Fig 2.31 Resonance curve

Key Features of the Resonance Curve:

1. **Current Amplitude**:

The vertical axis of the resonance curve represents the current amplitude in the circuit.The current amplitude is highest at the resonant frequency because the circuit's impedance is minimized.

2. **Shape of the Curve**:

The resonance curve typically has a **bell-shaped** or **peak** form, centered around the resonant frequency $\mathbf{f_r}$ .The peak of the curve represents the maximum current at resonance. On either side of the peak, the current decreases as the frequency deviates from $\mathbf{f_r}$.

3. **Bandwidth (BW)**:

The bandwidth of the resonance curve is the range of frequencies over which the circuit's current remains relatively high. It is defined by the frequencies at which the current falls to $\frac{1}{\sqrt{2}}$ (about 0.707) of its maximum value. Mathematically, the bandwidth is given by:

$$BW = \frac{R}{2\pi L}$$

A narrow bandwidth indicates a sharp resonance with high selectivity, while a wide bandwidth indicates a less selective resonance.

4. **Quality Factor (Q-Factor)**:

The Q-factor determines the sharpness of the resonance curve. A higher Q-factor corresponds to a sharper peak and narrower bandwidth, indicating that the circuit is more selective to a specific frequency. Q.F is defined as the voltage magnification in a series resonance (voltage resonance) circuit.

The Q-factor is given by:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega r}{\Delta f}$$

$$Q = \frac{\omega r}{\Delta \omega}$$

$$Q.F \ \alpha \ \frac{1}{Bandwidth}$$

If Δf is narrow then Q factor & selectivity will be high
If Δf is broad then Q factor & selectivity will be low.
High Q circuits are used in applications where precise frequency selection is critical, such as in filters and oscillators.
For large values of R, curve becomes more flatter.

Short Questions

Q1. A series circuit ha R = 10 Ω, L = 0.02 H, C = 3 µF. Calculate Q-factor of the circuit.
(AKTU 2018-2019)

Solution: Q-factor of the series circuit is,

$$Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q - factor = \frac{1}{10}\sqrt{\frac{0.02}{3 \times 10^{-6}}}$$

$$Q - factor = 8.165$$

Q2. In a series RLC circuit, $R = 2\ \Omega$, $L = 2\ mH, C = 10\ \mu F$. Find the resonant frequency and Q-factor. (AKTU 2023-2024)

Solution: f_r would be

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

Substitute the value of R,L,C in resonant frequency expression,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{2\times10^{-3}\times10\times10^{-6}}}$$

$$f_r = 1125.4\ Hz$$

Quality factor of the circuit at resonanse would be,

$$Q - factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q - factor = \frac{1}{2}\sqrt{\frac{2\times10^{-3}}{10\times10^{-6}}}$$

$$Q - factor = 7.07$$

2.16.3 BANDWIDTH OF SERIES RLC CIRCUIT

Bandwidth is defined as the band of frequencies that lie on either side of resonant frequency where current falls $\frac{1}{\sqrt{2}}$ tunes of its Max. value.

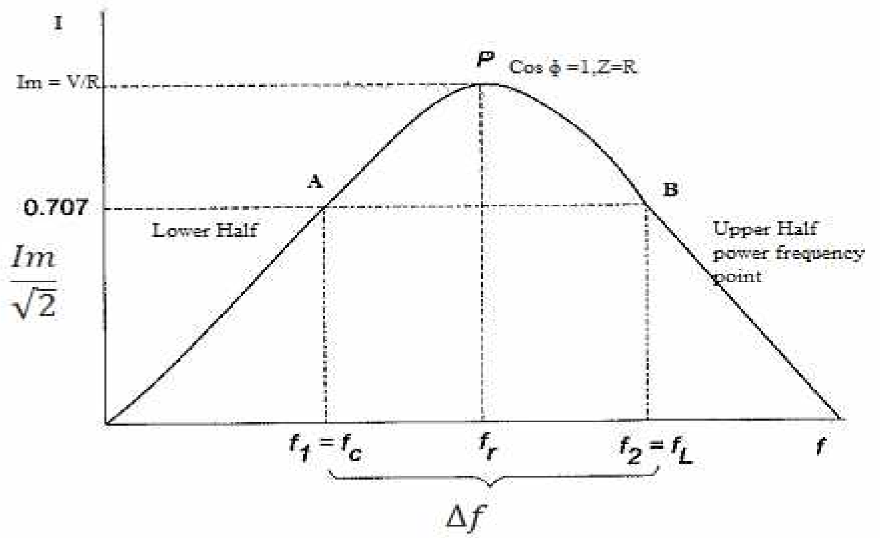


Fig 2.32 Bandwidth curve

At Half power point,

$$\text{Power} = P_A = P_B = \left(\frac{I_m}{\sqrt{2}}\right)^2 .R$$

$$\text{Power} = P_A = P_B = \frac{I_m^2}{\sqrt{2}}.R = \frac{P}{2}$$

Proof :- Impedance of R-L-C circuit is given by :-

$$Z = \sqrt{(R)^2 + (X_L - X_C)^2}\text{unit is ohm}(\Omega)$$

Current ,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R)^2 + (X_L - X_C)^2}}$$

At Half power points,

$$I = \frac{I_m}{\sqrt{2}} = \frac{V}{\sqrt{(R)^2 + (X_L - X_C)^2}}$$

As

$$I_m = \frac{V}{R} \text{ so, } = \frac{V}{\sqrt{2}.R} = \frac{V}{\sqrt{(R)^2 + (X_L - X_C)^2}}$$

Squaring both sides we get,

$$2R^2 = R^2 + (X_L - X_C)^2$$
$$R^2 = (X_L - X_C)^2$$
$$R = \pm (X_L - X_C)$$

Case 1:- When $R = +(X_L - X_C)$ (Ω)

$$R = (\omega L - \frac{1}{\omega C})$$

$$R = \frac{\omega^2 LC - 1}{\omega C}$$

$$\omega^2 LC - \omega CR - 1 = 0$$

$Ax^2 + BX + C = 0$
By shreedhar-acharya Formula,

$$\omega = \frac{+CR \pm \sqrt{c^2R^2 + 4LC}}{2LC}$$

$$\omega = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

$$\frac{1}{L} \ggg \frac{R^2}{4L^2} \text{ is neglected}$$

$$\therefore\ \omega = \frac{R}{2L} \pm \omega_r$$

$$\therefore\ \omega_r = \frac{1}{\sqrt{LC}} \quad \text{rad/sec}$$

Case 2:- When $R = -(X_L - X_C)$ (Ω)

Similarly as of case 1,

$$\omega = -\frac{R}{2L} \pm \omega_r$$

so there are 4 roots of ω,

$$\omega = \frac{R}{2L} + \omega_r\ ,\quad \omega = \frac{R}{2L} - \omega_r$$

$$\omega = -\frac{R}{2L} + \omega_r \ , \quad \omega = -\frac{R}{2L} - \omega_r$$

as ω_r cannot be negative

so, there are two roots of $'\omega'$.

$$\omega = \omega_r + \frac{R}{2L}$$

$$\omega = \omega_r - \frac{R}{2L}$$

Let, ω_1 = Lower half power frequency

ω_2 = Upper half power frequency

$$\omega_1 = \omega_r - \frac{R}{2L} , \omega_2 = \omega_r + \frac{R}{2L} \quad \text{rad/sec------------------------(1)}$$

$$f_1 = f_r - \frac{R}{4\pi L} , f_2 = f_r + \frac{R}{4\pi L} \quad \text{rad/sec------------------------(2)}$$

$$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\Delta f = f_2 - f_1 = \frac{R}{2\pi L} \text{ rad/sec} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Put eq 3 in eq 1 & eq 4 in eq 2

$$\omega_1 = \omega_r - \Delta\omega/2, \quad \omega_2 = \omega_r + \Delta\omega/2 \ldots\ldots\ldots\ldots(5)$$

$$f_1 = f_r - \frac{\Delta f}{2} \quad , f_2 = f_r + \frac{\Delta f}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Prove that

$$f_r = \sqrt{f1f2}$$

We have $X_L - X_C = \pm R$……………..(from eq A)

For upper half power frequency- $X_L - X_C = +R$

$$R = (\omega_2 L - \frac{1}{\omega 2\, C}) \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

For lower half power frequency-$X_L - X_C = -R$

$$-R = (\omega_1 L - \frac{1}{\omega 1\, C}) \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Adding eq 7& 8

$$(\omega_1 + \omega_2)L = \frac{1}{C}\left(\frac{\omega 1 + \omega 2}{\omega 1 \omega 2}\right)$$

$$\omega_1\ \omega_2 = \frac{1}{LC}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \quad (\text{Where } \omega_1\ \omega_2 = \omega_r)$$

$$f_1\ f_2 = f_r^2$$

This shows that resonant frequency is the geometric mean of two half-power frequencies.

Long Questions

Q1. The bandwidth of a series resonant circuit is 10 kHz and lower half power frequency is 120 KHz, find out the value of upper half power frequency and the quality factor of the circuit. (AKTU 2019-2020)

Solution: Given,

$Bandwidth = \Delta f = f_2 - f_1 = 10\ KHz$

where,

$f_1 = Lower\ halfpowerfrequency = 120\ KHz$

$f_2 = Upper\ halfpowerfrequency$

So

$$f_2 = \Delta f + f_1 = 10 + 120 = 130\ KHz$$

quality factor would be,

$$Q - factor = \frac{f_r}{\Delta f} \qquad (1)$$

Now the resonant frequency,

$$f_r = \sqrt{f_1 \times f_2} \qquad (2)$$

Substitute the value of $f_1 and\ f_2$ in the equation no. (2),Hence

$$f_r = \sqrt{120 \times 130} = 124.9 \text{ KHz}$$

Now Substitute the value of $f_r and\ \Delta f$ in the equation no. (1),Hence

$Q - factor = \frac{124.9}{10} = 12.49$

Q2. A series R-L-C circuit consists of R=1000 Ω, L=100 mH and C=10 µF. The applied voltage across the circuit is 100 V. (AKTU 2021-2022)
(i) Find the resonant frequency of the circuit.
(ii) Find the quality factor of the circuit at the resonant frequency.
(iii) At what angular frequencies do the half power point occur?
(iv) Calculate the bandwidth of the circuit.

Solution: Circuit shown in fig.2.34,

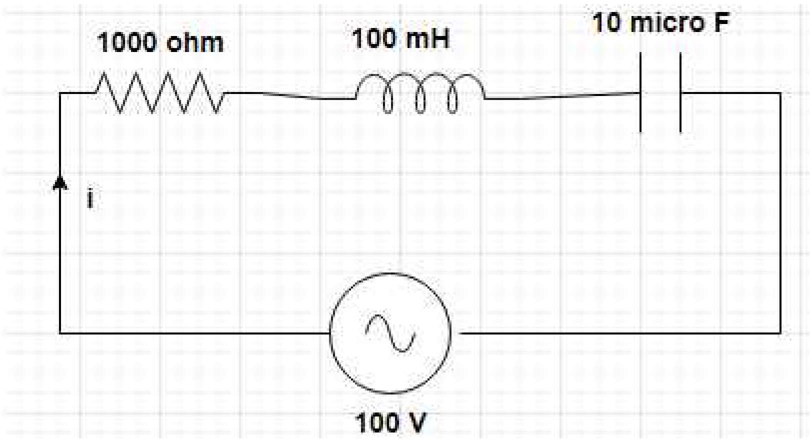


Fig.2.34

(i) f_r would be

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad (1)$$

Substitute the value of R,L,C in equation (1)

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{100\times10^{-3}\times10\times10^{-6}}}$$
$$f_r = 159.16\ Hz$$

(ii) Quality factor of the circuit at resonanse would be,

$$Q-factor = \frac{1}{R}\sqrt{\frac{L}{C}}$$
$$Q-factor = \frac{1}{1000}\sqrt{\frac{100\times10^{-3}}{10\times10^{-6}}}$$
$$Q-factor = 0.1$$

(iii)Angular frequencies of the given circuit is ω_1 *and* ω_2 then we have Relationship between quality factor and bandwidth would be,

$$Q-factor = \frac{f_r}{\Delta f}$$

So bandwidth should be,

$$\Delta f = \frac{f_r}{Q-factor}$$
$$\Delta f = \frac{159.16}{0.1} = 1591.6\ Hz$$
$$\Delta f = f_2 - f_1 = 1591.6\ Hz$$
$$f_2 = f_r + \frac{\Delta f}{2} = 159.16 + \frac{1591.6}{2} = 954.96\ Hz$$

Similarly,

$$f_1 = f_r - \frac{\Delta f}{2} = 159.16 - \frac{1591.6}{2} = -636.64\ Hz$$

(iv) Bandwidth would be,

$$\Delta f = \frac{f_r}{Q-factor}$$
$$\Delta f = \frac{159.16}{0.1} = 1591.6\ Hz$$

2.16.4 ACCEPTOR CIRCUIT:

As a series resonance circuit only functions on resonant frequency, this type of circuit is also known as an Acceptor Circuit because at resonance, the impedance of the circuit is at its minimum so easily accepts the current whose frequency is equal to its resonant frequency.

2.17 RESONANCE IN PARALLEL R-L-C CKT

Resonance is the condition in a parallel ckt (R L C) that at a particular frequency called resonant frequency (f_r) , the power
Factor of the ckt is unity. The supply voltage and that total current are in same phase and current is minimum.

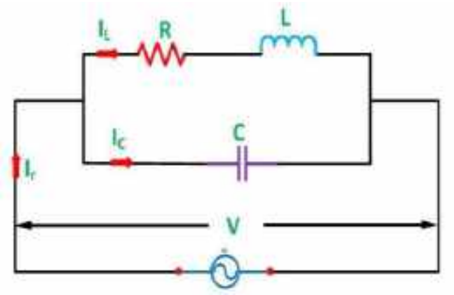


Fig 2.35 Parallel RLC circuit

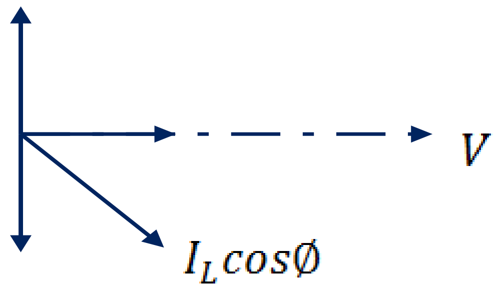


Fig 2.36 Phasor Diagram

Impedance of inductance branch;-

$Z_L = \sqrt{R^2 + X_L{}^2}$

Impedance of capacitive branch ;-X_C

From Phasor Diagram: -

'I_L' lags 'V' by $\emptyset_L$& I_C leads 'V' by 90°.

At resonant freq. F_r, the reactive component of current is Zero.

$\therefore I_C - I_L \sin \emptyset_L = 0$ ------- (1)

Total current I = $I_L \cos \emptyset_L$ ------(2)

From Impedance Δ :-

$\cos \emptyset_L = \frac{R}{\sqrt{R^2+X_L{}^2}}$, $\sin \emptyset_L = \frac{X_L}{\sqrt{R^2+X_L{}^2}}$

$\tan \emptyset_L = \frac{X_L}{R}$

$I_L = \frac{V}{Z_L} = \frac{V}{\sqrt{R^2+X_L{}^2}}$

$I_C = \frac{V}{X_C}$

From eqn (1) ,$I_C = I_L \sin \emptyset_L$

$\frac{V}{X_C} = \frac{V}{\sqrt{R^2+X_L{}^2}} \cdot \frac{X_L}{\sqrt{R^2+X_L{}^2}}$

Or $Z_L{}^2 = X_L . X_C$

$Z_L{}^2 = \omega_r L. \frac{1}{\omega_r C}$

Or $Z_L{}^2 = \frac{L}{C}$

Let ω_r= Resonant Frequency

$$R^2 + X_L{}^2 = \frac{L}{C}$$

$$R^2 + (\omega_r L)^2 = \frac{L}{C}$$

$$\omega_r{}^2 = \frac{L}{L^2 C} - \frac{R^2}{L^2}$$

$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ (rad/sec)}$$

Or $\omega_r = 2\pi F_r$

$$F_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ (Hz)}$$

if $\frac{1}{LC}$>>>>$\frac{R^2}{L^2}$, then $\frac{R^2}{L^2} \rightarrow$ neglected.

$$\therefore F_r = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz) (For Ideal Case).}$$

2.17.1 DYNAMIC IMPEDANCE;

The impedance offered by the parallel ckt at resonance is called dynamic impedance of the circuit.
At resonance,
Total current $I = I_L \cos\emptyset_L$

$$I = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$

$$I = \frac{V\,R}{{Z_L}^2} \quad (\text{as } {Z_L}^2 = \frac{L}{C})$$

$I = \frac{V}{\frac{L}{RC}}$ $\{\frac{L}{RC} \rightarrow Z_D(\,dynamic\ impedace)\}$

$$Z_D = \frac{L}{RC}\Omega$$

As I is minimum at resonance, Z_D is high.

