What do we study Introduction to Mechanics?

Force analysis and the resultant force is studied to understand how different forces interact within a system and predict the effects of these interactions on an object's motion or equilibrium.

Why do we need to study Introduction to Mechanics?

Force analysis involves identifying and quantifying the various forces acting on an object, such as gravity, friction, tension, and normal forces.

Where do we use Introduction to Mechanics?

The study of force analysis and resultant force is fundamental in solving practical problems by providing insights into how objects will respond to various forces and ensuring the reliability of designs in real-world applications.

3.1 Force: Force is an external agent, which when acting on a body changes or tends to change, the state of rest or uniform motion of the body.
In other word, the rate of change of momentum is called force. It is denoted by F.
According to Newton's second law of motion
Force= mass x acceleration

$$F = ma = m\frac{dv}{dt} = \frac{d}{dt}(mv),$$

$$F = \frac{dP}{dt} ....(1)$$

Where P is the momentum. From the above equation (1), It can be concluded that force is the rate of change of momentum. The concept of force is essential as an agency which changes or tends to change condition of rest or of uniform motion of the body.

In general, there are two types of forces:
(i) tensile force (ii) compressive force

(i) Tensile force: It is an axial force which is acting outward and coincides with the neutral axis of the component. Tensile force tries to elongate the component. It means tensile force increases the length of the component. Tensile force is positive force.

(ii) Compressive force: It is an axial force which is acting towards and coincides with the neutral axis of the component. Compressive force tries to contract the component. It means compressive force decreases the length of the component

3.2 Characteristics of Force:
A force is a vector quantity and it is completely defined by the following parameters
(i) Magnitude (ii) Point of application (iii) Line of action (iv) Direction

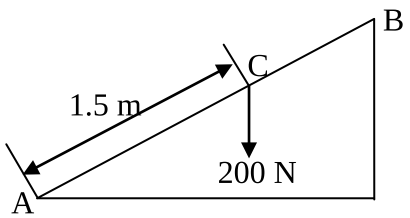


Fig 3.1 Ladder AB

With reference to above Fig. 3.1, ladder AB is at rest against a wall and a person of weight 200N stands at point c on the ladder. The force applied by the person on the has the following characteristics

(a) Magnitude is 200N

(b) The point of application is at C which is 1.5m from the floor along the ladder.

(c) The line of action of force is vertical

(d) The direction is downward

3.3 Principle of transmissibility:

The conditions of equilibrium or motion of a rigid body remains unchanged if a force acting on a given point of the rigid body is replaced (or shifted) by a force of same magnitude and direction but acting to any other point along the same line of action of the force. This principle is applicable only for rigid body

Let F be the force acting on a rigid body at point (a) as shown in Fig. 3.2 According to the law of transmissibility of forces, this force has the same effect on the state of the body if force F is applying at point (b). It means the force F has shifted from point ‘a’ to another point ‘b’ along the same line of action of force, it does not change the condition of equilibrium of body.

According to this principle, **any force acting at a point on a rigid body can be transmitted to act at any other point along its line of action without changing its effect on the rigid body**

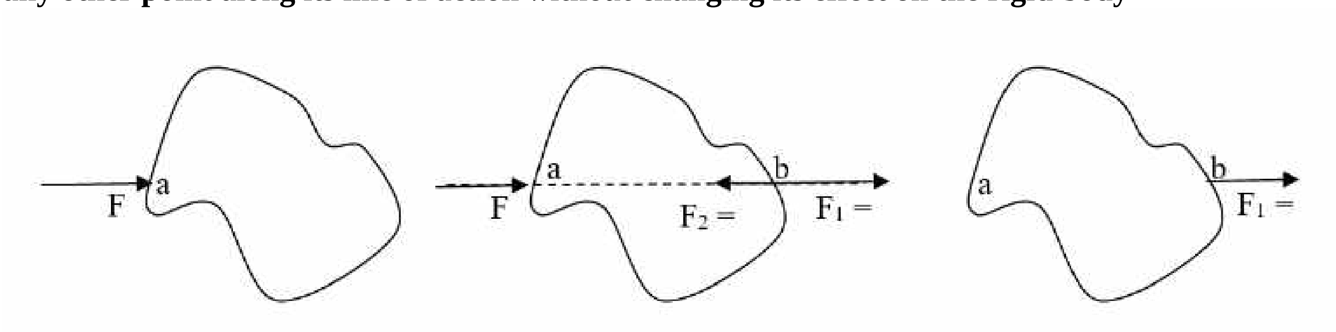


Fig 3.2 Principle of transmissibility

3.4 Force system

When several forces act on a body, then they are called a force system. Force systems are also known as system of forces. If all the forces in a system do not lie in a single plane they constitute the system of forces in space. A force system may be co planar or non-coplanar.

If in a system, all forces lie in the same plane then force system is called as coplanar force system. But if in a system, all the forces lie in a different plane, and then the force system is known as non co planar force system. Depending upon the orientation of the forces and position of the line of action of forces, system of forces can be classified as shown-

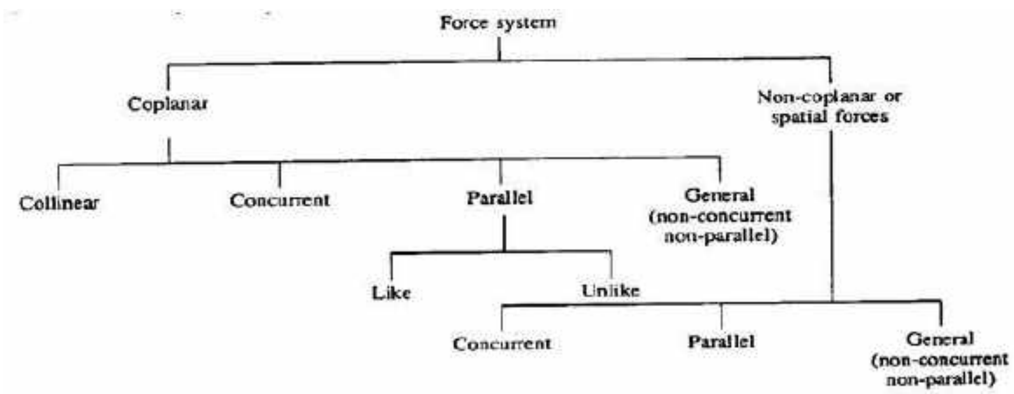


Fig. 3.3 Classification of Force System

- **(i)** **Coplanar forces:** If all the forces in a system lie in a single plane, it is called a coplanar force.
- **(ii)** **Non coplanar forces:** if all the forces in a system lie in different planes, it is called a non-coplanar force.
- **(iii)** **Concurrent forces:** If the lines of action of all the forces in a system pass through a single point it is called a concurrent force.
- **(iv)** **Non concurrent forces:** If the line of action of all the forces in a system does not meet at a single point, it is called non concurrent forces
- **(v)** **Co-linear forces:** The lines of action of all forces are acting along the same straight line
- **(vi)** **Coplanar concurrent forces:** all forces lie in the same plane, having different directions but their lines of action act at a single point
- **(vii)** **Coplanar non concurrent forces:** All the forces lie in the same plane but their lines of action do not pass through a single point
- **(viii)** **Non coplanar concurrent forces:** All the forces do not lie in the same plane but their line of action pass through a single point
- **(ix)** **Non coplanar & non concurrent forces:** All the forces do not lie in the same plane and their line of action do not pass through a single point
- **(x)** **Parallel forces:** If the lines of action of all the forces are parallel to each other. The force system is called parallel force system. If the line of action is in the same direction, such types of forces are called like parallel forces. If the lines of action of forces are in different direction, such type of forces is called unlike parallel force

3.5 Law of Forces:

The following laws are used to determine of the resultant of some forces acting simultaneously on a particle

3.5.1 Parallelogram law of forces:

It states as “If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.”

The value of resultant can be determined either graphically or analytically as below:

Let two forces *P* and *Q* acting simultaneously on a particle be represented in magnitude and direction by the adjacent sides *oa* and *ob* of a parallelogram *oacb* drawn from a point *o*, their resultant *R* will be represented in magnitude and direction by the diagonal *oc* of the parallelogram.

Draw vectors oa and ob to represent to some convenient scale the forces P and Q in magnitude and direction. Complete the parallelogram oacb by drawing ac parallel to ob and bc parallel to oa. The vector oc measured to the same scale will represent the resultant force R.

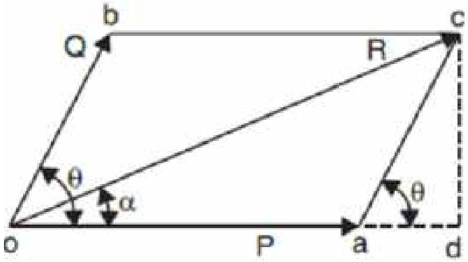


Fig 3.4 Parallelogram law

Resultant R of the forces will be calculated by $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

3.5.2. Triangle law of forces:

It states as under: "If two forces acting simultaneously on a body are represented in magnitude and direction by the two sides of triangle taken in order then their resultant may be represented in magnitude and direction by the third side taken in opposite order."

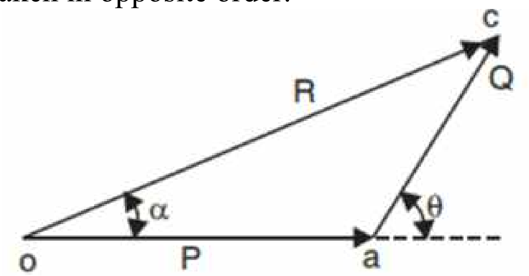


Fig 3.5 Triangle law of forces

$$\frac{P}{\sin(\theta-\alpha)} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\theta}$$

3.5.3. Polygon law of forces:

"If a number of coplanar concurrent forces, acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant may be represented in magnitude and direction by the closing side of a polygon, taken in the opposite order".

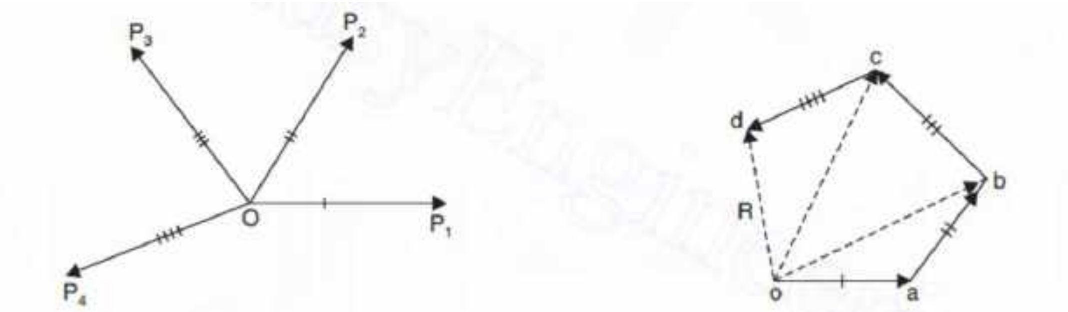


Fig 3.6 Polygon law of forces

3.6 Resultant of concurrent coplanar forces:

All the forces lie in the same plane and their line of action passes through a single point is called concurrent co planar forces. We will consider the following cases:

(i) When two forces act at a point
(ii) When more than two forces act at a point

When more than two forces act at a point:

a) **Analytical method:**

The resultant of forces can be calculated by method of resolution as discussed below

i. Resolve all the forces along horizontally and find out the algebraic sum of all the horizontal components i.e. ΣF_x
ii. Resolve all the forces along vertically and find out the algebraic sum of all the vertical components i.e. ΣF_Y
iii. Resultant (R) of the forces will be calculated by
$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
iv. The resultant force will be inclined at an angle α

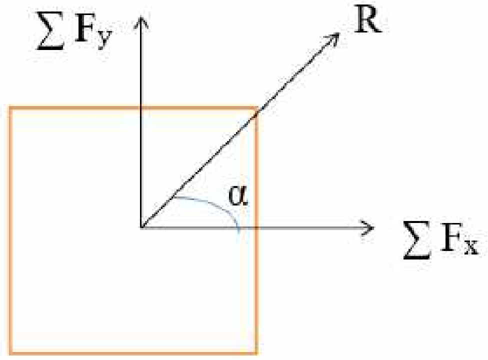


Fig 3.7 Resolution of Force

b) Graphical method:

The resultant of several forces acting at a point is found graphically with the help of the polygon law of forces. Polygon law states that "If several co planar forces are acting at appoint such that they can be represented in magnitude and direction by the sides of a polygon taken in same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in opposite order

Let the three forces F_1, $F_2$and F_3 act at a point. The resultant is calculated graphically by drawing polygon of forces by following steps

(i) Choose a suitable scale to represent the given forces

(ii) Take any point a. from a, draw a vector ab parallel to OF_1
Cut ab= force F_1 to the scale