Important points:

Impedance is max $Z_D = \frac{L}{RC}\Omega$,
current is minimum $I = \frac{V}{Z_D}$

power factor is unity ($\cos\phi = 1$).Admittance is minimum. Current magnification $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

Application:

1. R – F oscillators.
2. Complex communication circuit.
3. Impedance Transformation.
4. In filters

2.17.2 Q- FACTOR OF PARALLEL R-L-C CIRCUIT AT RESONANCE (OR) CURRENT MAGMIFICATION

$$\mathbf{Q} = \frac{current\ in\ inductive/capacitive\ branch}{total\ current\ at\ resonance}$$

$$\mathbf{Q} = \frac{I_L}{I} = \frac{\frac{V}{Z_L}}{\frac{V}{Z_D}}$$

$$\mathbf{Q} = \frac{Z_D}{Z_L} = \frac{\frac{L}{RC}}{\sqrt{\frac{L}{C}}}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

2.17.3 REJECTOR CIRCUIT:

As a parallel resonance circuit only functions on resonant frequency, this type of circuit is also known as an Rejecter Circuit because at resonance, the impedance of the circuit is at its maximum thereby suppressing or rejecting the current whose frequency is equal to its resonant frequency.
Means it rejects the signal of a particular frequency. Hence is called a Rejector circuit & is used in Filters.
As f increases, $Z_L=\sqrt{R^2 + XL^2}$ increases

$Cos\phi_L = R/Z_L$ Decreases, so that $I=I_L Cos\phi_L$ is minimum at f_r.

Short Questions

Q1. Why series resonant circuit is known as an acceptor circuit & parallel resonant circuit as a rejecter circuit? (AKTU 2017-2018)

2.18 COMPARISON BETWEEN SERIES RESONANCE AND PARALLEL RESONANCE

Parameters	**Series Resonance**	**Parallel Resonance**
Basic Circuit Configuration	Occurs in a series RLC circuit where the resistor (R), inductor (L), and capacitor (C) are connected in series.	Occurs in a parallel RLC circuit where the inductor (L) and capacitor (C) are connected in parallel with each other, and this combination is connected in series with a resistor (R).
Impedance Characteristics	• At resonance, the impedance of the circuit is at its minimum and equals the resistance R. • The circuit behaves like a pure resistor, and the current is at its maximum.	• At resonance, the impedance of the circuit is at its maximum • The current drawn from the source is at its minimum, ideally zero if there were no resistive losses.
Current Characteristics	The current is maximum at resonance because the circuit's impedance is minimum.	The current through the circuit is minimum at resonance because the impedance is maximum.
Voltage Characteristics	The voltage across the inductor and capacitor can be much higher than the supply voltage due to the high current flowing through the series circuit.	The voltage across the inductor and capacitor is equal to the supply voltage.
Power Factor	At resonance, the power factor is unity	The power factor approaches infinity, indicating a very high impedance and minimal real power consumption from the source.
Application	Used in applications where maximum current is desired at a specific frequency, such as in tuning circuits, filters, and impedance matching.	Commonly used in tank circuits for RF amplifiers and LC oscillators

2.19 AC PARALLEL CIRCUIT:

A Parallel circuit is one in which two or more impedance are connected in parallel across the supply voltage.

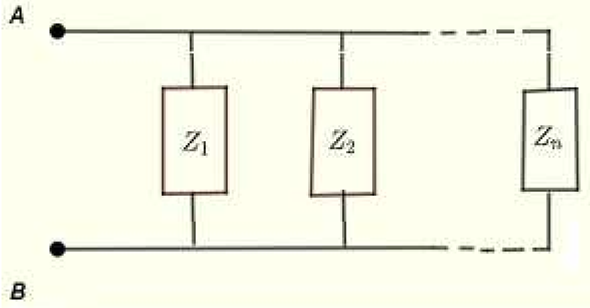


Fig.2.37 AC parallel circuit

$$\vec{I} = \vec{I_1} + \vec{I_2} + \vec{I_3}$$
$$\frac{\vec{V}}{\vec{Z}} = \frac{\vec{V}}{\vec{Z_1}} + \frac{\vec{V}}{\vec{Z_2}} + \frac{\vec{V}}{\vec{Z_3}}$$
$$\frac{1}{\vec{Z}} = \frac{1}{\vec{Z_1}} + \frac{1}{\vec{Z_2}} + \frac{1}{\vec{Z_3}}$$

Two Impedance in parallel

If two Impedance are in parallel and I_T is the total Current, then for calculating individual branch current, apply current division rule –

$$\vec{I_1} = \vec{I_T} \times \frac{\vec{Z_2}}{\vec{Z_1} + \vec{Z_2}}$$
$$\vec{I_2} = \vec{I_T} \times \frac{\vec{Z_1}}{\vec{Z_1} + \vec{Z_2}}$$

Admittance

Admittance is defined as the reciprocal of impedance
Let

$$Z = R \pm j X$$

Admittance(Y) = $\frac{1}{Z}$ ℧

$$Y = \frac{1}{Z} = \frac{1}{R \pm j X} \ ℧$$

Rationalizing the above expression;

$$Y = \frac{R \mp j X}{(R \mp j X)(R \mp j X)} = \frac{R \mp j X}{R^2 + X^2}$$
$$Y = \left(\frac{R}{R^2+X^2}\right) \mp j \left(\frac{X}{R^2+X^2}\right) = \frac{R}{Z^2} \mp j \frac{X}{Z^2}$$

$\therefore Y = G \mp j B$

Where G = Conductance = $\frac{R}{Z^2}$

B = Susceptance = $\frac{X}{Z^2}$

In polar form ,

$$Y = G + j B = |Y| \angle \emptyset \ siemens\ (℧)$$
$$|Y| = \sqrt{G^2 + B^2}$$

$\emptyset = \tan^{-1} \frac{B}{G}$

Applications of AC Parallel Circuits:

1. **AC Power Circuits**: Household and industrial wiring use parallel circuits to connect multiple devices and appliances to the same power supply.
2. **Signal Processing**: In electronics, parallel RC and RL circuits are used in filters and oscillators to shape and control AC signals.

Long Questions

Q1. Consider the circuit shown in figure below and calculate the following.
(AKTU2021-2022)

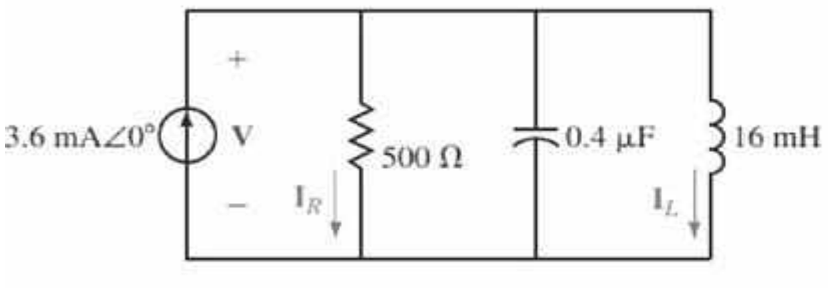


Fig.2.38

- **a. Determine the resonant frequencies, ω(rad/s) and f(Hz) of the tank circuit.**
- **b. Find the Q of the circuit at resonance.**
- **c. Calculate the voltage across the circuit at resonance.**
- **d. Solve for currents through the inductor and the resistor at resonance.**

Solution: Current distribution in the is shown in fig.

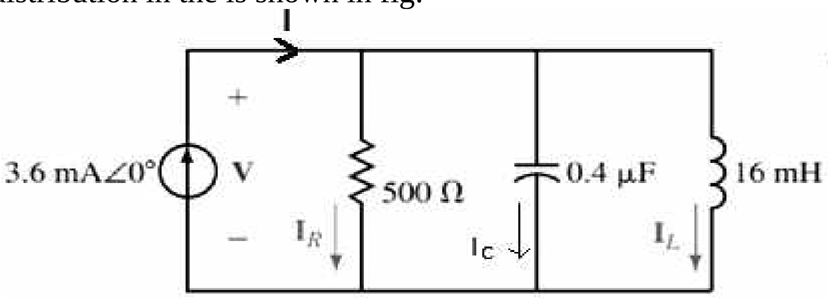


Fig.2.39

At resonance,
$I_L = I_C$

$$\frac{V}{X_L} = \frac{V}{X_C};\ X_L = X_C$$

So resonance frequency in red/sec.,

$$\omega_r = \sqrt{\frac{1}{L.C}} \qquad (1)$$

Given L=16 mH and C= 0.4 μF,

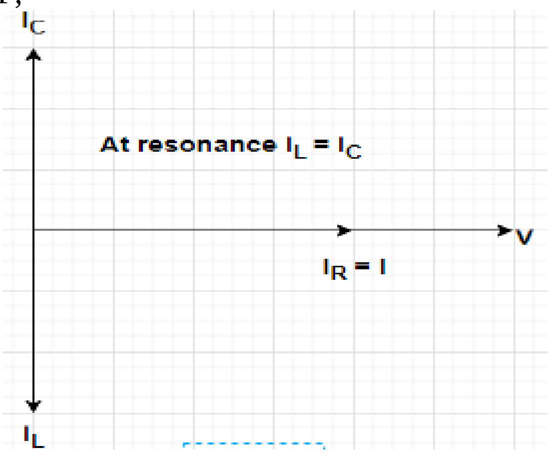


Fig.2.40

(a) Substitute the value of L and C in the equation (1)

$$\omega_r = \sqrt{\frac{1}{16 \times 10^{-3} \times 0.4 \times 10^{-6}}} = 12.5 \times 10^3 \frac{rad}{sec}$$

resonance frequency in Hz would be,

$$f_r = \frac{\omega_r}{2\pi} = \frac{12.5 \times 10^3}{2\pi} = 1.99\ kHz$$

(b) Quality factor of the circuit at resonance,

$$Q - factor = R\sqrt{\frac{C}{L}} = 500\sqrt{\frac{0.4 \times 10^{-6}}{16 \times 10^{-3}}}$$

$$Q - factor = 2.5$$

(c)At resonance,

$$I_R = I = 3.6 \times 10^{-3}\ A$$

Voltage across the circuit,

$$V = I_R \times R = 3.6 \times 10^{-3} \times 500$$

$$V = 1.8\ Volt$$

d) Current through resistor and inductor would be,
At resonance,

$$I_R = I = 3.6 \times 10^{-3}\ A$$

and I_L

$$I_L = \frac{V}{X_L} = \frac{1.8}{2\pi f L} = \frac{1.8}{2\pi \times 1.99 \times 1000 \times 0.016}$$

$$I_L = 8.998\ mA$$

2.20 POWER FACTOR

Power factor (p.f) is a measure of how effectively electrical power is being used in an AC electrical system.
It is defined as the ratio of real power (P) that does useful work to the apparent power (S) that is supplied to the circuit.

PF = cosϕ

)

$$PF = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X^2}}$$

$$\boldsymbol{Power\ Factor = \frac{Real\ Power\ (p)}{Apparant\ Power\ (s)}}$$

Where:

Real Power (P): Measured in watts (W), it is the actual power consumed by the load to do useful work.
Apparent Power (S): Measured in volt-amperes (VA), it is the product of the voltage and current in the circuit.
The power factor value ranges from 0 to 1:
PF = 1 (or 100%) means all the power is effectively used (purely resistive loads).
PF < 1 indicates inefficiency in power usage, meaning more apparent power is required to do the same amount of real work.

2.20.1 Causes of Low Power Factor

Low power factor typically arises due to the presence of inductive loads in the system, which cause the current to lag behind the voltage. Common causes include:

1. Inductive Loads:

Electric Motors: Especially under light load conditions, motors draw more reactive power, leading to a low power factor.

Transformers: Transformers introduce inductance, which can lower the power factor.

2. **Fluorescent Lighting:** Fluorescent lamps and other discharge lamps, without proper compensation, can lower the power factor.

2.20.2 Disadvantages of Low Power Factor

1. Increased Energy Costs:

Utilities often charge penalties for low power factor because it means more apparent power is required to deliver the same amount of real power.Low power factor leads to higher current draw, which increases energy losses in the system.

2. Increased Losses:

Higher I^2R Losses: With increased current flow, the resistive losses (I^2R) in the conductors increase, leading to more heat and energy waste.Thermal Stress on Equipment: The additional heat generated due to higher current can lead to thermal stress on electrical components, reducing their lifespan.

2.20.3 Power Factor Improvement

Improving the power factor involves reducing the phase difference between the voltage and current, primarily by compensating for the reactive power in the system. Here are common methods:

1. **Capacitor Banks:**

Function: Capacitors provide leading reactive power, which counteracts the lagging reactive power from inductive loads, improving the power factor.

Applications: Commonly used in industrial settings with a high concentration of inductive loads like motors and transformers.

2. **Synchronous Condensers**:

Function: A synchronous condenser is a synchronous motor that runs without a mechanical load and can supply or absorb reactive power as needed.

Applications: Used in power plants and large electrical grids where dynamic power factor correction is necessary.

3. **Phase Advancers:**

Function: Phase advancers are used to improve the power factor of induction motors by advancing the phase of the current in the stator windings.

Applications: Suitable for large induction motors operating under partial load conditions.

<u>Short Questions</u>

Q1. Define power factor. **(AKTU 2021-2022)**

Q2. What is power factor of a circuit having impedance of $3 + j4\ ohms$? **(AKTU 2017-2018)**

Soultion: Power factor of the circuit would be,

$$p.f = \frac{R}{Z} = \frac{3}{\sqrt{3^2 + 4^2}}$$

$$p.f = \frac{3}{5} = 0.6\,(lagging)$$

Long Questions

Q3. Refer to the circuit shown in fig. find: (AKTU 2018-2019)

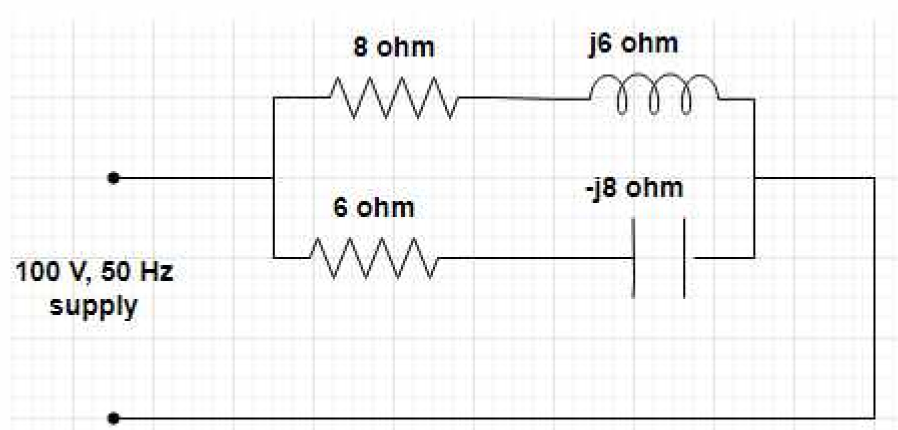


Fig 2.41

(i) R.M.S line current.
(ii) Power dissipated in each branch.
(iii) Power factor.
(iv) Reactive power in each branch.
(v) Total apparent power

Solution: different parameter of the given problem shown in fig.2.42.

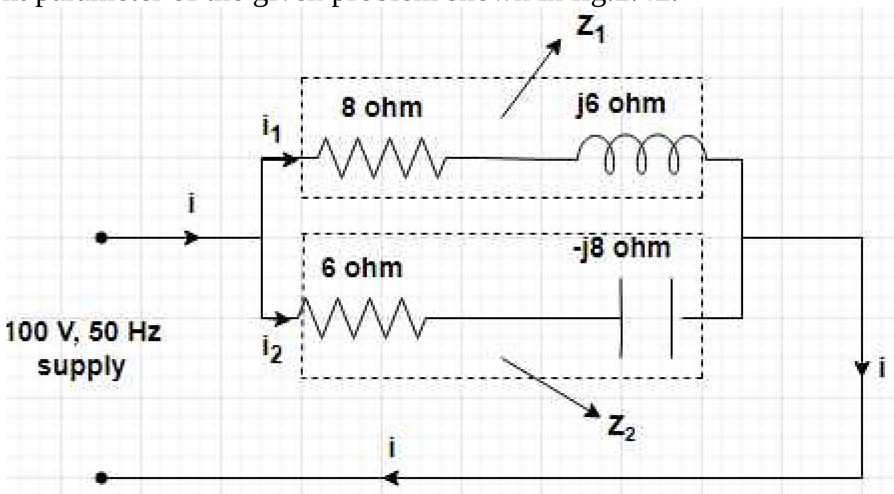


fig 2.42

So Z_1,

$$Z_1 = 8 + j6 = 10\,\angle 36.87^0 \Omega$$

So Z_2,

$$Z_1 = 6 - j8 = 10\,\angle -53.13^0 \Omega$$

Both impedance are in parallel so equivalent impedance would be,

$$Z = \frac{Z_1 \times Z_2}{Z_1 + Z_2} \qquad (1)$$

Substitute the value of Z_1 and Z_2 in equation (1) then we have,

$$Z = \frac{10\,\angle 36.87^0 \times 10\,\angle -53.13^0}{(8 + j6) + (6 - j8)}$$

$$Z = \frac{100 \angle -16.26^0}{14 - j2} = \frac{100 \angle -16.26^0}{14.12 \angle -8.13^0}$$
$$Z = 7.08 \angle -8.13^0$$

(i) R.M.S current in the circuit would be,

$$i_{r.m.s} = \frac{V}{Z} = \frac{100 \angle 0^0}{7.08 \angle -8.13^0} = 14.12 \angle 8.13^0 A$$

(ii) Current in each branch,

$$i_1 = \frac{V}{Z_1} = \frac{100 \angle 0^0}{10 \angle 36.87^0} = 10 \angle -36.87^0 A$$
$$i_2 = \frac{V}{Z_2} = \frac{100 \angle 0^0}{10 \angle -53.13^0} = 10 \angle 53.13^0 A$$

Now power dissipated in first branch would be,

$$P_1 = i_1^2 \times R_1 = 10^2 \times 8 = 800\ W$$

similarly power dissipated in second branch would be,

$$P_2 = i_2^2 \times R_2 = 10^2 \times 6 = 600\ W$$

(iii) Supply current i leads by 8.13^0 with reference of supply voltage so power factor would be,

$$p.f = \cos\emptyset = \cos 8.13 = 0.99\ (Leading)$$

(iv) Reactive power in first branch i.eQ_1

$$Q_1 = i_1^2 \times X_L = 10^2 \times 6 = 600\ VAR$$

Reactive power in second branch i.eQ_2

$$Q_2 = i_2^2 \times X_C = 10^2 \times 8 = 800\ VAR$$

(v) Total apparent power is,

$$S = V_{r.m.s} \times i_{r.m.s} = 100 \times 14.12 = 1412\ VA$$

Q4. Two coil having resistance 5 Ω and 10 Ω and inductances 0.04 H and 0.05 H respectively are connected in parallel across a 200 V, 50 Hz supply. Calculate:
(i) Conductance, susceptance and admittance of each coil.
(ii) Total current drawn by hte circuit and its power factor.
(iii) power absorbed by the circuit.