(iii) From point b, draw line bc parallel to OF_2. Cut bc=Force F_2

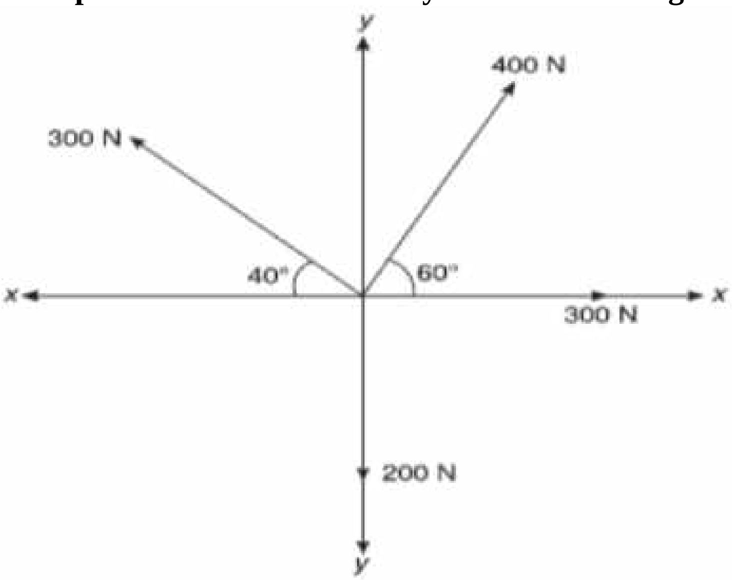
(iv) From point c, draw line cd parallel to OF_3. Cut cd=force F_3

(v) Join point a tod. This is closing side of the polygon. Hence ad represents the resultant of forces of same magnitude and direction
Magnitude of resultant R=length ad x suitable scale

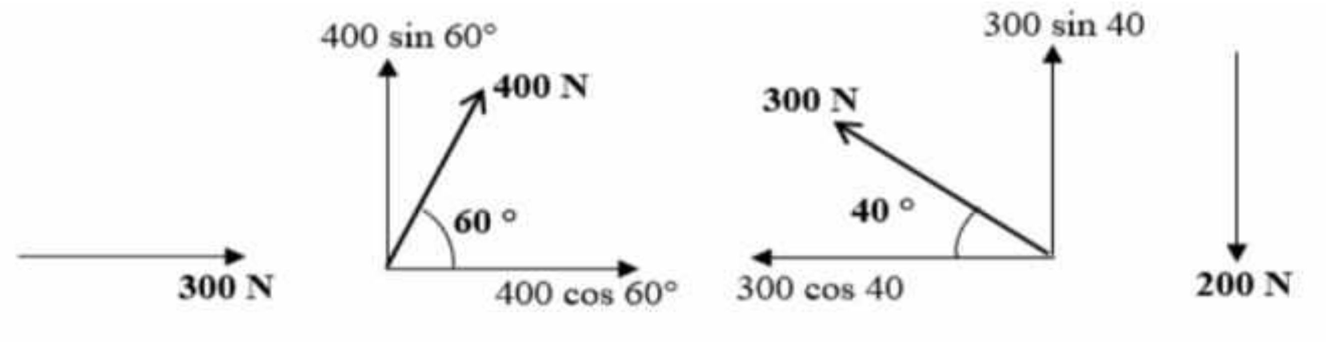
The resultant is acting from a to e

3.6.1 Numerical Problem based on resultant of coplanar con-current forces

1. Find the resultant of the coplanar concurrent force system shown in Figure



Step 1: Resolution of Forces



Step 2: Summation of Forces along X and Y axis (Σ Fx & Σ Fy)

$\Sigma F_x = 300 + 400 \cos 60 - 300 \cos 40$
$\Sigma F_x = 270.19$ N

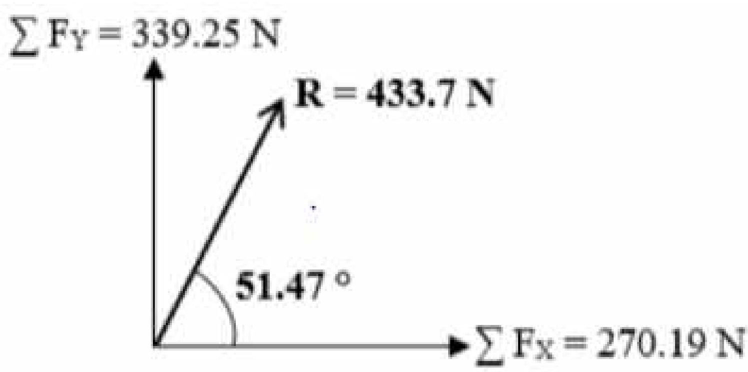
$\Sigma Fy = 400 \sin 60 + 300 \sin 40 - 200$
$\Sigma Fy = 339.25$ N

Step 3: Determination Magnitude of Resultant force,

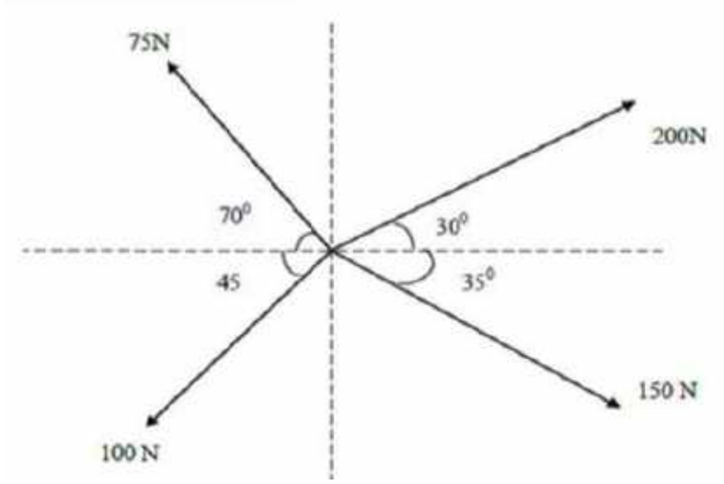
$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$
$R = \sqrt{(270.19)^2 + (339.25)^2}$
Magnitude of Resultant, R = 433.70 N

Direction of Resultant,
$\alpha_x = \tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right)$
$\alpha_x = \tan^{-1}\left(\frac{339.25}{270.19}\right)$
$\alpha_x = 51.47^o$

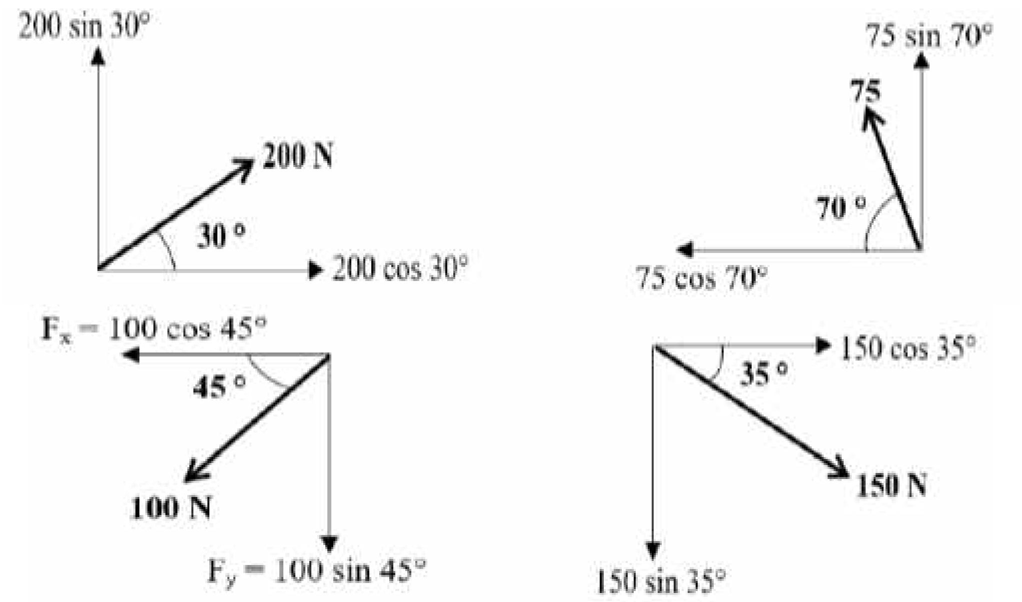
Step 4: Final Figure



2. Determine the magnitude & direction of the resultant of the coplanar concurrent force system shown in figure below.



Step 1: Resolution of Forces



Step 2: Summation of Forces along X and Y axis (Σ Fx & Σ Fy)

$Fx = 200\cos 30 – 75 \cos 70 – 100 \cos 45 + 150\cos 35$
$\Sigma Fx = 199.72$ N

$Fy = 200\sin 30 + 75 \sin 70 – 100 \sin 45 – 150 \sin 35$
$\Sigma Fy = 13.73$ N

Step 3: Determination of Magnitude (R) and direction of Resultant (α_X)

Magnitude of Resultant force, $R = \sqrt{(\Sigma Fx)^2+(\Sigma Fy)^2}$

$R = \sqrt{(199.72)^2+ (13.73)^2}$

Magnitude of Resultant, R = 200.19 N

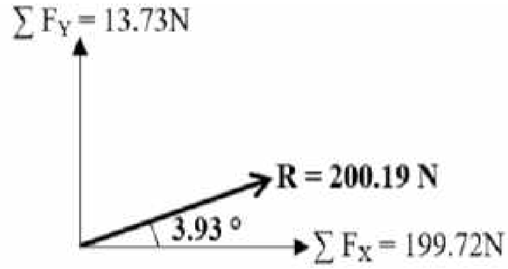
Direction of Resultant,

$\alpha_x = \tan^{-1}(\frac{\Sigma Fy}{\Sigma Fx})$

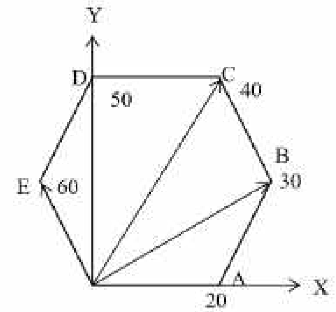
$\alpha_x = \tan^{-1}(\frac{13.73}{199.72})$

$\alpha_x = 3.93^o$

Step 4: Final Figure



3. Five forces are acting on a regular hexagon at an angular point as shown in Fig. Calculate their resultant in magnitude and direction. [AKTU 2022-23]



Solution:

Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis.

Included angle for regular polygon of n sides = $\frac{2n-4}{n} \times 90^0$

For regular hexagon n= 6, then included angle between two adjacent sides =(8/6) x 90^0 = $120^{0,}$

i.e Angle between OA and AB =120^0,

$\angle AOB = \angle ABO = \angle BOC = \angle COD = \angle DOE = 30^0$,

Resolving all the inclined forces along X and Y Axis,

$\Sigma F_X = 20 + 30\cos 30^0 + 40\cos 60^0 - 60\cos 60^0$

$= 20 + 30 \times \frac{\sqrt{3}}{2} + 40 \times \frac{1}{2} - 60 \times \frac{1}{2} = 20 + 15\sqrt{3} + 20 - 30 = 15\sqrt{3} + 10 = 35.98N$,

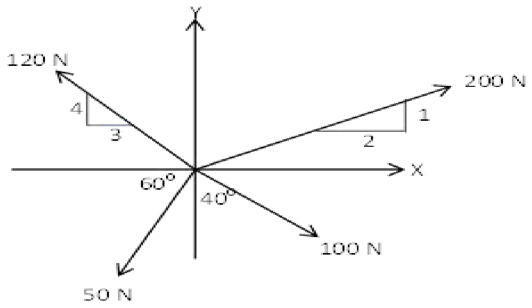
$\Sigma F_Y = 30\sin 30^0 + 40\sin 60^0 + 50 + 60\sin 60^0$

$= 30 \times \frac{1}{2} + 40 \times \frac{\sqrt{3}}{2} + 50 + 60 \times \frac{\sqrt{3}}{2} = 15 + 20\sqrt{3} + 50 + 30\sqrt{3} = 65 + 50\sqrt{3} = 151.6N$

$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(35.98)^2 + (151.60)^2} = 155.8N$,

$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{151.60}{35.98} = 4.213$, $\alpha = Tan^{-}(4.213) = 76.648^0$

4. A system of four forces acting on a body is as shown in Fig. Determine the resultant.

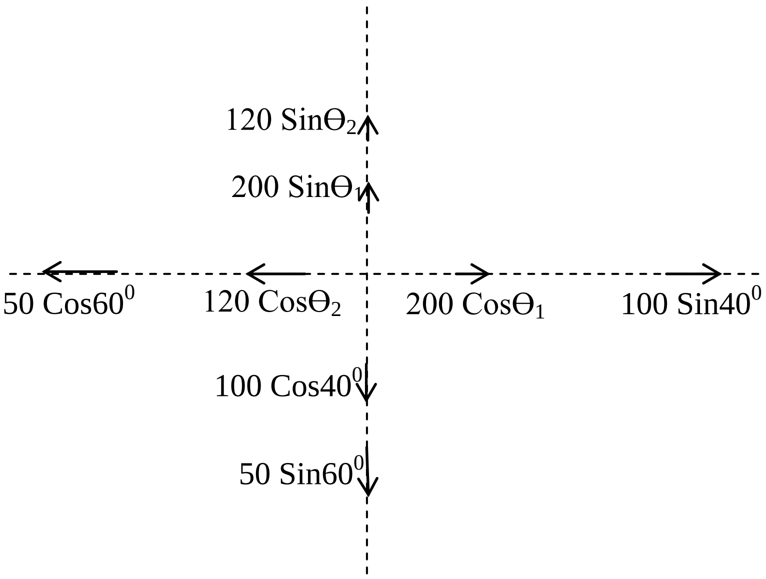


Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis. Resolving all the forces along x and y axis is as shown in Fig.