(AKTU 2021-2022)

Solution: Circuit diagram is shown in fig.2.43

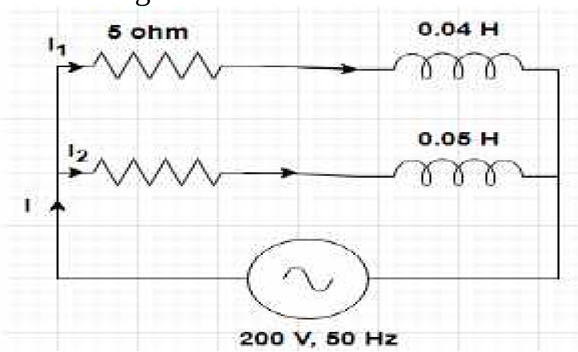


Fig.2.43

Z_1

$$Z_1 = R_1 + jX_{L1} = 5 + j2\pi fL = 5 + j2\pi \times 50 \times 0.04$$
$$Z_1 = 5 + j12.57\ ohms$$

Z_2

$$Z_2 = R_2 + jX_{L2} = 10 + j2\pi \times 50 \times 0.05$$
$$Z_2 = 10 + j15.71\ ohms$$

(i) Admittance of first coil,

$$Y_1 = \frac{1}{Z_1} = \frac{1}{5+j12.57} = \frac{5}{183.01} - j\frac{12.57}{183.01}$$
$$Y_1 = 0.027 - j0.069\ \ \Omega^{-1}$$

Conductance of the first coil,

$$G_1 = 0.027\ \Omega^{-1}$$

Susceptance of the first coil,

$$B_1 = 0.069\ \Omega^{-1}$$

Admittance of Second coil,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{10 + j15.71} = \frac{10}{346.8} - j\frac{15.71}{346.8}$$

$$Y_2 = 0.029 - j0.045\ \ \Omega^{-1}$$

Conductance of the second coil,

$$G_2 = 0.029\ \Omega^{-1}$$

Susceptance of the second coil,

$$B_2 = 0.045\ \Omega^{-1}$$

(ii) Equivalent impedance of the parallel circuit,

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

Reciprocal of impedance is known as admittance so equivalent admittance would be,

$$Y = Y_1 + Y_2 \qquad (1)$$

Substitute the value of Y_1 *and* Y_2 in equation (1) so,

$$Y = 0.027 - j0.069 + 0.029 - j0.045$$

$$Y = 0.056 - j0.114\ \Omega^{-1}$$

Equivalent impedance of the parallel circuit,

$$Z = \frac{1}{Y}$$

$$Z = \frac{1}{0.056 - j0.114} = \frac{0.056 + j0.114}{(0.056)^2 + (0.114)^2}$$

$$Z = \frac{0.056}{0.016} + j\frac{0..144}{0.016} = 3.5 + j9\ \Omega$$

Total current drawn by the circuit,

$$I = \frac{V}{Z} = \frac{200}{\sqrt{3.5^2 + 9^2}} = \frac{200}{9.66} = 20.71\ A$$

Power factor of the circuit,

$$p.f = \frac{R}{Z} = \frac{3.5}{9.66} = 0.36\ (lagging)$$

(iii)Current in the first coil,

$$I_1 = \frac{V}{Z_1} = \frac{200}{\sqrt{5^2 + 12.57^2}} = \frac{200}{13.53} = 14.78\ A$$

Power absorbed in the first coil,

$$P_1 = {I_1}^2 \times R_1 = 14.78^2 \times 5 = 1092.24\ W$$

Current in the second coil,

$$I_2 = \frac{V}{Z_2} = \frac{200}{\sqrt{10^2 + 15.71^2}} = \frac{200}{18.62} = 10.74\ A$$

Power absorbed in the first coil,

$$P_2 = {I_2}^2 \times R_2 = 10.74^2 \times 10 = 1153.48\ W$$

Beyond the syllabus

2.21 THREE-PHASE AC CIRCUIT

2.21.1 Advantages of 3-Φ System over a 1-Φ System

A poly phase or three phase power supply has the following advantages over a single phase power supply system.

1. To transmit a specific power over a specific distance at a given rated voltage, a three phase system needs less conductor material as compared to the single phase system.
2. The size of a three-phase system operated machine is less than the machine operated at single phase voltage having the same output rating.
3. In a three phase power supply system, the less voltage drop occurs from source to the load points,
4. A three phase supply produces uniform rotating magnetic field therefore, three phase motors are simpler in construction, small in size and can be started automatically with smooth operation.
5. A poly phase system produces power at a constant rate in the load.
6. A three phase system can transmit more power as compared to a single phase system.
7. The efficiency of three phase operated devices and appliances is higher than the single phase operated machines.
8. Three phase machines are less costly and more efficient.
9. The output rating of machines can be increased by increasing the number of phases in a system.
10. A three phase machine having the same rating occupies less space as compared to the single phase machine.
11.A three phase supply can be easily converted to a single phase supply while a complex system is needed to convert the single phase supply into a three phase supply system
12. If a fault occurs on a single phase line, the whole system will have to shut down. In case of three phase single line fault, the other two lines provide the power supply to other single phase load points connected to them.
13.Three phase motors have better power factor as compared to single phase motors.

Short Questions

Q1. Write four advantages of three phase system. **(AKTU 2016-2017)**

2.21.2 Terms related to Three-Phase AC Systems:

1.**Three-Phase AC:** An electrical power system with three alternating currents of the same frequency but with phase differences of 120°.
2.Phase: Each of the individual currents or voltages in a three-phase system.
3.Line Voltage: The voltage measured between any two of the three phases in a three-phase system.
4.Phase Voltage: The voltage measured between any one phase and neutral in a three-phase system.
5.Balanced Load: A situation where all three phases in a three-phase system have equal current and voltage magnitudes.
6.Unbalanced Load: When the current or voltage magnitudes differ among the three phases.
7.Delta Connection (Δ): A method of connecting a three-phase system where the end of each phase is connected to the beginning of the next, forming a closed loop.
8.Wye Connection (Y or Star): A method of connecting a three-phase system where each phase is connected to a common neutral point.
9.Neutral: A conductor that carries the return current in a wye-connected three-phase system.

2.21.3 **Star-Connected System:(Wye-connected system)**

In a star connection, each of the three-phase windings (or loads) is connected to a common central point called the **neutral**.

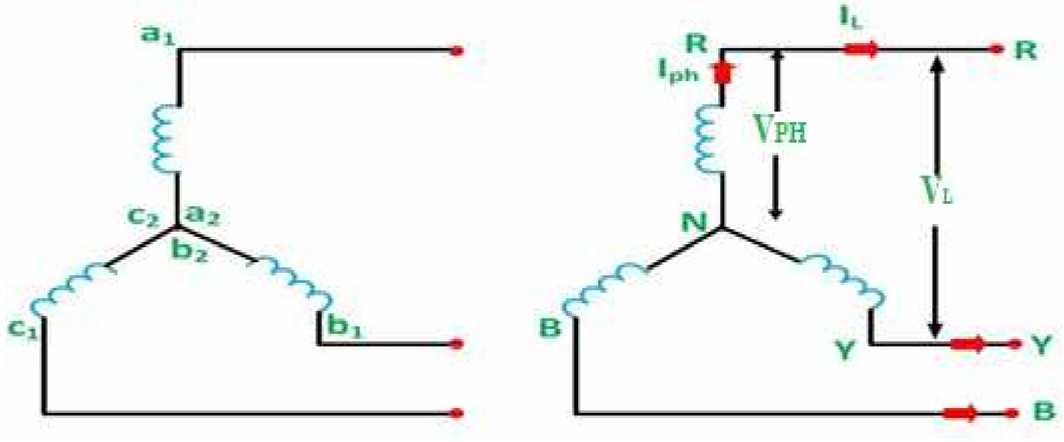


Fig 2.44 Star connection

Star connected 3 phase balanced system will have,

Line voltage (V_L) = $\sqrt{3}$ Phase voltage (V_P).

Also Line current (I_L) = Phase current (I_P)

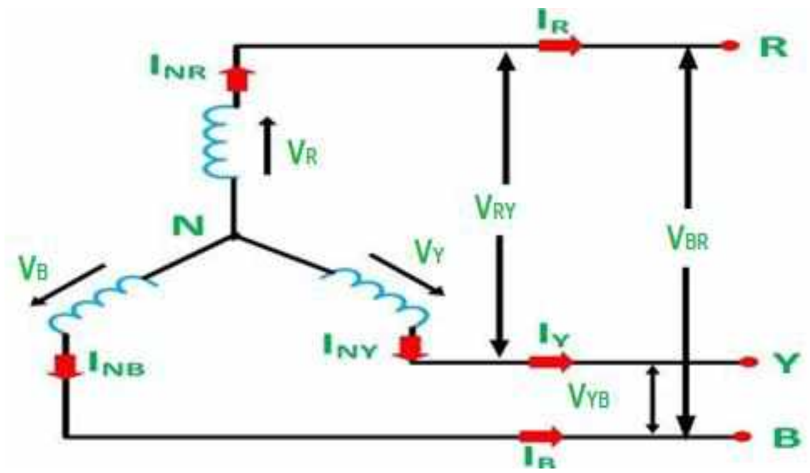


Fig 2.45 Star connection

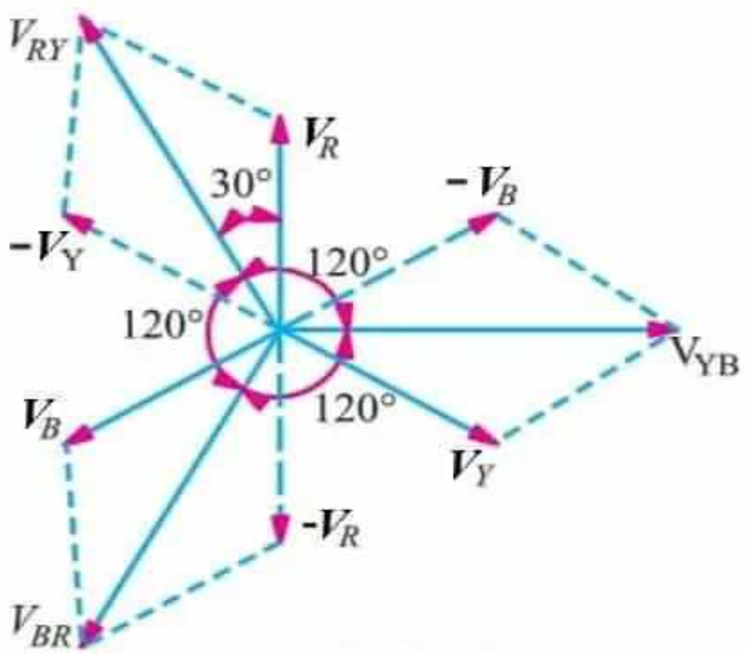


Fig 2.46 Phasor Diagram

Let V_R, V_Y, V_B are phase voltages &V_{RY},V_{YB},V_{BR} are line voltages respectively.

$$V_{RY} = V_R - V_Y$$
$$V_{YB} = V_Y - V_B$$
$$V_{BR} = V_B - V_R$$

From phasor-Apply parallelogram law in any of the one parallelogram-

$$|V_{RY}| = \sqrt{|V^2{}_R| + |V^2{}_Y| + 2|V_R||V_Y|\cos 60}$$

$$|V_{RY}| = \sqrt{V_P^2 + V_P^2 + 2 \times V_P^2 \times \frac{1}{2}}$$

$$V_L = \sqrt{3}V_P$$

For star connection

$$I_L = I_P$$

Line current = Phase current

2.21.4 Delta-Connected System:

In a delta connection, the three-phase windings or loads are connected in a closed loop, forming a triangle (or delta) shape, where each phase is connected end-to-end.

Delta connected 3 phase balanced system will have,

Line current (I_L)=$\sqrt{3}$ Phase current (I_P).

Also Line voltage (V_L)= Phase voltage (V_P)

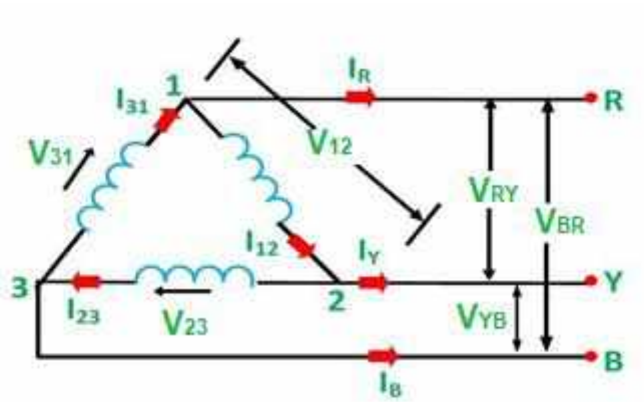


Fig 2.47 Delta connection

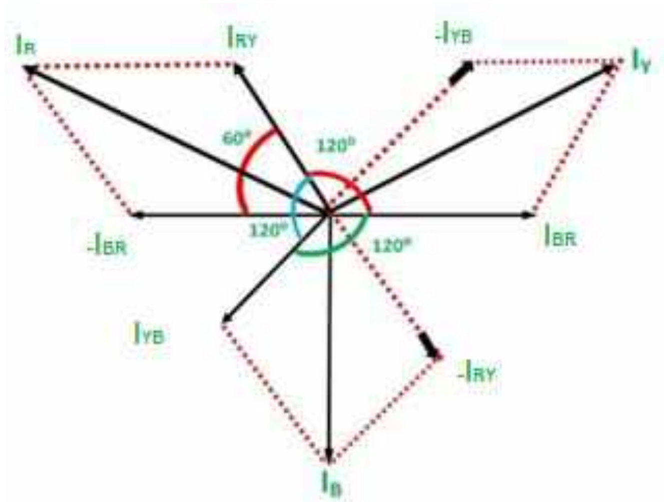


Fig 2.48 Phasor diagram

Let I_R, I_Y, I_B are line currents & I_{RY},I_{YB},I_{BR} are phase currents respectively.
Applying KCL at node 1,2,3 respectively.
At node 1-

$$I_R + I_{BR} = I_{RY}$$
$$I_R = I_{RY} - I_{BR} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

At node 2-

$$I_B + I_{RY} = I_{YB}$$
$$I_B = I_{YB} - I_{RY} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

At node 3-

$$I_Y + I_{YB} = I_{BR}$$
$$I_Y = I_{BR} - I_{YB} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

From phasor diagram applying parallelogram law-

$$|I_Y| = \sqrt{|I^2{}_{BR}| + |I^2{}_{YB}| + 2|I_{BR}||I_{YB}|\cos 60}$$
$$I_L = \sqrt{I_P^2 + I_P^2 + 2 \times I_P^2 \times \frac{1}{2}}$$
$$I_L = \sqrt{3} I_P$$

One phase comes across the two lines

$$V_L = V_P$$

Line voltage = Phase voltage.

2.21.5 Three-Phase Power:

Active Power (P): The actual power consumed by the load, measured in watts (W).

$$P = \sqrt{3}\ V_L \times I_L \cos\phi$$

Reactive Power (Q): The power that oscillates between the source and the load, measured in volt-amperes reactive (VAR).

$$Q = \sqrt{3}\ V_L \times I_L \sin\phi$$

Apparent Power (S): The product of the RMS values of voltage and current, representing the total power in the system, measured in volt-amperes (VA).

$$S = \sqrt{3} \times V_L \times I_L$$

2.22 MAGNETIC CIRCUIT:-

Any current carrying conductor produces a magnetic field around the conductor. Magnetic field comprises the magnetic Lines of forces which passes through the magnetic material in a closed path.
The closed path followed by the magnetic Lines of forces is called magnetic circuit. All electrical devices Like as transformer, generator etc. work by the magnetic circuit.

2.22.1 IMPORTANT TERMS RELATED TO MAGNETIC CIRCUIT:-

(1) Magneto motive force [M.M.F.]

Magnetic flux generator is called magneto motive force. It is a some sort of magnetic pressure which set-up a magnetic flux in a magnetic circuit.
It depends upon the following factors:-
(a) No. of turns in a coil (N)
(b) Intensity of a current (I).
Hence,

$$\text{M.M.F.} = N \times I \quad \text{Ampere-turn}$$

(2) Reluctance (S):-

Resistance offered by magnetic circuit is called the reluctance. It opposes the magnetic flux in a magnetic flex circuit. Its unit is ampere turns- per weber. It depends upon the following factors :-

(a) Length of the core materials(L).
(b) Cross-section area of the core (A).
Hence, Reluctance
Mathematically it can be expressed as

$$S = \frac{L}{\mu A} \quad \frac{AT}{Weber}$$

$\mu = \mu_o \mu_r$

$$S = \frac{L}{\mu_0 \mu_r A}$$

where,
L = length of the magnetic path in Meters
μ_o= Permeability of free space (vacuum), $\frac{Henry}{meter}$

μ_r = Relative permeability of a magnetic material
A= Cross sectional area in square meters (m^2)`
"Reluctance of the core can also be obtained by the ratio of M.M.F. to magnetic flux"

$$S = \frac{M.M.F}{Magneticflux}$$

$$S = \frac{M.M.F}{\Phi} \frac{AT}{Weber}$$

Reluctance is analogous of resistance in electric circuit.

(3) Permeance (P):

Permeance is the reciprocal of reluctance of the material and it is measured in $\frac{Weber}{AT}$.

Hence, Permeance

$$P = \frac{1}{Reluctance}$$

$$P = \frac{1}{S} \quad \frac{Weber}{AT}$$

$$P = \frac{A\mu_0\mu_r}{l} \quad \frac{Weber}{AT}$$

Permeance is the analogous of the conductance in an electric circuit.

(4) Magnetic flux (Φ):

Number of magnetic line of force passes through any cross section is called magnetic flux.
It's unit is weber. Hence Magnetic flux

$$\Phi = B.A \ \ Weber$$

where,
B= Magnetic field in Tesla
A= Cross-section area in metre^2
b) Let us consider a toroidal ring of ferromagnetic material of mean radius R and circular cross-section of diameter d. The core of the ring is excited by a coil with N turns carrying a current I amperes. Magnetic flux is established in the core & forms a closed path,

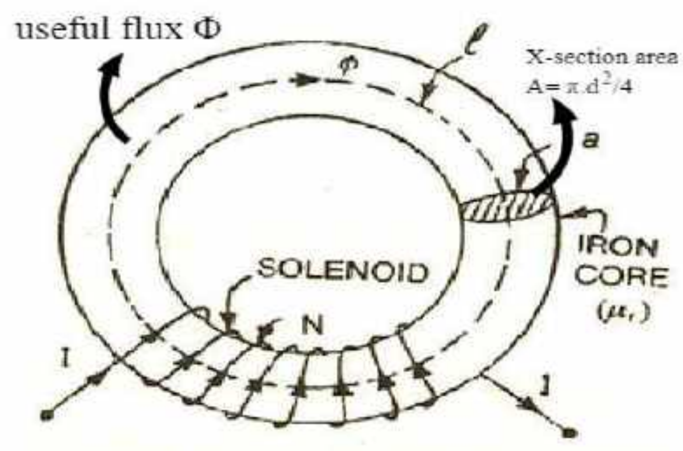


Fig.-2.54 Toroidal ring

Let , l= mean length of the magnetic circuit in meter =2πR

A = Cross section area of the core ,

$$A = \frac{\pi d^2}{4} \text{meter}^2$$

μ_r= Relative permeability of the core

We know that

$$B.\, l = \mu N.I$$

$$B = \frac{\mu.N.I}{l}$$

Since, $\mu = \mu_o \, \mu_r$

$$B = \frac{\mu.N.I}{l} \qquad (1)$$

Flux density in the core material,

$$B = \frac{\text{Flux}}{\text{cross−section area}}$$

$$B = \frac{\varphi}{A} \qquad (2)$$

On comparing the above two equations we get,

$$\frac{\varphi}{A} = \frac{\mu_o \mu_r N I}{l}$$

$$\therefore \; NI = M.M.F$$

$$\Phi = \frac{\mu_o \mu_r NIA}{l}$$

$$\Phi = \frac{NI}{\frac{l}{\mu_o \mu_r A}}$$

$$\Phi = \frac{M.M.F.}{\text{Reluctance of magnetic path}}$$

$$\Phi = \frac{NI}{S} \quad \text{weber.}$$

From the above it is clear that intensity of flux is inversely proportional to the Reluctance of the core,Hence ,

$$\Phi \propto \frac{1}{S}$$

$$\Phi = \frac{NI}{S} \qquad \text{(Analogous to ohm's law)}$$

In magnetic circuit , Total Flux produced (Φ_T) by the toroidal ring is categorised into two parts:-
(i) Linkage Flux or useful flux (Φ_U)
(ii) Leakage Flux (Φ_L)

(i) Linkage Flux or useful flux (Φ_U) :- Linkage flux is that flux which is linked with the magnetic core i.e. Φ_U

(ii) Leakage Flux (Φ_L) :- The Magnetic flux which does not follow the intended path in a magnetic circuit is called leakage flux i.e Φ_L.

$$[\Phi_T = \Phi_U + \Phi_L]$$

(iii) Leakage coefficient or Leakage factor (λ) :-

The ratio of total flux produced to the useful flux is called leakage coefficient or leakage factor. It is indicated by the symbol (λ). Hence,

$$\text{Leakage factor} = \frac{\text{Totl flux}}{\text{useful flux}} = \frac{\varphi_T}{\varphi_U}$$

The value of leakage factor is always greater than unity. Practically, it will be lie between 1.12 to 1.25.

Hence, $1.12 < \lambda < 1.25$.

2.22.2 Fringing :-

Air gaps are provided in many practical magnetic circuit. Consider a ring provided with an air gap as shown in the figure below :-
When the flux lines cross the air-gaps they tend to spreading at the edges of the air-gap. So, the effective area of the air-gap increases and flux density decreases. This effect is called Fringing.

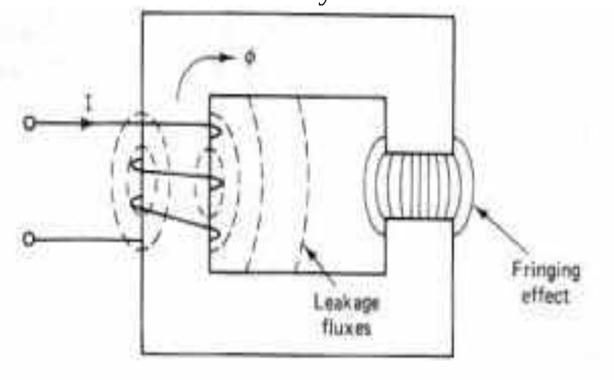


Fig.- 2.55 Fringing effect

If the air-gap is small as possible then we will assume, fringing flux neglected .If it is not neglected then Effective cross-section area will increase.

2.22.3 Comparison between Magnetic and Electric Circuit :-

	Electric circuit	Magnetic circuit
1.	A closed path followed by electric current is known as electric circuit.	A closed path followed by magnetic field line or magnetic flux is known as magnetic circuit.
2.	The electric current is the quantity which flows in an electric circuit.	The magnetic flux is the quantity which flows in a magnetic circuit.

3.	For electric circuit, Ohm's law is as Current, $I=\frac{EMF}{Resistance}$	For magnetic circuit, the expression of Ohm's law is -Magnetic flux, $\phi=\frac{MMF}{Reluctance}$
4.	The resistance of the electric circuit opposes the electric current flowing in the circuit.	The reluctance of the core of magnetic circuit opposes the flow of magnetic flux in the circuit.
5.	In an electric circuit, the resistance produces the opposition whose reciprocal is known as conductance and is given by,Conductance, $G=\frac{1}{R}$	In a magnetic circuit, the opposition to the magnetic flux is reluctance whose reciprocal is known as permeance, and is given by,Permeance = $\frac{1}{Reluctance}$
6.	The electric current flows in an electric circuit whose density (called current density) is given by, $J=\frac{I}{A}$ A/m^2	Magnetic flux flows in a magnetic circuit whose density, called magnetic flux density, is given by, $B=\frac{\Phi}{A}$ Wb/m^2
7.	In an electric circuit, electric field exists whose field intensity is given by, $E=\frac{V}{d}$	In a magnetic circuit, there is a magnetic field whose intensity is $H=\frac{NI}{l}$
8.	The Ohm's law for electric circuit, KVL and KCL are followed in an electric circuit.	The Ohm's law for magnetic circuit, Kirchhoff's MMF law and Kirchhoff's flux law are followed in a magnetic circuit.
9.	The direction of electric field lines is from positive terminal to the negative terminal in the electric circuit, i.e. the electric field lines starts from positive charge and ends on the negative charge.	The magnetic field lines starts from north pole and ends on the south pole.
10.	In an electric circuit, the energy must be expended continuously as long as the electric current flows.	In a magnetic circuit, once the magnetic flux is set up in the circuit, no energy need to be expended.

2.22.4 Combination of Magnetic circuits :-

There are two combinations used which are as follows:-
(i) Series magnetic circuit
(ii) Parallel magnetic circuit

(i) Series magnetic circuit:-

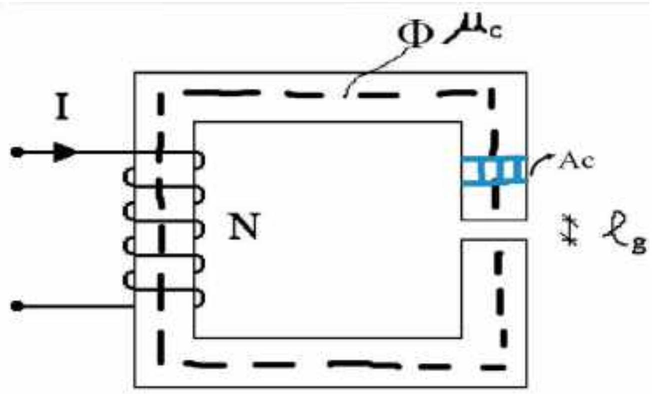


Fig.- 2.56 Series magnetic circuit

Total Reluctance of the magnetic circuit

$$S= S_c + S_g$$

Where
S_c = Reluctance of core
S_g = Reluctance of air-gap

$$S_C = \frac{l_c}{\mu_o \mu_{rA_c}} \frac{AT}{Weber}$$

$$S_g = \frac{l_g}{\mu_o \mu_r A_g} \frac{AT}{Weber}$$

Substituting the values of S_c and S_g, we get

$$S = S_c + S_g$$

$$S = \frac{l_c}{\mu_o \mu_{rA_c}} + \frac{l_g}{\mu_o \mu_r A_g}$$

Total m.m.f. can be obtained by the following expression :-

$$M.M.F = \Phi \times \text{Total reluctance } (S)$$

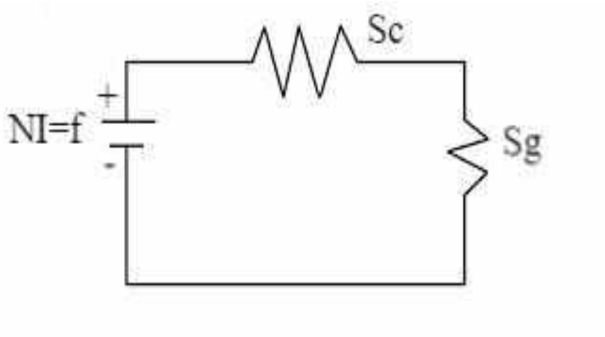


Fig.-2.57 Total reluctance

(ii) Parallel magnetic circuit:-

In parallel magnetic circuit two or more magnetic path are connected in such a manner that the magnetic flux is different in each magnetic path.Parallel magnetic circuit is shown below:-

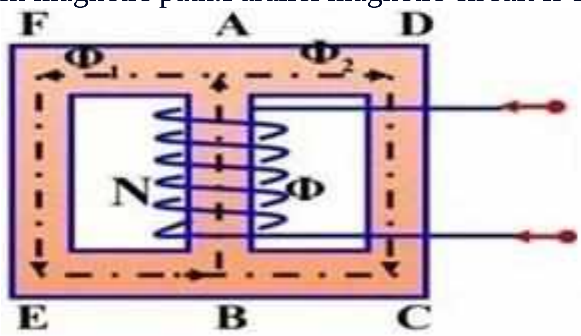


Fig.2.58 Parallel magnetic circuit

In the above circuit the current carrying coil is wound on the central limb AB. The coil sets up a magnetic flux Φ in the central limb which is further divided into two parts, i.e. ;

(i) Flux Φ_1 flows in the path of AFEB

(ii) Flux Φ_2 flows in the path of ADCB

<u>Short Questions</u>

Q1. Define magnetomotive force (MMF). (AKTU 2016-2017)

Q2. How MMF is related to reluctance? Explain. (AKTU 2019-2020)

2.22.5 What is Hysteresis

Hysteresis is the common property of ferromagnetic substances. when the magnetization of ferromagnetic materials lags behind the magnetic field, this effect can be described as the hysteresis effect. Hysteresis is characterized as a lag of magnetization lntensity (B) behind the magnetic field intensity (H).

B-H Curve:-

If an alternating magnetic field is applied to the material, its magnetization will trace out a Loop Called a hysteresis a Hysteresis loop or B-H curve.

B→ Magnetic flux density
H→ Magnetizing force.

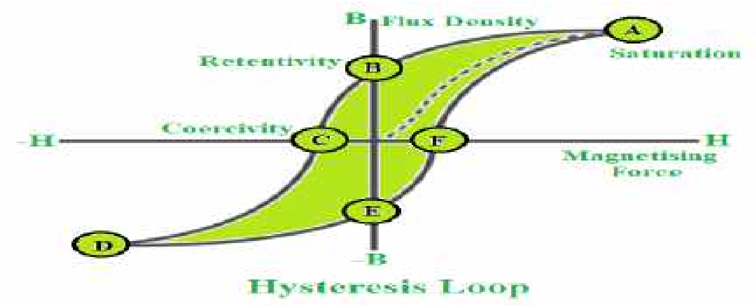


Fig.- 2.59 B-H curve

a) Coercivity:- It is the minimum external electric field required to destroy the residual magnetism is called Coercivity.

b) Retentivity: It is the value of intensity of magnetization retained by the ferromagnetic substance when the magnetizing field is switched off.

Materials used to make Permanent magnets should have high value of retentivity and Coercivity.
Material used to make electromagnets have high retentivity and low coercitity.

2.23 Why we need transformer

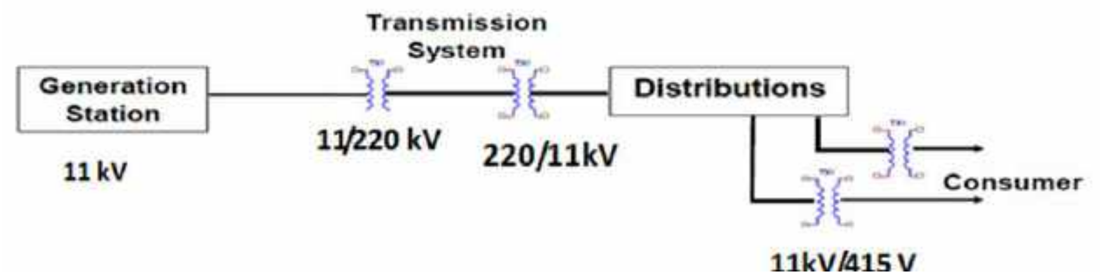


Fig.-2.60 Single line diagram of transformer

Now three phase power = $\sqrt{3}\, V_L\, I_L \cos\Phi$

$I_L \propto 1/V_L$ { as P and Cos Φ are constant} if $V_L\uparrow$, than $I_L\downarrow$

IF $I_L \downarrow = I_2$ loss low, size of conductor in transmission line are small, efficiency will increase, voltage drop in line reduces.

- ❖ The power transmission system using transformers has been shown above in figure 3.7.
- ❖ Transformers help improve safety and efficiency of power systems by raising and lowering voltage levels as and when needed.
- ❖ They are used in a wide range of residential and industrial applications, primarily and perhaps most importantly in the distribution and regulation of power across long distances.

2.23.1 Transformer :-

Transformer is a static device i.e. no rotating part or no moving part , which transfers electrical energy from one electrical circuit to another one with the desired change in voltage and current via magnetic flux and without change in frequency.

2.23.2 Principle of Transformer:-

Transformer works on the principle of mutual Induction which States that “when two coils are inductively coupled and if current in one coil is changed uniformly then an EMF gets Induced in the other coil”. Transformer has two windings (Primary (N_1turns) winding and Secondary (N_2 turns) winding.

1. For Step up transformer (N_1>N_2)
2. For Step down transformer (N_1> N_2)

2.23.3 Working of a Transformer

when primary winding is excited by an A.C. Voltage, due to Current I_1, & N_1, an MMF (N_1,I_1) is produced which circulates flux (Φ) in the core. Due to alternating current, flux changes at primary and an EMF (Statically) gets induced in the primary by Faraday's law of Electromagnetic Induction.

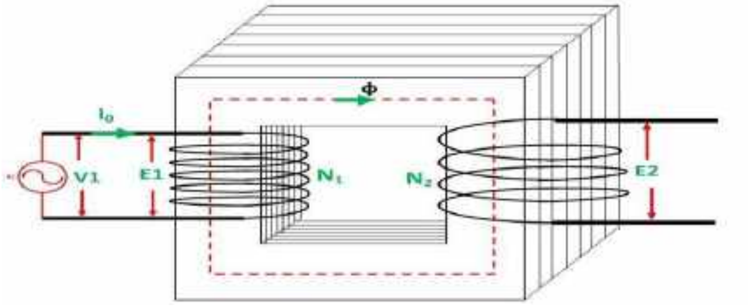


Fig.-2.61 Transformer

The flux is alternating in nature and it links with the secondary , So, due to mutual Induction an emf gets induced in the secondary winding . There is no electrical connection between the primary and secondary windings.

2.23.4 What will happen if Transformer will operate on DC

Transformer Cannot work on DC, because in DC Supply flux produced in the core is not of alternating nature but Flux is of constant Nature. As there is no change in flux so, no EMF is induced. If forcefully DC is given to the primary, Core saturates and it will draw excessively large current which may burn the winding .