If Θ_1 is the inclination of 200 N force to x axis, then $Tan\theta_1 = \frac{1}{2}, Sin\theta_1 = \frac{1}{\sqrt{5}}, Cos\theta_1 = \frac{2}{\sqrt{5}},$

Let Θ_2 is the inclination of 120 N force to x axis, then $Tan\theta_2 = \frac{4}{3}, Sin\theta_2 = \frac{4}{5}, Cos\theta_2 = \frac{3}{5},$

$$\Sigma F_X = 200\cos\theta_1 + 100\sin 40^0 - 120\cos\theta_2 - 50\cos 60^0$$
$$= 200\times\frac{2}{\sqrt{5}} + 100\times 0.642 - 120\times\frac{3}{5} - 50\times\frac{1}{2}$$
$$= 178.88 + 64.2 - 72 - 25 = 146.08N$$



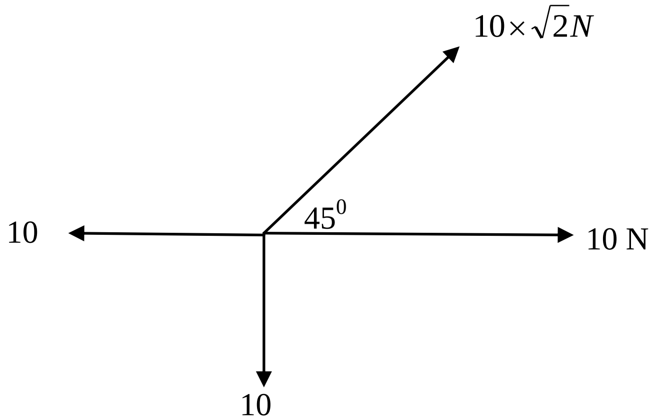
$$\Sigma F_Y = 120\sin\theta_2 + 200\sin\theta_1 - 100\cos 40^0 - 50\sin 60^0$$
$$= 120\times\frac{4}{5} + 200\times\frac{1}{\sqrt{5}} - 100\times 0.766 - 50\times\frac{\sqrt{3}}{2}$$
$$= 96 + 89.44 - 76.6 - 43.3 = 65.54N$$

$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(146.08)^2 + (65.54)^2} = 160.10N,$$

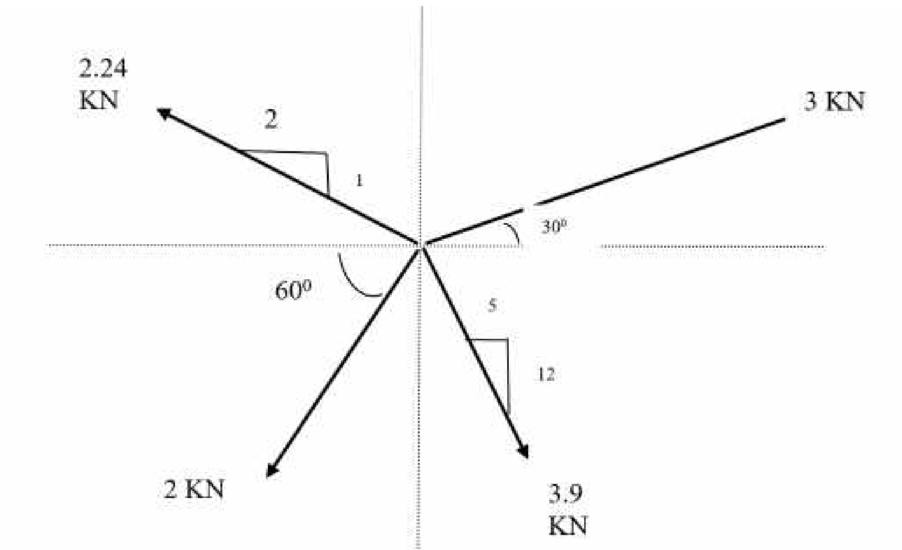
$$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{65.54}{146.08} = 0.4486, \alpha = Tan^{-}(0.4486) = 24.16^0$$

Long Types Question Problems

1. Find the magnitude and direction of the resultant of the system of coplanar forces as shown in Fig [Ans. R = 0 N]



2. Determine the resultant, both in magnitude and direction of the four forces acting on the body as shown in the figure below. (Ans: R = 3.034 KN, $\alpha = 68.86^0$ (Anticlockwise with respect. to x axis)



3. Find the resultant of forces 2, √3, 5, √3 and 2 N that act at an angular point of a regular hexagon towards the other angular points taken in order. (Ans: R = 10 N, $\alpha = 60^0$)

3.7 Moment of a force or simply Moment:

Moment of a force about a point is the product of the magnitude of the force applied at a point and the perpendicular distance between the force and a point about which the moment is calculated. It is represented by 'M'.
Mathematically,

$$M_A = F \times d \text{ } \mathbf{N\text{-}m}$$

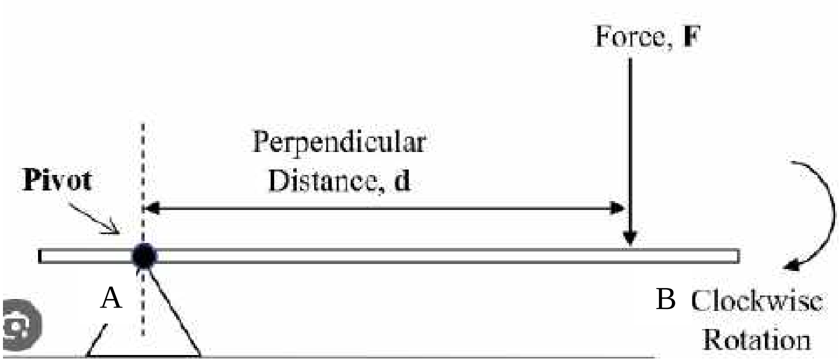


Fig. 3.8 Moment of Force

When a force acts on a body, it causes or tends to cause a change of state of rest or of uniform motion of the body. The action of the moment tends to cause a rotational motion of the body. Moment is the turning effect produced by the force on which it acts. The tendency of rotation or turning of the body due to moment of force may be clockwise or anticlockwise. In general, clockwise moment is taken as positive. And anticlockwise moment is taken as negative.
Moment of forces has following significant aspects:

(i) Moment is the turning effect produced by a force on which it acts
(ii) The moment of the force is equal to the product of force and the perpendicular distance between point and line of action of force
(iii)Moment of a force is represented by M

M_A =force x perpendicular distance

$= F \times d$

3.8 Couple:

Two equal and opposite forces are separated by a finite distance form a couple. The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces.