Short Questions

Q1. Why transformer is not used on D.C. AKTU(2017-2018,2020-2021)
Q2. What will happen if the primary of a transformer is connected to D.C supply. AKTU(2018-2019)
Q3. Explain why transformer cannot be operated on D.C. AKTU (2018-2019)

2.24 Types of Transformer:-

EXPLANATION:- Various types of Transformers on the basis of construction are core Type Transformer and Shell Type Transformer.

2.24.1 CORE TYPE TRANSFORMER :-

*Core of this transformer is in the form of rectangular frame and made up of laminations to reduce eddy current loss.
* Core is made up of high grade silicon steel to minimize hysteresis loss.
* Laminated rectangular core provides a single magnetic circuit.
The primary and secondary windings are uniformly distributed on two limbs of the core.

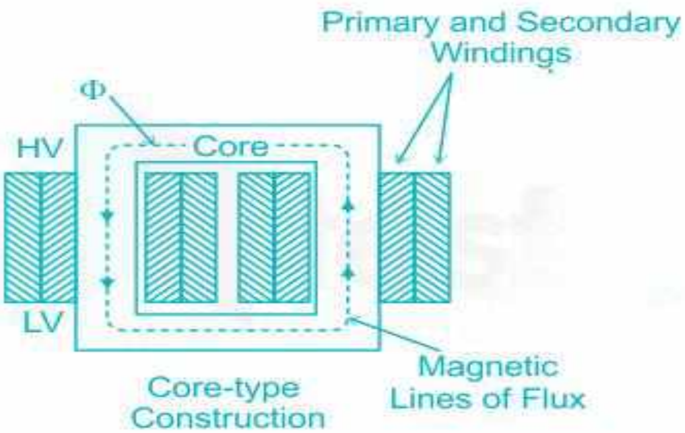


Fig.-2.62 core type transformer

*Low Voltage winding is placed near the core so that less insulation is required.
* Better cooling as more surface is exposed to the atmosphere.
*The coils can be easily removed by removing the laminations of the type of yoke for maintenance.
* It is used for low voltage transformer as well as for high voltage transformers in power system.

2.24.2 SHELL TYPE TRANSFORMER:

*The Primary and secondary windings are placed on the central limb of the Core.
* The high voltage and low Voltage Windings are of sandwich-type, which are in the form of interleaved - -pan cakes.
* This type of Core provides double magnetic circuit. This type of core provides a better mechanical Support and protection for the windings.

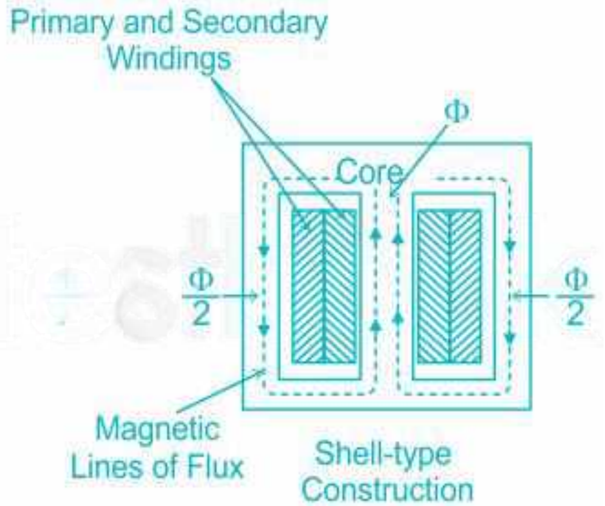


Fig.-2.63 Shell type transformer

*The core surrounds the windings, cooling is not very effective.
For removing any winding for maintenance, large no. of laminations are required to be removed .
* This construction is used for very high voltage transformers.

2.24.3 Difference between core type and shell type transformer:-

Sr. No	Core Type Transformer	Shell Type Transformer
1.	The core has only one window.	The core has two windows.
2.	Winding encircles the core.	Core encircles the windings.
3.	Cylindrical windings are used.	Sandwich type windings are used.
4.	Easy to repair.	It is not so easy to repair.
5.	Better cooling since more surface is exposed to the atmosphere.	Cooling is not very effective.

2.25 E.M.F. equation of transformer:

Let, Φ = Flux in the core of transformer in weber
Φ_m: The maximum amount of flux is presented in weber
f: The source frequency in Hz

$$T=\frac{1}{f} = \text{Time period (seconds)}$$

N_1: Turns number in the first winding
N_2: Turns number in the second winding

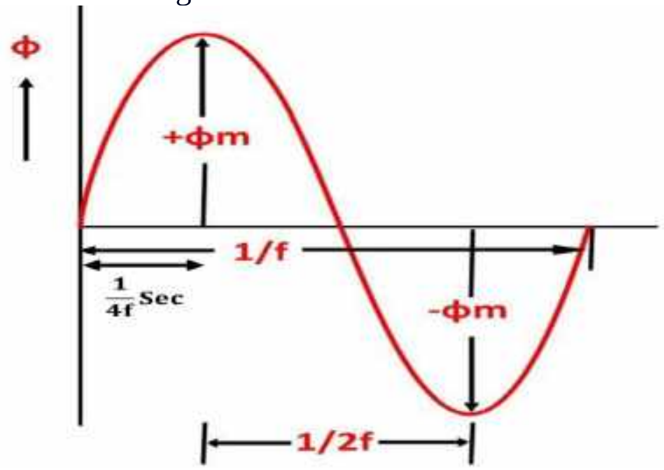


Fig.-2.64 A.c. waveform

E_1 = RMS value of primary induced emf
E_2 = RMS value of secondary induced emf
B_m = Max. Flux density (Wb/m^2)
According to Faraday's law of electromagnetic induction , the average EMF induced in each turn is proportional to the Average rate of change of flux.

$$\text{Average EMF per turn} \propto \frac{d\varphi}{dt}$$

Therefore,

$$\frac{d\varphi}{dt} = \frac{\text{Change in flux}}{\text{Time required for change in flux}}$$

Consider $\frac{1}{4^{th}}$cycle Φ changes from 0 to Φ_m.

Therefore,

$$\frac{d\varphi}{dt} = \frac{\varphi_m}{\frac{T}{4}}$$

$$\text{Average EMF per turn} = 4f\ \Phi_m$$

$$\frac{\text{E.M.F.induced}}{\text{Turn}} = 4f \times \Phi_m \text{ Volt}$$

$$\text{Form factor} = \frac{\text{R.M.S.value}}{\text{Average value}}$$

$$\text{Form factor} = 1.11$$

$$\text{RMS value} = \text{Form Factor} \times \text{Average Value}$$

$$\frac{\text{E.M.F.induced}}{\text{Turn}}(\text{rms}) = (1.11) \times (4f\ \Phi_m) = 4.44\ f\ \Phi_m \quad \text{Volts}$$

RMS value of emf induced in primary:-

$$E_1 = 4.44\ f\ \Phi_m \times N_1$$

$$E_1 = 4.44\ f\ B_m\ A\ N_1 \quad \text{Volts}$$

Similarly, RMS value of emf induced in secondary:-

$$E_2 = 4.44\ f\ \Phi_m \times N_2$$

$$E_2 = 4.44\ f\ B_m\ A\ N_2 \quad \text{Volts}$$

2.25.1 Voltage ratio :-

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

K= Transformation ratio

$$K = \frac{N_2}{N_1}$$

Thus,
1. If $N_2 > N_1$ i.e. $k > 1$, $E_2 > E_1 \rightarrow$ Step up transformer
2. If $N_2 < N_1$ i.e. $k < 1$, $E_2 < E_1 \rightarrow$ Step down transformer
3. If $N_2 = N_1$ i.e. $k = 1$, $E_2 = E_1 \rightarrow$ Isolation or 1:1 transformer

2.25.2 Current ratio :-

For an Ideal transformer there are no losses. So input equal to the output (VA)

Input (VA) = Output (VA)

Therefore

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

2.25.3 Rating of Transformer:-

The rating of transformer is in Volt-Ampere or kVA/MVA. While designing the transformer, there is no idea about load and its nature so its rating is expressed in VA/kVA/MVA. Moreover, Cu loss is directly proportional to I^2 and core loss is directly proportional to the V_{supply}. These losses doesn't depend on load power factor. So rating of transformer is in VA/kVA/MVA not in kW/MW.

<u>Short Questions</u>

Q1. Why transformers are rated in kVA ? (AKTU 2016-17)

Soultion: Copper loss of a transformer depends on current i.e,

$$P_{cu} = {I_1}^2 R_{01} = {I_2}^2 R_{02}$$

and iron loss depends on voltage,

$$P_i = P_h + P_e$$

$$P_i = K_h V^x f^{1-x} + K_e V^2$$

Hence total losses depend on Volt- Ampere and not on the power factor of the load. That is why the rating of transformers is in KVA and not in KW.

2.25.4 Full load currents:-

KVA rating of transformer = $\frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$

$$(I_1)_{f.l.} = \frac{KVA_{Rating} \times 1000}{V_1}$$

$$(I_2)_{f.l.} = \frac{KVA_{Rating} \times 1000}{V_2}$$

Where I_1 and I_2 are the full load primary and secondary currents.
It is the safe maximum value of current which transformer can bear.

Long Question

Q1. A single phase, 50 Hz core type transformer has square cores of 20 cm. side, permissible maximum flux density is $1\ Wb/m^2$. Calculate the number of turns per limb on the high and low voltage sides for a 300/220 $V\ ratio.$

(AKTU 2017-2018)

Solution: Given,
$Frequency, f = 50\ Hz$
$Area\ of\ the\ core, A = 20 \times 20 \times 10^{-4} m^2 = 4 \times 10^{-2} m^2$
$Maximum\ flux\ density, B_m = 1\ Wb/m^2$
$Primary\ voltage, E_1 = 300\ V$
$Secondary\ voltage, E_2 = 220\ V$
$No.of\ limb\ in\ core\ type\ transformer = 2$
So the value of maximum flux in the core,

$$\varphi_m = B_m \times A = 1 \times 4 \times 10^{-2} = 4 \times 10^{-2}\ Wb.$$

Induced e.m.f in the transformer would be,

$$E = 4.44 \times f \times \varphi_m \times N\ \ volt$$

Induced e.m.f in the primary winding of transformer would be,

$$E_1 = 4.44 \times f \times \varphi_m \times N_1\ \ volt$$

So,

$$N_1 = \frac{E_1}{4.44 \times f \times \varphi_m} = \frac{300}{4.44 \times 50 \times 4 \times 10^{-2}} = 33.78$$

Hence no. of turns per limb would be,

$$No.of\ turns\ in\ the\ primary\ winding\ per\ limb = \frac{N_1}{2} = \frac{33.78}{2} = 16.89 \cong 17$$

Similarly Induced e.m.f in the secondary winding of transformer would be,

$$E_2 = 4.44 \times f \times \varphi_m \times N_2\ \ volt$$

So,

$$N_2 = \frac{E_2}{4.44 \times f \times \varphi_m} = \frac{220}{4.44 \times 50 \times 4 \times 10^{-2}} = 24.78$$

Hence no. of turns per limb would be,

$$No.of\ turns\ in\ the\ secondary\ winding\ per\ limb = \frac{N_2}{2} = \frac{24.78}{2} = 12.39 \cong 12$$

Unsolved Problems:

Q1. A single phase transformer has 350 primary turns and 1050 secondary turns. The net cross sectional area of the core is 55 cm^2. If the primary winding be connected to a 400 V, 50 Hz single phase supply. Calculate (i) maximum value of the flux density in the core and (ii) The voltage induced in the secondary winding. **[7Marks]**

[i) 0.936 T (ii) 1200 V]

Q2. A single phase 2200/400 V, 50 Hz transformer has core area 3600 mm^2 and the maximum flux density 1.6 T. Determine the number of turns of primary and secondary windings

[2 Marks].

[172,1720]

Q3. A 1100/400 V, 50 Hz single phase transformer has 100 turns on the secondary winding. Calculate the number of turns on its primary. **[2 Marks].**

[275]

2.26 Ideal Transformer:-

What do you understand by the term "ideal transformer" ?

Definition: A transformer that doesn't have any losses like Copper and core is known asan ideal Transformer. Ideal transformer has the following properties-

1. No losses (Iron and copper)
2. Primary and secondary winding resistances are zero
3. Leakage flux is zero i.e. 100%, flux Produced by primary links with the secondary.
4. Permeability of the core is so high.
5. Efficiency (η) = 100%

6. As $R_1$and R_2 are zero. So, $V_1= E_1$ and $E_2=V_2$

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

2.26.1 Ideal Transformer on No Load:-

For Ideal Transformer on no load has the following properties :-

I_c will be zero (I_c=0)

No Core loss, No cu loss

Primary draws a current which is just necessary to set up flux in the core.

$I_0= I_m$ = Magnetizing current (Sets up flux in the core)

As

$$R_1=0,\ R_2 = 0$$

$$V_1=E_1 \text{ and } E_2= V_2$$

E_1 opposes V_1,

By Lenz's Law (Induced E.M.F)

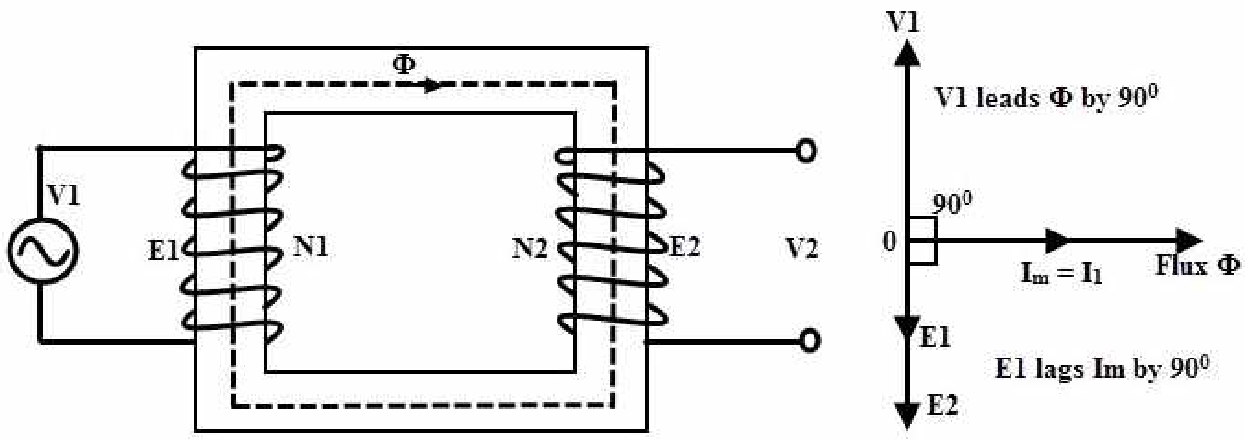


Fig.-2.65 Ideal transformer on no load

2.26.2 Phasor diagram of Ideal Transformer on No-Load:-

Take Φ as reference

I_m Sets up Φ, so is in Phase with Φ.

I_m Lags V_1 by 90° as winding is purely Inductive.

E_1 and E_2 are induced emfs. (in same phase) but oppose V_1, (Lenz's Law)

$$P_{in} = V_1 I_m \cos 90 = 0,\ P_{in} = P_{out} = 0$$

Short Questions

Q1. Draw the no load phasor diagram of a transformer (AKTU 2021-2022)
Q2. Draw the phasor diagram for ideal transformer on no load? (AKTU 2021-2022)
Q3. Draw the phasor diagram of a practical two-winding transformer in no-load condition. (AKTU 2020-2021,2022-2023)

Long Question

Q1. A 25 kVA, 3300/230 V, 50 Hz, 1-phase transformer draws a no-load current of 15 A when excited on load voltage side and consumes 350 watt. Calculate two components of current.

[I_w=1.521 A, I_m=14.922 A]

2.27 PRACTICAL TRANSFORMER ON NO LOAD:

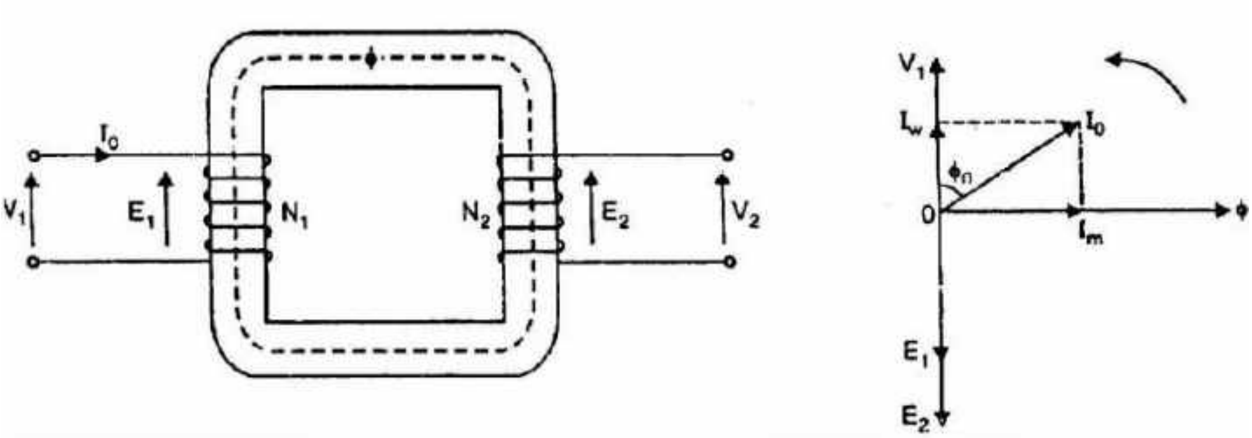


Fig.-2.66 Practical transformer on load

❖ On no load, In practical transformer has iron losses (hysteresis loss) and eddy current loss as it is subjected to the alternating flux.

❖ The no load current I_0 has two components:

1. I_m - Magnetising Component –Sets up flux in the core
2. I_w - Active component – Supplies for core loss

No load input power,

$$W_0 = V_1 I_0 \cos\Phi_0$$

As seen from the phasor diagram, the no-load primary current I_0 can be resolved into two rectangular components viz. I_w and I_m

(i) The component I_w in phase with the applied voltage V_1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_w = I_0 \cos\Phi_0$$

(ii) The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is the component which produces the mutual flux Φ in the core.

$$I_m = I_0 \sin\Phi_0$$

Clearly, I_0 is phasor sum of I_m and I_w.

$$I_0 = \sqrt{(I_m^2 + I_w^2)}$$

No load power factor,

$$\cos\Phi_0 = I_w/I_0$$

It is emphasized here that no load primary copper loss (i.e. $\mathbf{I_0^2 R_1}$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,
No load input power, W_0 = Iron loss

Note:

At no load, there is no current in the secondary so that $V_2 = E_2$. On the primary side, the drops in R_1 and X_1, due to I_0 are also very small because of the smallness of I_0.
Hence, we can say that at no load, $V_1 = E_1$.

2.27.1TRANSFORMER ON LOAD:-

(MMF Balancing on Load)

❖ When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding.

❖ The secondary current induces the MMF, N_2I_2 on the secondary winding of the transformer.

❖ This force set up the flux φ_2 in the transformer core. The flux φ_2 opposes the flux φ, according to **Lenz's law**.

❖ As the flux φ_2 opposes the flux φ, the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1.

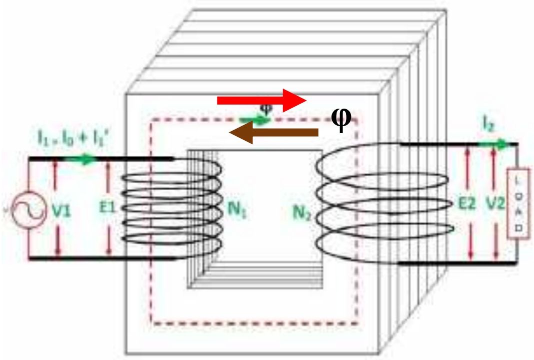


Fig.-2.67 Transformer on load

❖ Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply.

❖ The additional current is used for restoring the original value of the flux in the core of the transformer so that $V_1 = E_1$.

❖ The primary current I'_1 is in phase opposition with the secondary current I_2. Thus, it is called the primary counter-balancing current.

TRANSFORMER ON LOAD

(MMF Balancing on Load)

❖ The additional current I'_1 induces the MMF $N_1I'_1$. And this force set up the flux φ_1

❖ The direction of the flux is the same as that of the φ and it cancels the flux φ_2 which induces because of the MMF N_2I_2

Now,

$$N_1I_1' = N_2I_2$$

Therefore,

$$I_1' = I_2\left(\frac{N_2}{N1}\right) = KI_2$$

❖ The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.

❖ The power factor of the secondary side depends upon the type of load connected to the transformer.

❖ If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the currents I_0 and I_1'. i.e

$$\vec{I} = \vec{I_0} + \vec{I_1'}$$

- ❖ When the load is connected ; secondary current I_2 flows.
- ❖ To counter balance its effect , current I_1' flows in primary.
- ❖ Hence resultant current in primary;
- ❖ Current I_o is hardly 2 to 5 % of full load primary current.
- ❖ Copper losses also occur in addition with iron losses.

2.27.2 STEPS TO DRAW THE PHASOR DIAGRAM:-

- ❖ Take flux ϕ, a reference
- ❖ Induces EMF E_1 and E_2 lags the flux by 90 degrees.
- ❖ The component of the applied voltage to the primary equal and opposite to induced EMF in the primary winding. E_1 is represented by V_1'.
- ❖ Current I_0 lags the voltage V_1' by 90 degrees.
- ❖ The power factor of the load is lagging. Therefore current I_2 is drawn lagging E_2 by an angle ϕ_2.
- ❖ The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phase difference of E_2 and voltage drop.
- ❖ $V_2 = E_2$ – voltage drops $I_2 R_2$ is in phase with I_2 and I_2X_2 is in quadrature with I_2.
- ❖ The total current flowing in the primary winding is the phasor sum of I_1' and I_0.
- ❖ Primary applied voltage V_1 is the phasor sum of $V_1'$and the voltage drop in the primary winding.
- ❖ Current I_1' is drawn equal and opposite to the current I_2
- ❖ $V_1 = V_1'$+ voltage drop I_1R_1 is in phase with I_1 and I_1X_I is in quadrature with I_1.
- ❖ The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- ❖ The power factor of the secondary side depends upon the type of load connected to the transformer.
- ❖ If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading.
- ❖ Where I_1R_1 is the resistive drop in the primary windings I_2X_2 is the reactive drop in the secondary winding

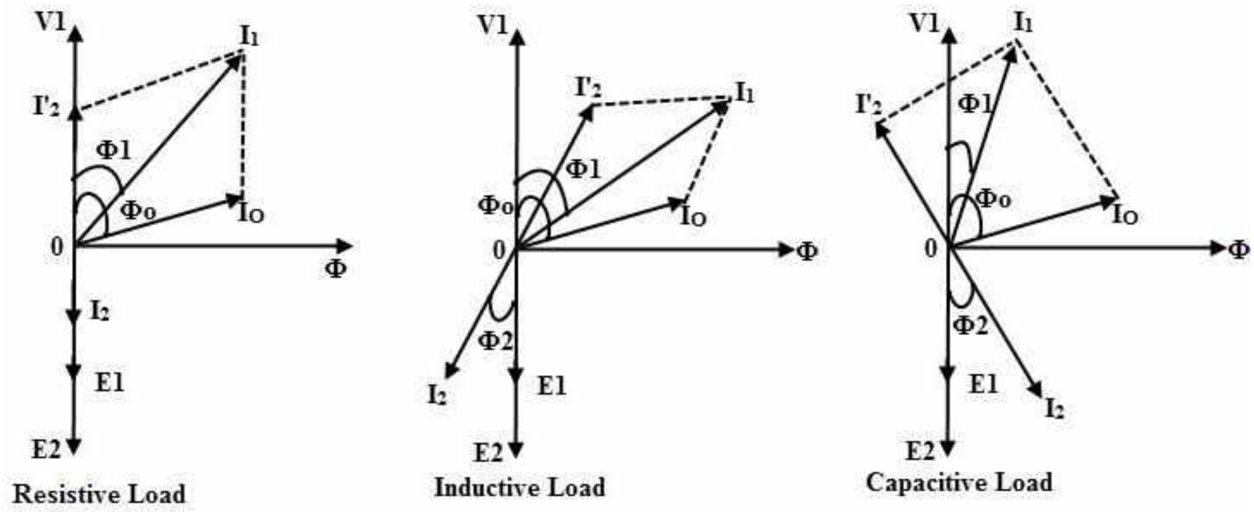


Fig.-2.68 Phasor diagram on full load

2.28 EQUIVALENT CIRCUIT

- ❖ Equivalent impedance of transformer is essential to be calculated because the electrical power transformer is an electrical power system equipment for estimating different parameters of the electrical power system which may be required to calculate the total internal impedance of an electrical power transformer, viewing from primary side or secondary side as per requirement.

❖ This calculation requires equivalent circuit of transformer referred to the primary or equivalent circuit of transformer referred to secondary sides respectively. Percentage impedance is also a very essential parameter of the transformer.

❖ Special attention is to be given to this parameter during installing a transformer in an existing electrical power system. Percentage impedance of different power transformers should be properly matched during parallel operation of power transformers.

❖ The percentage impedance can be derived from the equivalent impedance of the transformer so, it can be said that the equivalent circuit of the transformer is also required during the calculation of the % impedance.

2.28 1 Exact Equivalent Circuit

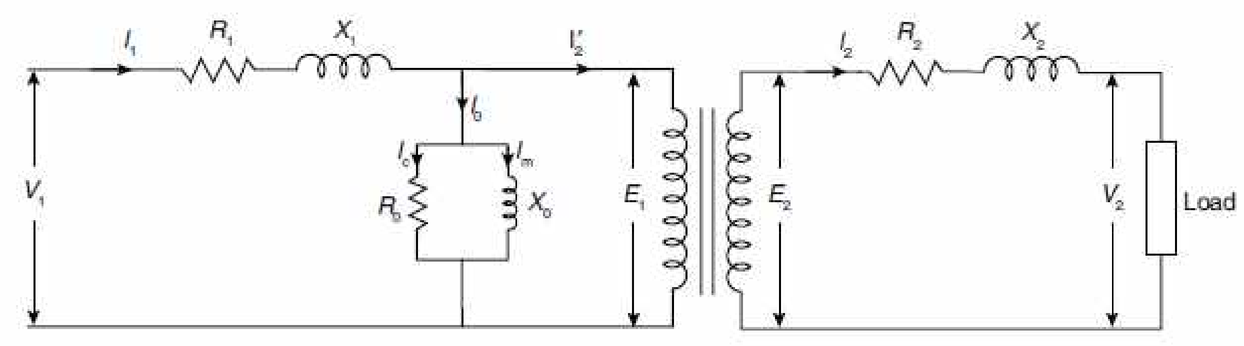


Fig.-2.69 Exact equivalent circuit

- ❖ Exact equivalent circuit is drawn as shown in figure below.
- ❖ Primary & secondary windings are electrically separated.
- ❖ For analysis purpose it is electrically connected by referring one side to another side.
- ❖ Generally primary is referred to secondary.
- ❖ Secondary values get changed when referred to primary.
- ❖

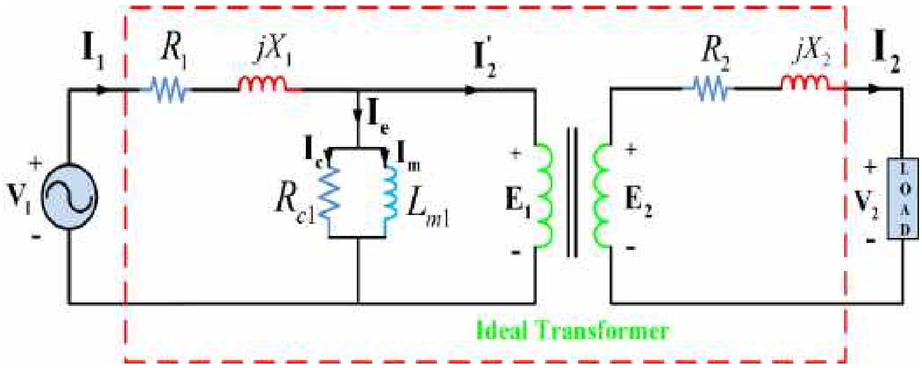


Fig.-2.70 Exact equivalent circuit referred to primary

Exact Equivalent Circuit Referred to Primary

In this case, to draw the equivalent circuit of the transformer all the quantities are to be referred to the primary as shown in the figure below:

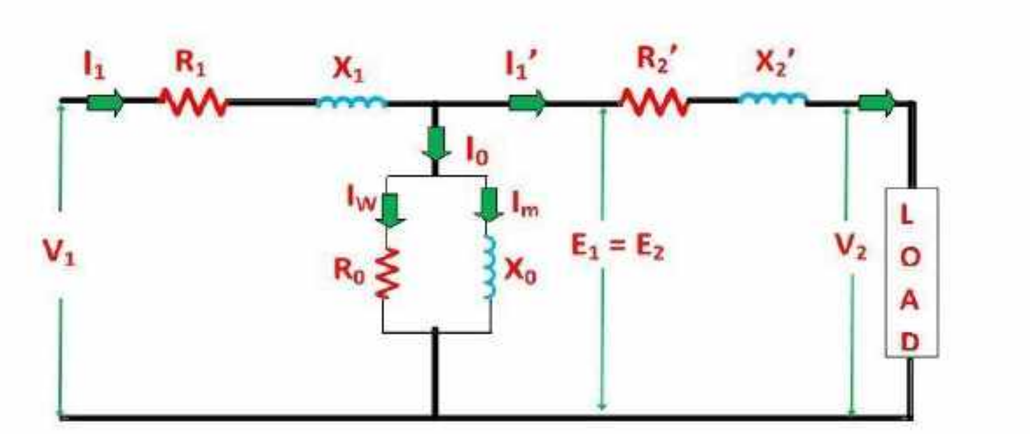


Fig.-2.71 Exact equivalent circuit referred to primary

The following are the values of resistance and reactance given below: –

- ❖ Secondary resistance referred to the primary side is given as:

$$R_2' = \frac{R_2}{K^2}$$

- ❖ The equivalent resistance referred to the primary side is given as:

$$R_{eq} = R_1 + R_2'$$

- ❖ Secondary reactance referred to the primary side is given as:

$$X_2' = \frac{X_2}{K^2}$$

- ❖ The equivalent reactance referred to the primary side is given as:

$$X_{eq} = X_1 + X_2'$$

Exact Equivalent Circuit Referred to Secondary:-

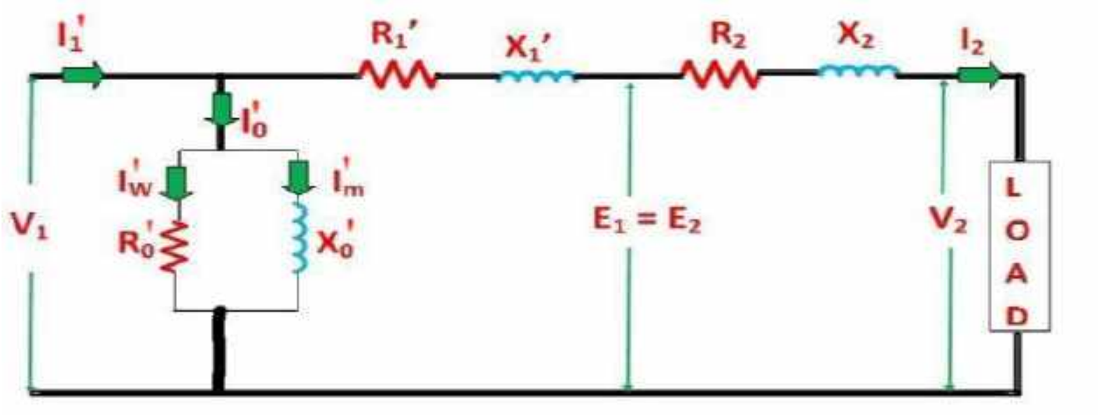


Fig.-2.72 Exact equivalent circuit referred to secondary

<u>Exact Equivalent Circuit Referred to Secondary</u>

The following are the value of the resistance and reactive gives below:

- Primary resistance referred to the secondary side is given as:

$$R_1' = K^2 R_1$$

- The Equivalent resistance referred to secondary side is given as:

$$R_{2eq} = R_2 + R_1'$$

- Primary Reactance referred to secondary side is given as:

$$X_1' = K^2 X_1$$

- The Equivalent reactance referred to secondary side is given as:

$$X_{eq} = X_2 + X_1'$$

NOTE: No Load current I_0 is hardly 3 to 5% of full load rated current, the parallel branch consisting of resistance R_0 and reactance X_0 can be omitted without introducing any appreciable error in the behavior of transformer under the loaded condition.

Long Questions

Q1. A 400/200 V single phase transformer has primary winding resistance 1.0 Ω and secondary winding resistance 0.2 Ω. What will be total resistance of the transformer referred to the primary side?

(AKTU 2015-2016)

Soultion: $R_1 = 1.0 \ \Omega$, $R_2 = 0.2 \ \Omega$

Transformation ratio,

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{200}{400} = 0.5$$

Secondary winding resistance referred to primary side,

$$R_2' = \frac{R_2}{K^2} = \frac{0.2}{(0.5)^2}$$

$$R_2' = 0.8 \ \Omega$$

Total winding resistance of the transformer referred to primary side,

$$R_{01} = R_1 + R_2'$$

$$R_{01} = 1.0 + 0.8 = 1.8 \ \Omega$$

Q2. A 100 KVA, 2400/240 V, 50 Hz, single phase transformer has the following parameters-primary winding (H.V side); resistance $R_1 = 2.4\,\Omega$, leakage reactance $X_1 = 6.0\,\Omega$. Secondary winding(L.V side):resistance $R_2 = 0.03\,\Omega$, leakage reactance $X_2 = 0.07\,\Omega$. Find the equivalent resistance & leakage reactance referred to secondary.

(AKTU 2018-2019)

Solution: Primary resistance and leakage reactance are shifted to secondary side of the transformer are shown in fig.2.73.

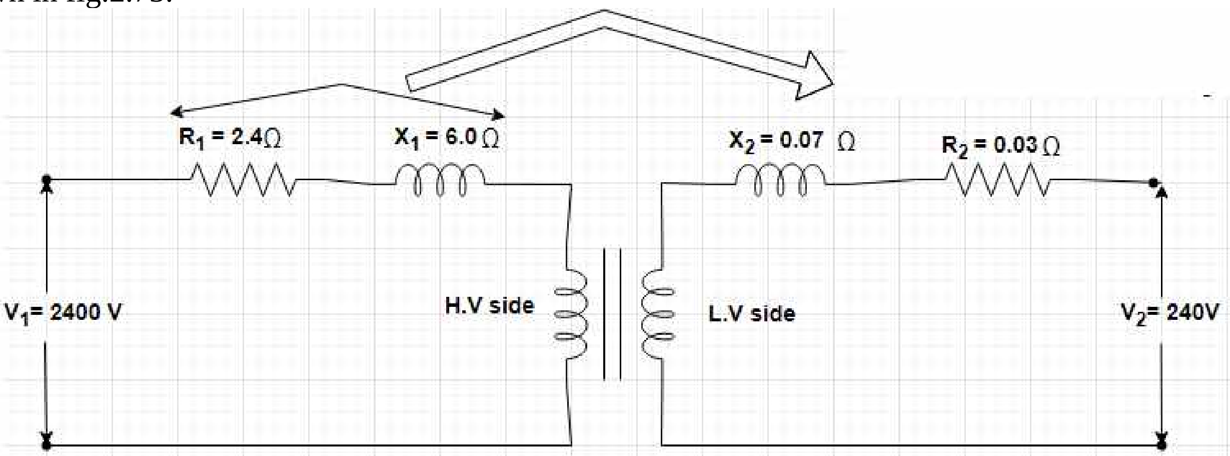


fig.2.73.