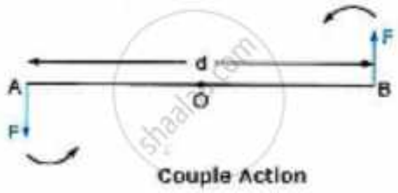


Fig. 3.9 couple

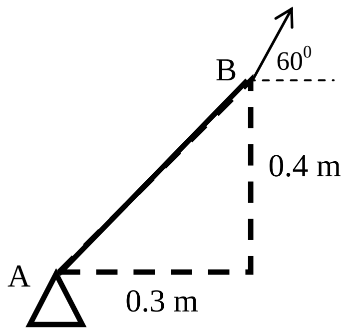
Two equal and opposite force of magnitude F is separated by a distance d and acting opposite direction is as shown in Fig. 3.9 These two forces form a couple. Mathematically it can be written as

$$C = F \times d \text{ } \mathbf{N\text{-}m}$$

Where C is representing the Couple of the system.

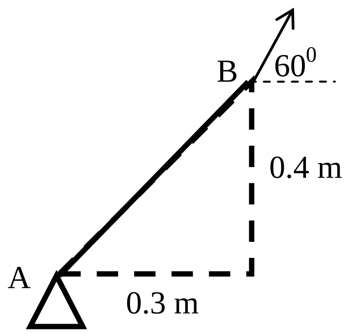
3.9 Numerical Problem based on Moment of Forces

1. **A force of 200 N is acting at point B of a lever AB which is hinged at its lower end as shown in Fig. 1.05. Find the moment of force about the hinged end A.** [AKTU 2022-23]

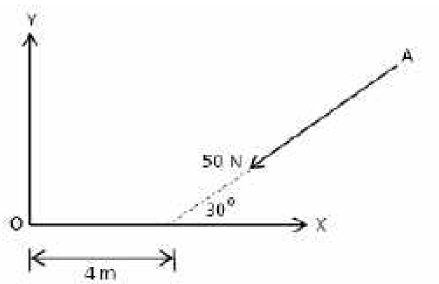


Solution:
The horizontal and vertical components of Force 200 N are

$F_X = 200Cos60^0 = 100N, F_Y = 200Sin60^0 = 173.2N$
Moment of Force 200 N about hinged end A
$= 200Cos60^0 \times 0.4 + 200Sin60^0 \times 0.3 = -11.96Nm(Anti-clockwise))$



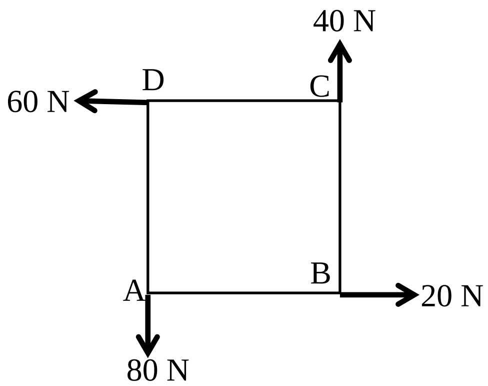
2. **A force of 50 N is acting at a point A as shown in Fig. Determine the moment of this force about point O.**



Solution:
Resolve the 50 N force along X and Y axis. Component of 50 N force along X and Y Axis are 50 $Cos30^0$ and 50 $Sin30^0$ respectively. Moment of Force 50 N about O is

$$\Sigma M_A = 50Sin30^0 \times 4 = 100Nm(Clockwise)$$

3. **Four forces of magnitude 20 N, 40 N, 60 N and 80 N are acting respectively along the four sides of square ABCD as shown in Fig. Determine the moment about point A. It is given that each side of square is 2 m.**



Solution:

Let us assumed that clockwise moment is taken as positive.

$\Sigma M_A = -40 \times 2 - 60 \times 2 = -200 Nm(Anti-clockwise)$

3.10 Varignon'sTheorem:

Varignon's theorem states that 'the algebraic sum of moments of two forces about any point in their plane is equal to the moment of their resultant about the same point'.Varignon's theorem is also known as law of moment.

3.10.1 Proof of Varignon'sTheoram: Let R be the resultant of forces P_1 and $P_2$and O be the moment center as shown in Fig.1.9. Let d, d_1 and d_2 be the moment arms of the forces R,P_1 and P_2 respectively. According to Varignon's theorem

$Rd = P_1 d_1 + P_2 d_2 ....(1)$

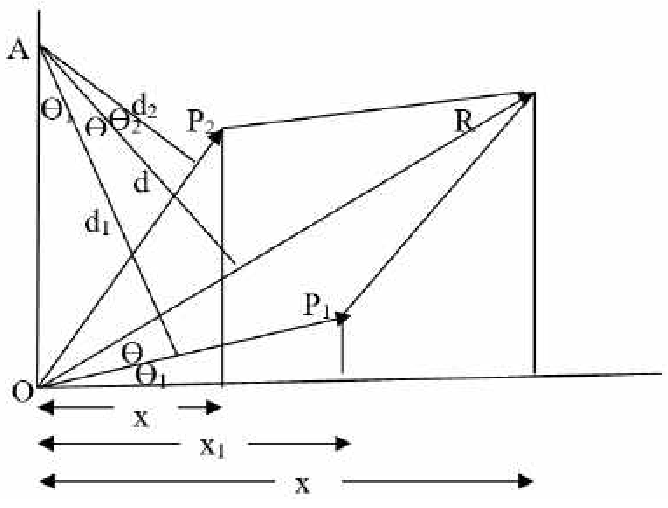


Fig 3.10 Varignon'sTheorem

Moment of force at A due to $P_1 = P_1 \times d_1 = P_1 \times (OA \cos\theta_1) = OA \times (P_1 \cos\theta_1) = OA \times x_1 .....(2)$, Moment of force at A due to $P_2 = P_2 \times d_2 = P_2 \times (OA\cos\theta_2) = OA \times (P_2\cos\theta_2) = OA \times x_2 .....(3)$,

Adding equation (2) and (3), we get

Moment of force at A due to force P_1 and P_2= $OA \times (x_1 + x_2).....(4)$,

From geometry, $x = x_1 + x_2$, Putting the value of $x_1 + x_2$ in equation (4), we get

Moment of force at A due to force P_1 and P_2= $OA \times (x_1 + x_2). = OA \times x....(5)$,

Moment of force at A due to $R = R \times d = R \times (OA\cos\theta) = OA \times (R\cos\theta) = OA \times x.....(6)$,

From equation(5) and (6),

Moment of force at A due to force P_1 and P_2 = Moment of force at A due to R,

$$Rd = P_1 d_1 + P_2 d_2$$

Varignon's theorem is applicable when two or more than two coplanar forces are acting on the body. If a system of forces consists of more than two forces then according to Varignon's theorem if a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point

3.11 Resultant of co planar non concurrent system:

Resultant of a force system is single force. It may be a pure moment of force or couple. But they have the same rotational and translatory effect as the given system of forces.