Resistance of the primary winding,

$$R_1 = 2.4 \ \Omega$$

Resistance of the secondary winding,

$$R_2 = 0.03 \ \Omega$$

Reactance of the primary winding,

$$X_1 = 6.0 \ \Omega$$

Reactance of the secondary winding,

$$X_2 = 0.07 \ \Omega$$

Transformation ratio of the transformerwould be,

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{240}{2400} = 0.1$$

Equivalent resistance of primary winding referred to secondary side,

$$R_1' = K^2 R_1$$
$$R_1' = (0.1)^2 \times 2.4 \ \Omega$$
$$R_1' = 0.024 \ \Omega$$

Equivalent resistance referred to secondary side,

$$R_{02} = R_2 + R_1'$$
$$R_{02} = 0.03 + 0.024 = 0.054 \ \Omega$$

Equivalent reactance of primary winding referred to secondary side,

$$X_1' = K^2 X_1$$
$$X_1' = (0.1)^2 \times 6 \ \Omega$$
$$X_1' = 0.06 \ \Omega$$

Equivalent reactance referred to secondary side,

$$X_{02} = X_2 + X_1'$$
$$X_{02} = 0.07 + 0.06 = 0.13 \ \Omega$$

Unsolved Problem

Q1. A 100 kVA, 1100/220V, single phase 50Hz transformer has a impedance (0.1+j0.4) Ω for H.V. winding and (0.006+j0.015) Ω for L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.

$$[R_{01} = 0.775\ \Omega, X_{01} = 0.775\ \Omega, Z_{01} = 0.8143\ \Omega, R_{02} = 0.01\ \Omega, X_{02} = 0.031\ \Omega, Z_{02} = 0.326\ \Omega]$$

2.29 Losses in a Transformer:

The losses which occur in an actual transformer are:

(i) Iron or core losses

(ii) Copper losses

Core or iron losses:

When AC supply is given to the primary winding of a transformer an alternating flux is set up in the core, therefore, hysteresis and eddy current losses occur in the magnetic core.

(a) Hysteresis loss:

When the magnetic material is subjected to reversal of magnetic flux, it causes a continuous reversal of molecular magnets. This effect consumes some electric power which is further dissipated in the form of heat as loss. This loss is known as hysteresis loss. This loss can be minimized by using silicon steel material for the construction of core.

$$P_h = K_h V f B_m^{1.6}$$

(b) Eddy current loss:

Since flux in the core of a transformer is alternating, it links with the magnetic material of the core itself also. This induces an emf in the core and circulates eddy currents. Power is required to maintain these eddy currents. This power is dissipated in the form of heat and is known as eddy current loss. This loss can be minimized by making the core of thin laminations. The flux set up in the core of the transformer remains constant from no-load to full load. Hence, iron loss is independent of the load and is known as constant losses.

$$P_e = K_e V f^2 t^2 B_m^2$$

Copper losses:

Copper losses occur in both the primary and secondary windings due to their ohmic resistance. If I_1, I_2 are the primary and secondary currents and R_1, R_2 are the primary and secondary resistances, respectively.

Then total copper losses = $I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{ep} = I_2^2 R_{es}$

Where

R_p = equivalent resistance referred to primary side

R_s = equivalent resistance referred to secondary side

The currents in the primary and secondary winding vary according to the load; therefore, these losses vary according to the load and are known as variable loss.

Long Questions

Q1. Classify the losses in transformer ? (AKTU 2020-2021)

Q2. A 20 KVA, 2000/200 V, single phase, 50 Hz transformer has a primary resistance of 1.5 Ω and leakage reactance of 2 Ω. The secondary resistance and leakage reactance are 0.015 Ω and 0.02 Ω. The no-load current of transformer is 1 A at 0.2 power factor. Determine:

(i) Equivalent resistance, leakage reactance and impedance referred to primary

(ii) Supply current

(iii) Total copper loss

Draw approximate equivalent circuit.

(AKTU 2021-2022)

Solution: Power rating of the transformer,

$$S = 20\ KVA$$

Resistance of the primary winding,

$$R_1 = 1.5\ \Omega$$

Resistance of the secondary winding,

$$R_2 = 0.015\ \Omega$$

Reactance of the primary winding,

$$X_1 = 2\ \Omega$$

Reactance of the secondary winding,

$$X_2 = 0.02\ \Omega$$

Transformation ratio of the transformerwould be,

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

Equivalent resistance of Secondary winding referred to primary side,

$$R_2' = \frac{R_2}{K^2} = \frac{0.015}{(0.1)^2}$$

$$R_2' = 1.5\ \Omega$$

Equivalent reactance of Secondary winding referred to primary side,

$$X_2' = \frac{X_2}{K^2} = \frac{0.02}{(0.1)^2}$$

$$X_2' = 2\ \Omega$$

Equivalent resistance of the transformer referred to primary side,

$$R_{01} = R_1 + R_2'$$

$$R_{01} = 1.5 + 1.5 = 3\ \Omega$$

Equivalent leakage reactance of the transformer referred to primary side,

$$X_{01} = X_1 + X_2'$$

$$X_{01} = 2 + 2 = 4\ \Omega$$

Equivalent impedance of the transformer referred to primary side,

$$Z_{01} = R_{01} + jX_{01}\Omega$$

Substitute the value ofR_{01} *and* X_{01} then equivalent impedance of the transformer referred to primary side would be,

$$Z_{01} = 3 + j4\ \Omega$$

So its magnitude would be,

$$Z_{01} = \sqrt{(3)^2 + (4)^2} = 5\ \Omega$$

(ii) Supply current,

$$S = V_1 \times I_1$$

So,

$$I_1 = \frac{S}{V_1} = \frac{20 \times 10^3}{2000} = 10\ A$$

(iii) The total copper loss load would be,

$$P_{cu} = I_1^2 \times R_{01}$$

So,

$$P_{cu} = (10)^2 \times 3 = 300\ W$$

No-load will be conducted from low voltage side so the shunt parameter of the transformer are shown in fig.2.74.

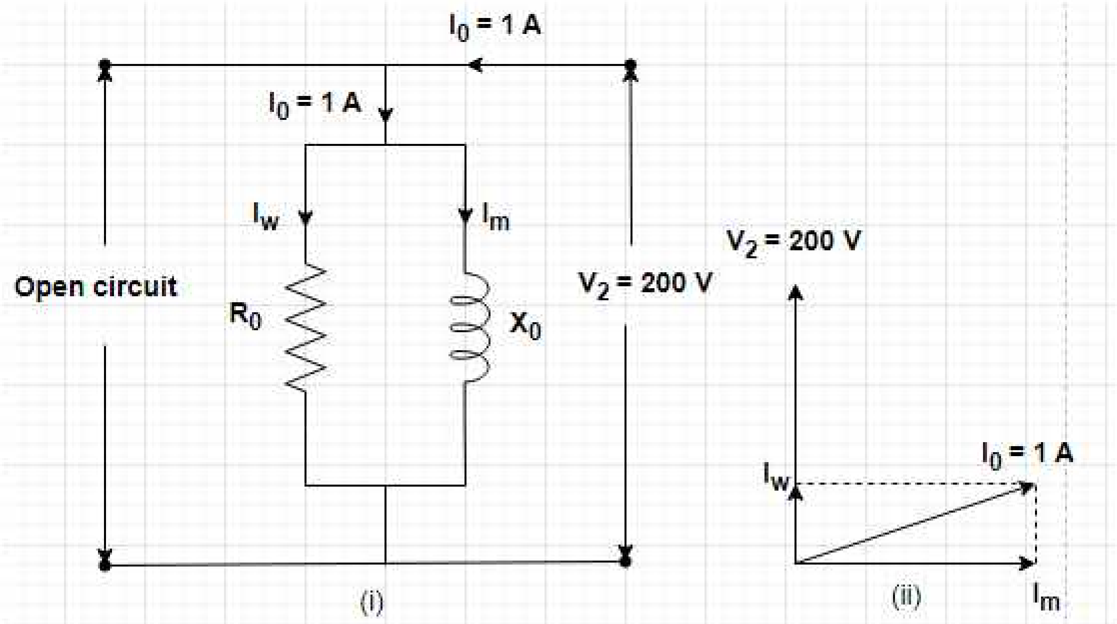


Fig.2.74

Supply voltage at no-load is,

$$V_0 = 200\ V$$

Supply current at no-load is,

$$I_0 = 1\ A$$

Power factor at no-load is,

$$p.f = \cos\varphi_0 = 0.2\ (lagging)$$

From the phasor diagram of the transformer at no-load which is shown in fig.3.21(ii),Core loss current would be,

$$I_w = I_0 \cos\varphi_0$$

So its value would be,

$$I_w = 1 \times 0.2 = 0.2\ A$$

Similarly magnetizing current would be,

$$I_m = I_0 \sin\varphi_0 = I_0\sqrt{1 - cos^2\varphi_0}$$

$$I_m = 1\sqrt{1 - (0.2)^2}$$

$$I_m = 0.98\ A$$

No-load circuit parameters are $R_0 and\ X_{0,}\ hence$

$$R_0 = \frac{V_0}{I_w} = \frac{V_0}{I_0\cos\varphi_0} = \frac{200}{0.2} = 1000\ \Omega$$

Similarly X_0would be,

$$X_0 = \frac{V_0}{I_m} = \frac{V_0}{I_0 \sin \varphi_0} = \frac{200}{0.98} = 204.08\ \Omega$$

Shunt parameter referred to primary side,

$$R_0' = \frac{R_0}{K^2} = \frac{1000}{(0.1)^2}$$

$$R_0' = 100000\ \Omega$$

similarly

$$X_0' = \frac{X_0}{K^2} = \frac{204.08}{(0.1)^2}$$

$$X_0' = 20408\ \Omega$$

Secondary voltage referred to primary side,

$$V_2' = \frac{V_2}{K} = \frac{200}{0.1} = 2000\ V$$

Approximate equivalent circuit diagram referred to primary side is shown in fig.2.75

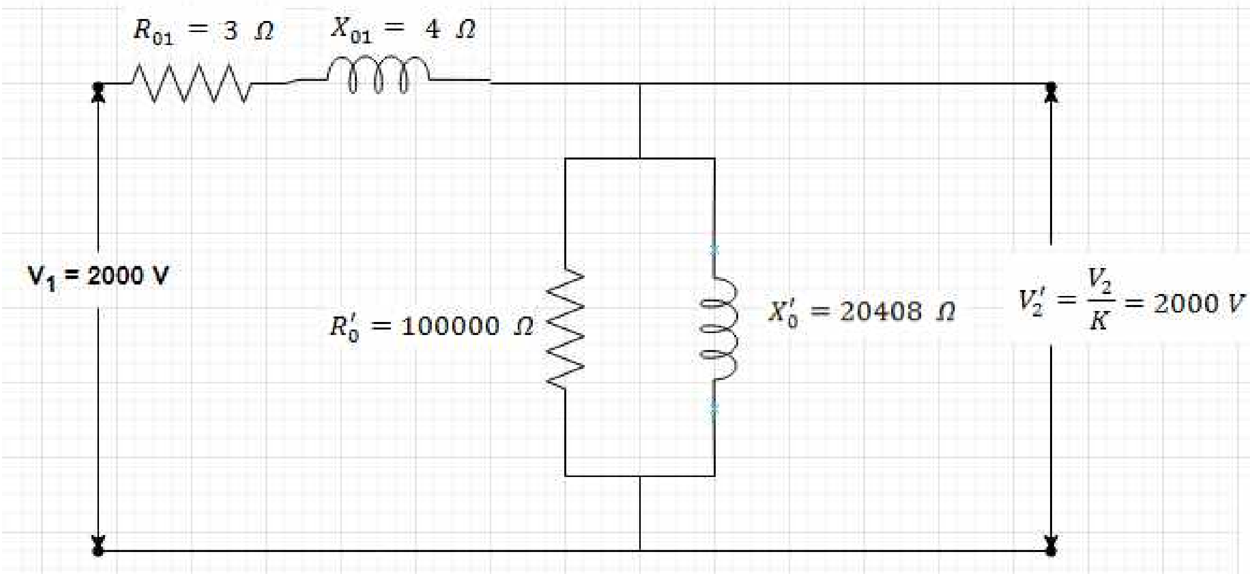


fig 2.75

Q3. A 20 KVA, 2000/200 V, 1-Φ, 50 Hz transformer has a primary resistance of 1.5 Ω and reactance of 2 Ω. The secondary resistance and reactance are 0.015 Ω and 0.02 Ω. The no-load current of transformer is 1 A at 0.2 power factor. Determine:

(i) Equivalent resistance, reactance referred to primary

(ii) Total copper loss

(AKTU 2023-2024)

Solution: Power rating of the transformer,

$$S = 20\ KVA$$

Resistance of the primary winding,

$$R_1 = 1.5\ \Omega$$

Resistance of the secondary winding,

$$R_2 = 0.015\ \Omega$$

Reactance of the primary winding,

$$X_1 = 2\ \Omega$$

Reactance of the secondary winding,

$$X_2 = 0.02\ \Omega$$

Transformation ratio of the transformerwould be,

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

Equivalent resistance of Secondary winding referred to primary side,

$$R_2' = \frac{R_2}{K^2} = \frac{0.015}{(0.1)^2}$$

$$R_2' = 1.5 \ \Omega$$

Equivalent reactance of Secondary winding referred to primary side,

$$X_2' = \frac{X_2}{K^2} = \frac{0.02}{(0.1)^2}$$

$$X_2' = 2 \ \Omega$$

Equivalent resistance of the transformer referred to primary side,

$$R_{01} = R_1 + R_2'$$

$$R_{01} = 1.5 + 1.5 = 3 \ \Omega$$

Equivalent leakage reactance of the transformer referred to primary side,

$$X_{01} = X_1 + X_2'$$

$$X_{01} = 2 + 2 = 4 \ \Omega$$

Rated primary current,

$$S = V_1 \times I_1$$

So,

$$I_1 = \frac{S}{V_1} = \frac{20 \times 10^3}{2000} = 10 \ A$$

(ii) The total copper loss load would be,

$$P_{cu} = I_1^2 \times R_{01}$$

So,

$$P_{cu} = (10)^2 \times 3 = 300 \ W$$

Unsolved Problem

Q1. A 15 kVA, 220/110V Single phase transformer has R_1=1.75 Ω, R_2=0.0045 Ω. The leakage reactance are X_1=2.6 Ω, X_2=0.0075 Ω. Calculate:

a) Equivalent resistance and reactance referred to primary.
b) Equivalent resistance and reactance referred to secondary.
c) Equivalent impedance referred to primary and secondary
d) Total copper loss

[(a)3.55 Ω,5.6 Ω (b)0.0089 Ω, 0.014 Ω(c) 6.63 Ω, 0.017 Ω (d)165.027 W]

2.30 Efficiency of a Transformer:

The efficiency of a transformer is defined as the ratio of output to the input power, the two being measured in same units (either in watts or in kW).

Transformer efficiency,

η=output power/input power

$$\eta = \frac{(\text{Output power})}{(\text{output power} + \text{losses})}$$

$$\eta = \frac{(\text{Output power})}{(\text{output power} + \text{iron loss} + \text{copper loss})}$$

$$\eta = \frac{V_2 I_2 \cos\varphi}{(V_2 I_2 \cos\varphi + P_i + P_{cu})}$$

Where,

V_2=Secondary terminal voltage

I_2=Full load secondary current
$\cos\emptyset$ = power factor of load

P_i= Iron loss = Hysterisis losses + eddy current losses
P_{cu}= Full load copper loss = $I^2{}_2R_{eq}$
If x is the fraction of the full load, the efficiency of the transformer at this fraction is given by the relation:
η= x.output at full load / (x.output at full load + P i+ x^2P_{cu})

$$\eta = \frac{x\, V_2 I_2 \cos\varphi}{x\, V_2 I_2 \cos\varphi + P_i + x^2 P_{cu}}$$

The copper losses vary as the square of the fraction of the load

2.30.1 Condition for Maximum Efficiency:

As secondary voltage of transformer is approximately constant for given load power factor,so only secondary current is variable and x is variable part of secondary current, hence differentiating efficiency with respect to x and equating to zero .

$$\eta = \frac{x\, V_2 I_2 \cos\varphi}{x\, V_2 I_2 \cos\varphi + P_i + x^2 P_{cu}}$$

$$\frac{d\eta}{dx} = 0$$

Hence finally

$$\mathbf{x^2P_{cu} = P_i}$$

Copper loss = iron loss.