Let F_1,F_2 and F_3 constitute a system of forces acting on the body which are the non-concurrent but co planar . Each force can be replaced by a force of the same magnitude and acting in the same direction at point O and a couple of magnitude $M_i = F_i d_i$ where d_i is the perpendicular distance between the line of action of force F_i and point O.

Following steps may be used to find out the resultant of non-concurrent coplanar forces:

(i) To determine the value of ΣF_x and ΣF_y

(ii) To determine the value of resultant by equation (1)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} .....(1)$$

(iii) To determine the direction of R w.r.t x axis by using equation (2)

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_y} .....(2)$$

(iv) To determine the algebraic sum of the moments of all forces about any given point O i.e. ΣM_O

(v) Mark the position of the resultant such that it produces the same direction of moment about point O

(vi) Applying varignon's theorem to find the position of the resultant i.e.

$$\Sigma M_O = R \times d$$

(vii) To determine the x and y intercepts of the line of action of resultant

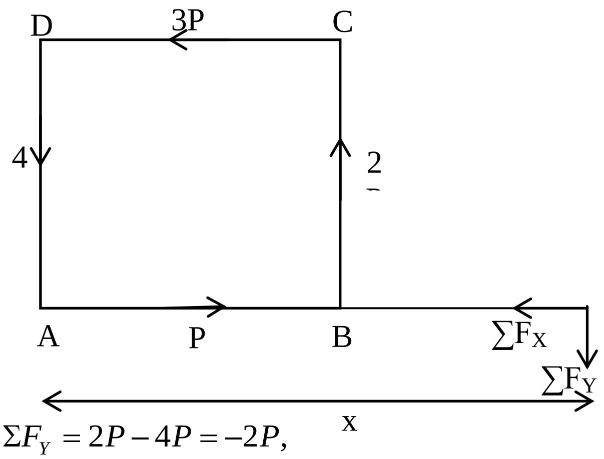
$$x = \frac{d}{\sin\theta}, y = \frac{d}{\cos\theta}$$

3.11.1 Numerical Based on Resultant of co planar non concurrent system:

1) A square ABCD is subjected to forces equal to P, 2P, 3P and 4P along the sides AB, BC, CD and DA. Determine the magnitude, direction, and line of action of the resultant.

Solution:

Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis is as shown



$$\Sigma F_X = P - 3P = -2P, \Sigma F_Y = 2P - 4P = -2P,$$

$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(-2P)^2 + (-2P)^2} = 2\sqrt{2}P,$$

$$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{-2P}{-2P} = 1, \alpha = 45^0, (180^0 + 45^0 = 225^0)$$

Since ΣF_x and ΣF_y are both negative, therefore α lies in the 3rd quadrant, so α=225^0

Let resultant act at a distance of x from point A. For finding the value of line of action of resultant(x), apply Varignon's Theorem. According to this theorem the algebraic sum of moment due to indivisible forces is equal to moment due to resultant force. Let a is side of square.

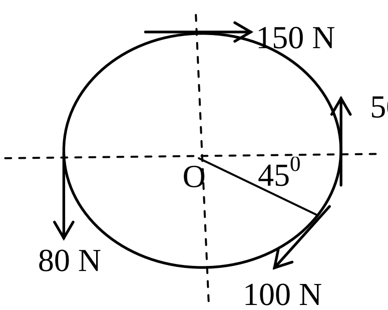
Moment due to indivisible forces about A $= -2P \times a - 3P \times a = -5Pa$

Moment due to resultant force about A $= \Sigma F_Y \times x = 2P \times x$,

$$-5Pa = 2P \times x, x = \frac{-5Pa}{2P} = -2.5a$$

Negative value indicate that resultant is acting at a distance of 2.5a from point A along negative X Axis (along left from point A).

2) **Determine the resultant of the four forces acting tangentially to a circle of radius 3m as shown in Fig. What will be the location of the resultant with respect to centre of the circle?**



Solution: Resolving all the inclined forces along X and Y Axis. Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis.
100 N force is inclined force. Then component of 100 N force along x direction =100 cos45^0 ,
Component of 100 N force along y direction = 100 sin45^0

$$\Sigma F_X = 150 - 100\cos 45^0 = 150 - 70.71 = 79.29N, \Sigma F_Y = 50 - 80 - 100\sin 45^0 = -100.71N,$$

$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(79.29)^2 + (-100.71)^2} = 128.18N,$$

$$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{-100.71}{79.29} = -1.27, \alpha = 51.8^0$$

Since ΣF_x is positive and ΣF_y is negative, therefore resultant lies in the fourth quadrant.
Moment of all the indivisible forces about the centre $= -50\times 3 + 150\times 3 - 80\times 3 + 100\times 3 = 360Nm$
If d is the perpendicular distance of resultant from the centre, then

$$R\times d = 360, d = \frac{360}{128.18} = 2.795m$$

The resultant will act a perpendicular distance of 2.795 m from the centre.

3.12 Free body diagram:
It is a diagram of the body in which the body under consideration is free from the entire contact surface and all the forces acting on it (both applied and not applied) are drawn. In this diagram, all the supports like walls, floors, hinges, etc. are removed and replaced by the reactions which these supports exert on the body. For the analysis of problem based on equilibrium, it is necessary to isolate the body from all contact surfaces and draw all forces acting on it. While drawing the free body diagrams, it includes reaction from other contact surface, internal forces, and external forces.
Some examples of system with their free body diagram is as shown in Fig. 3.12

(i) A sphere resting on a frictionless plane surface

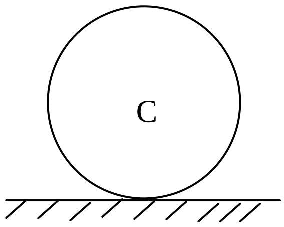


Fig. 3.12

Free body diagram of sphere is representation in Fig. 3.13

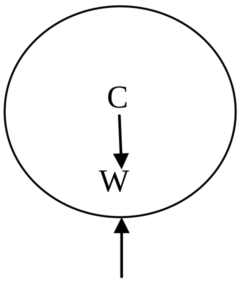


Fig. 3.13

Lami's Theorem:

Lami's Theorem states that if three coplanar forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.