Also, corresponding load for max. efficiency;

$$\boldsymbol{x} = \sqrt{\frac{P_i}{P_{cu}}}$$

Thus, the efficiency of a transformer will be maximum when copper (or variable) losses are equal to iron (or constant) losses.
If x is the fraction of full load kVA at which the efficiency of the transformer is maximum. Then, copper losses = x^2 P_{cu} (where P_{cu} is the full load Cu losses)
Maximum efficiency of a transformer would be,

$$\%\eta_{max} = \frac{x.V_2I_2cos\varphi_2}{x.V_2I_2cos\varphi_2 + 2P_i} \times 100$$

Short Questions

Q1. The efficiency of a transformer is always higher than that of rotating electric machines, why? (AKTU 2019-2020)

Soultion: In rotating electric machines there is additional mechanical losses (frictional and wind age losses) due to the rotating parts. As there is no rotating part in transformer so the efficiency of transformer is always higher than rotating electric machine.

Long Questions

Q1. A 25 KVA, 2000/200 V transformer the iron and copper losses are 200 W and 400 W respectively. Calculate the efficiency at half load and 0.8 p.f lagging. Also determine the maximum efficiency and corresponding load.

(AKTU 2016-2017,2018-2019)

Soultion: Given,
$S = V_2 I_2 = 25\ kVA, \quad P_i = 200\ W, \ P_{cu} = 400\ W, \quad p.f = cos\Phi = 0.8\ (lagging)$
General expression for finding the efficiency of the transformer,

$$\eta = \frac{x\ V_2 I_2 \cos\varphi}{x\ V_2 I_2 \cos\varphi + P_i + x^2 P_{cu}}$$

$$\eta = \frac{xS \cos\varphi}{xS \cos\varphi + P_i + x^2 P_{cu}}$$

In percentage it would be,

$$\%\eta = \frac{xS \cos\varphi}{xS \cos\varphi + P_i + x^2 P_{cu}} \times 100$$

Where x is the load,
efficiency at half load where $x = 1/2\ so,$

$$\eta = \frac{(1/2) \times 25 \times 10^3 \times 0.8}{(1/2) \times 25 \times 10^3 \times 0.8 + 200 + (1/2)^2 \times 400}$$

$$\eta = 0.9709$$

In percentage,

$$\%\ \eta = 0.9709 \times 100 = 97.09\ \%$$

The maximum efficiency of the transformer,

$$\eta_{max.} = \frac{xS \cos\varphi}{xS \cos\varphi + 2P_i}$$

here x equal to,

$$x = \sqrt{\frac{P_i}{P_{cu}}}$$

So load at which transformer gave the maximum eficiency,

$$x = \sqrt{\frac{200}{400}} = 0.707$$

Hence the maximum efficiency of the transformer,

$$\eta_{max.} = \frac{0.707 \times 25 \times 10^3 \times 0.8}{0.707 \times 25 \times 10^3 \times 0.8 + 2 \times 200}$$

$$\eta_{max.} = 0.9725$$

$$\%\ \eta = 0.9755 \times 100 = 97.25\ \%$$

corresponding load at which transformer gave the maximum eficiency in kVA,

$$Load\ in\ KVA = x.S = 0.707 \times 25 = 17.675\ KVA$$

Q2. The maximum efficiency of a 100 KVA, 1100/440 V, 50 Hz transformer is 96%. This occur at 75% of full load at 0.8 p.f lagging. Find the efficiency of transformer at 3/4 full load at 0.6 p.f leading.
(AKTU 2018-2019)

Solution: Given,
$Power\ rating\ of\ transformer, S = 100\ KVA$
$Maximum\ efficiency, \eta_{max.} = 0.96$
$Power\ factor, p.f = 0.8$
$Load\ at\ maximum\ efficiency, x = 0.75$
The maximum efficiency of the transformer,

$$\eta = \frac{x\ V_2 I_2 \cos\varphi}{x\ V_2 I_2 \cos\varphi + 2P_i}$$

$$\eta_{max.} = \frac{xS\cos\varphi}{xS\cos\varphi + 2P_i}$$

So iron loss would be,

$$xS\cos\varphi + 2P_i = \frac{xS\cos\varphi}{\eta_{max.}}$$

$$2P_i = \frac{xS\cos\varphi}{\eta_{max.}} - xS\cos\varphi$$

$$2P_i = \frac{0.75 \times 100 \times 10^3 \times 0.8}{0.96} - 0.75 \times 100 \times 10^3 \times 0.8 = 62.5 \times 10^3 - 60 \times 10^3$$

$$2P_i = 2.5 \times 10^3$$

$$P_i = \frac{2.5 \times 10^3}{2} = 1.25 \times 10^3\ W = 1.25\ KW$$

Now the full load copper would be,

$$x^2.P_{cu} = P_i$$

$$P_{cu} = \frac{P_i}{x^2} = \frac{1.25}{(0.75)^2} = 2.22\ KW$$

General expression for finding the efficiency of the transformer,

$$\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2P_{cu}}$$

In percentage it would be,

$$\%\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2P_{cu}} \times 100$$

Efficiency at 3/4 load at 0.6 leading power factor, here $x = 3/4$ *so,*

$$\eta = \frac{(3/4) \times 100 \times 10^3 \times 0.6}{(3/4) \times 100 \times 10^3 \times 0.6 + 1250 + (3/4)^2 \times 2222}$$

$$\eta = 0.9474$$

In percentage,

$$\%\,\eta = 0.9474 \times 100 = 94.74\ \%$$

Q3. A 25 KVA, 2000/200 V transformer, the constant and variable losses are 350 W and 400 W respectively. Calculate the efficiency on unity power factor at (i) Full load (ii) Half load.

(AKTU 2019-2020)

Solution: Given,

$S = 25\ KVA,\quad P_i = constant\ loss = 350\ W,\ \ P_{cu} = variable\ loss = 400\ W,$

The efficiency of the transformer,

$$\eta = \frac{x\,V_2I_2\cos\varphi}{x\,V_2I_2\cos\varphi + P_i + x^2P_{cu}}$$

$$\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2P_{cu}}$$

In percentage it would be,

$$\%\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2P_{cu}} \times 100$$

Where x is the load,

(i) Efficiency at full load at unity power factor, where $x = 1$ *so,*

$$\eta = \frac{(1) \times 25 \times 10^3 \times 1}{(1) \times 25 \times 10^3 \times 1 + 350 + (1)^2 \times 400}$$

$\eta = 0.9709$

In percentage,

$$\% \, \eta = 0.9709 \times 100 = 97.09 \, \%$$

(ii) Efficiency at half load at unity power factor where $x = 1/2$ so,

$$\eta = \frac{(1/2) \times 25 \times 10^3 \times 1}{(1/2) \times 25 \times 10^3 \times 1 + 350 + (1/2)^2 \times 400}$$

$$\eta = 0.9653$$

In percentage,

$$\% \, \eta = 0.9653 \times 100 = 96.53 \, \%$$

Q4. A 100 KVA, 6.6 KV/230 V, 50 Hz transformer has 90% efficiency at 0.8 lagging power factor at full load and also at half load. Determine iron loss and copper loss at full load for transformer.

(AKTU 2020-2021)

Solution: Given,

$S = 100 \, KVA,$ $P_i = Iron \ loss,$ $P_{cu} = Copper \ loss \ at \ full \ load$, $\%\eta_{FL} = \%\eta_{HL} = 90\%$

The efficiency of the transformer,

$$\eta = \frac{x \, V_2 I_2 \cos\varphi}{x \, V_2 I_2 \cos\varphi + P_i + x^2 P_{cu}}$$

In percentage it would be,

$$\%\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2 P_{cu}} \times 100$$

Where x is the load,

Efficiency at full load at 0.8 lagging power factor, where $x = 1$ so,

$$\%\eta_{FL} = \frac{(1) \times 100 \times 10^3 \times 0.8}{(1) \times 100 \times 10^3 \times 0.8 + P_i + (1)^2 P_{cu}} \times 100$$

$$90 = \frac{(1) \times 100 \times 10^3 \times 0.8}{(1) \times 100 \times 10^3 \times 0.8 + P_i + (1)^2 P_{cu}} \times 100$$

Rearrange the above equation,

$$(1) \times 100 \times 10^3 \times 0.8 + P_i + (1)^2 P_{cu} = \frac{(1) \times 100 \times 10^3 \times 0.8}{90} \times 100$$

$$P_i + (1)^2 P_{cu} = \frac{(1) \times 100 \times 10^3 \times 0.8}{90} \times 100 - (1) \times 100 \times 10^3 \times 0.8$$

$$P_i + (1)^2 P_{cu} = \frac{(1) \times 100 \times 10^3 \times 0.8}{90} \times 100 - (1) \times 100 \times 10^3 \times 0.8$$

$$P_i + P_{cu} = 88.888 \times 10^3 - 80 \times 10^3$$

$$P_i + P_{cu} = 8.888 \times 10^3$$

Hence,

$$P_i + P_{cu} = 8.888 \, KW \qquad (1)$$

Efficiency at half load at 0.8 lagging power factor, where $x = 1/2$ so

$$\%\eta_{HL} = \frac{(1/2) \times 100 \times 10^3 \times 0.8}{(1/2) \times 100 \times 10^3 \times 0.8 + P_i + (1/2)^2 P_{cu}}$$

$$90 = \frac{(1/2) \times 100 \times 10^3 \times 0.8}{(1/2) \times 100 \times 10^3 \times 0.8 + P_i + (1/2)^2 P_{cu}} \times 100$$

Rearrange the above equation,

$$(1/2) \times 100 \times 10^3 \times 0.8 + P_i + (1/2)^2 P_{cu} = \frac{(1/2) \times 100 \times 10^3 \times 0.8}{90} \times 100$$
$$(1/2) \times 100 \times 10^3 \times 0.8 + P_i + (1/2)^2 P_{cu} = 44.444 \times 10^3$$
$$P_i + (1/2)^2 P_{cu} = 44.444 \times 10^3 - 40 \times 10^3$$
$$P_i + (1/2)^2 P_{cu} = 4.444 \times 10^3$$

Hence,

$$P_i + \frac{1}{4} P_{cu} = 4.444\ KW \qquad (2)$$

By solving equation (1) and equation (2),Then we have,

$$P_i = 2.963\ KW\ \& P_{cu} = 5.925\ KW$$

Q5. A 100 KVA single phase transformer has an iron loss of 600 W and a copper loss of 1.5 KW at full load current. Calculate the efficiency at
(i) Full load and 0.8 lagging p.f, and
(ii) Half load at unity p.f

(AKTU 2022-2023, 2022-2023,2023-2024)

Solution: Given,
$S = 100\ KVA,\quad P_i = 600\ W,\ P_{cu} = 1.5\ KW = 1500\ W$

$$\eta = \frac{x\ V_2 I_2 \cos\varphi}{x\ V_2 I_2 \cos\varphi + P_i + x^2 P_{cu}}$$
$$\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2 P_{cu}}$$
$$\%\eta = \frac{xS\cos\varphi}{xS\cos\varphi + P_i + x^2 P_{cu}} \times 100$$

Where x is the load,
(i) Efficiency at full load at 0.8 lagging power factor, where $x = 1\ so,$

$$\eta = \frac{(1) \times 100 \times 10^3 \times 0.8}{(1) \times 100 \times 10^3 \times 0.8 + 600 + (1)^2 \times 1500}$$
$$\eta = 0.9744$$

In percentage,

$$\%\ \eta = 0.9744 \times 100 = 97.44\ \%$$

(ii) Efficiency at half load at unity power factor where $x = 1/2\ so,$

$$\eta = \frac{(1/2) \times 100 \times 10^3 \times 1}{(1/2) \times 100 \times 10^3 \times 1 + 600 + (1/2)^2 \times 1500}$$
$$\eta = 0.9809$$

In percentage,

$$\%\ \eta = 0.9809 \times 100 = 98.09\ \%$$

Unsolved Problems

Q1. The efficiency of a 400kVA, single phase transformer is 98.77% at full load 0.8 power factor lagging and 99.13% at half load unity power factor.
Find:
(i) Iron losses at full load and at half load.
(ii) Cu loss at full load and at half load.

[(i) $P_{1(FL)} = P_{1(HL)} = 1012.022$ W (ii) $P_{cu\,(FL)} = 2972.99$ W, $P_{cu\,(HL)} = 743.25$ W]

Q2. A 250 kVA single phase transformer has Iron loss of 1.8 kW. The full load copper loss is 2000 Watt. Calculate
(i) Efficiency at full load, 0.8 lagging P.F.

(ii) Efficiency at half load, 0.8 lagging P.F.
(iii) kVA supplied at maximum efficiency
(iv) Maximum efficiency of the transformer

[(i) 98.14% (ii)97.57% (iii) 237.17 KVA (iv) 98.18%]

Q3. A transformer is rated at 100kVA, at full load its copper loss, 1400 Watt and iron losses are 940 Watt. Calculate:
(i) The efficiency at full load, unity power factor.
(ii) The efficiency at half-full load, the same power factor and 0.8 power factor lagging.
(iii) The load kVA at which maximum efficiency will occur

[(i) 97.72% (ii)97.49% (iii) 81.94 KVA]

2.31 Voltage regulation:

When a transformer is loaded, with a constant supply voltage, the terminal voltage changes due to voltage drop in the internal parameters of the transformer i.e., primary and secondary resistances and inductive reactance. The voltage drop at the terminals also depends upon the load and its power factor. The change in terminal voltage from no-load to full-load at constant supply voltage with respect to no-load voltage is known as voltage regulation of the transformer.

Let,
E_2 = Secondary terminal voltage at no-load.
V_2 = Secondary terminal voltage at full-load.
Then,

$$\text{voltage regulation} = \frac{E_2 - V_2}{E_2} \; (per\ unit)$$

In the form of percentage,

$$\%\ \text{Reg} = \frac{E_2 - V_2}{E_2} \times 100$$

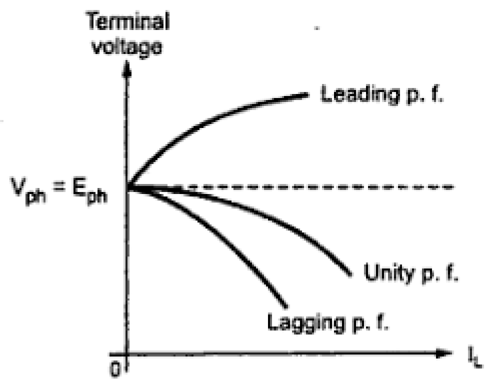


Fig 2.76 Voltage regulation curve

Approximate Expression for Voltage Regulation:-

(i) For resistive load % Regulation = % resistance drop
(ii) For capacitive load % Regulation = % resistance drop × cos ϕ_2 - % reactance drop × sin ϕ_2
(iii) For inductive load % Regulation =% resistance drop × cos ϕ_2 + % reactance drop × sin ϕ_2

Note: For resistive load voltage regulation is positive.
For inductive load voltage regulation is positive.
For capacitive load voltage regulation can be zero or negative.

Impoerant Question: 7 Marks

Q1. A $\frac{250}{125}$ V, 5 KVA single phase transformer has primary resistance of 0.2 Ω and a reactance of 0.75 Ω. The secondary resistance is 0.05 Ω and reactance is 0.2 Ω. Determine:-
(i) Its voltage regulation while supplying full load on 0.8 leading power factor.
(ii) The secondary terminal voltage on full load and 0.8 leading power factor.
(iii) Determine % resistive and reactive drops? (AKTU 2017-18)

(Solution) Given :- R_1= 0.2 Ω, X_1= 0.75 Ω, R_2=0.05 Ω, X_2= 0.2 Ω, cosφ= 0.8 (leading)

$$(I_{2F.L.}) = \frac{KVA}{V_2} = \frac{5\times 10^3}{125} = 40\ A$$

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{125}{250} = 0.5$$

For leading power factor $E_2 < V_2$.

$$R_{2e}=R_2+K^2R_1 = 0.05 + (0.5)^2x0.2 = 0.1\ \Omega$$

$$X_{2e}=X_2+K^2X_1 = 0.2 + (0.5)^2x0.75 = 0.3875\ \Omega$$

(i) cosφ= 0.8

$$\%\ V.R.= \frac{I_2[R_{2e}\cos\emptyset - X_{2e}\sin\emptyset]}{V_2}\times 100$$

$$\%\ V.R. = \frac{40[0.1\times 0.8 - 0.3875\times 0.6]}{125}\times 100$$

%V.R.= -4.88%

(ii) For leading power factor

$$E_2 = V_2 + I_2R_{2e}\cos\varphi - I_2X_{2e}\sin\varphi$$

$$V_2 = E_2 - I_2R_{2e}\cos\varphi + I_2X_{2e}\sin\varphi$$

V_2= 125 - 40[0.1x0.8 – 0.3875x0.6]

V_2= 125 - 40[-0.1525]

V_2= 125 – (-6.1)

V_2= 131.1 V

(iii) % Resistive drop

$$\%\ R = \frac{I_2R_{2e}\cos\emptyset}{V_2}\times 100$$

$$\%\ R = \frac{40\times 0.1\times 0.8}{125}\times 100$$

$$\%\ R = 2.56\ \%$$

% Reactive drop

$$\%\ X = \frac{I_2X_{2e}\cos\emptyset}{V_2}\times 100$$

$$\%\ X = \frac{40\times 0.3875\times 0.6}{125}\times 100$$

$$\%\ X = 7.44\ \%$$

Short Questions

Q1. Define the voltage regulation of a transformer. (AKTU 2019-2020)

Q2. What is the nature of load for negative voltage regulation in the transformer? (AKTU2021-2022)

Unsolved Problem

Q1. Calculate the regulation of a transformer in which ohmic losses are 1%and reactance drop 5% respectively. When the power factor is (i) 0.8 lagging (ii) 0.8 leading power factor (iii) unity power factor.

[(i) 3.8% (ii) -2.2%(iii)1%]