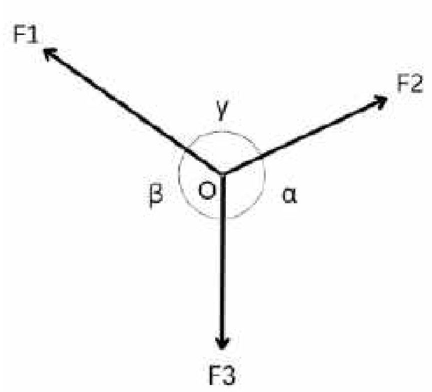


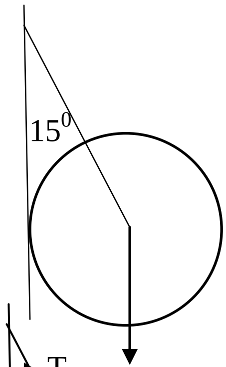
Fig. 3.14

Mathematically, if forces F_1, F_2, and F_3 are in equilibrium, and the angles opposite them are α, β, and γ respectively, then

$$\frac{F1}{\sin\alpha} = \frac{F2}{\sin\beta} = \frac{F3}{\sin\gamma}$$

3.12.1 Numerical Problem based on Free Body Diagram

1. A sphere of weight 100 N is tied to a smooth wall by a string as shown in Fig. Find the tension T in the string and reaction R of the wall.



Solution:

Free Body Diagram (FBD) of sphere

Applying Condition of equilibrium,

$\Sigma F_x = 0, \Sigma F_y = 0$

$\Sigma F_x = 0$

R- T $\sin 15^0$ = 0, R = T sin 15^0 …..(i)

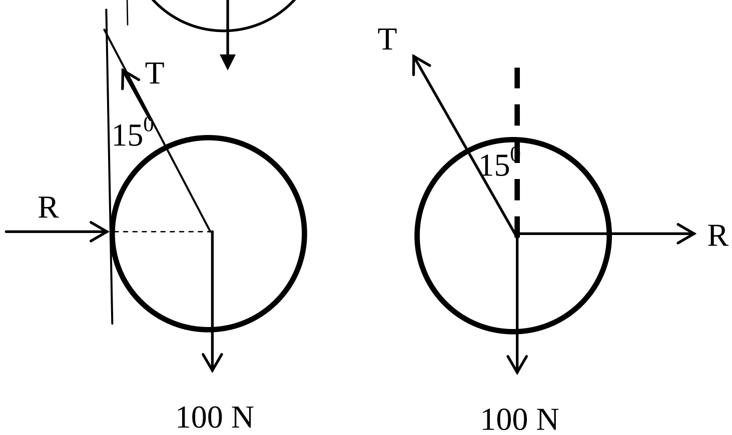
$\Sigma F_y = 0$

T cos 15^0 – 100 = 0,

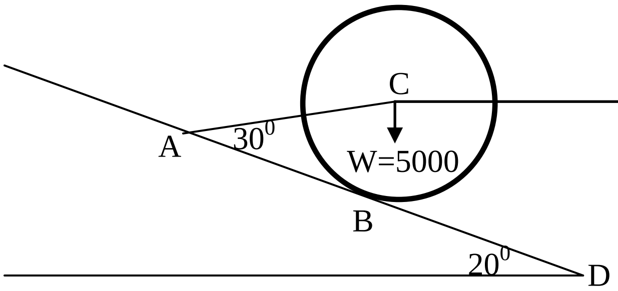
$$T = \frac{100}{Cos15^0} = \frac{100}{0.9659} = 103.53N$$

Putting the value of T in equation (i), we get

R = T sin 15^0 = 103.53 × 0.2588 = 26.79 N

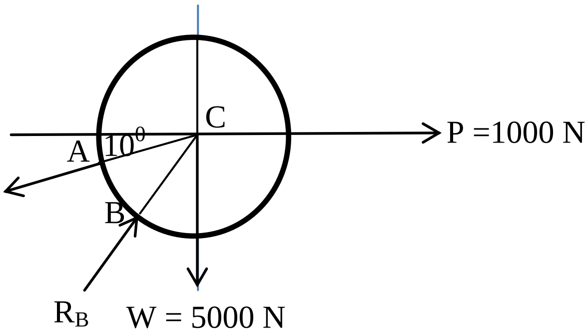


1. A right circular roller of weight 5000 N rest on a smooth inclined plane and is held in position by a cord AC as shown in Fig. Find the tension in the cord if there is a horizontal force of magnitude 1000 N acting at C.



Solution:

Free body diagram (FBD) of circular roller of weight W=5000 N is drawn is as shown in Fig.



Resolve all the inclined forces along X and Y axis. Appling condition of equilibrium, we get

$\Sigma F_X = 0,$

$1000 - T\cos 10^0 + R_B \sin 20^0 = 0................(1)$

$\Sigma F_Y = 0,$

$R_B \cos 20^0 - T\sin 10^0 - 5000 = 0..................(2)$

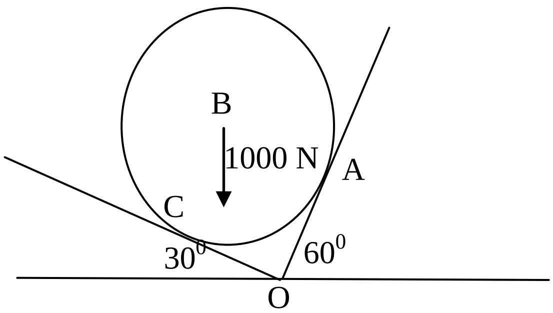
Solving equation (1) and (2), we get

$T = 3059.58N, R_B = 5885.90N$

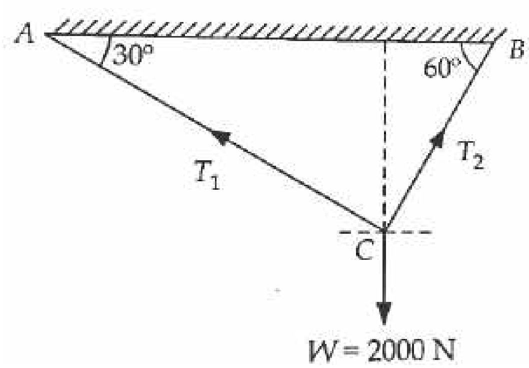
Tension in the cord is 3.059 KN.

Long Types Question Problems

Q.1 A smooth circular cylinder of weight 1000 N and radius 10 cm rests in a right-angled groove whose sides are inclined at an angle of 30^0 and 60^0 to the horizontal as shown in Fig. Determine the reaction R_A and R_C at the points of contact. [Ans. R_A = 500 N, R_C = 866.6 N]



Q.2 A weight of 2000 N is supported by two chains AC and BC as shown in figure. Determine the tension in each chain.



3.13 Condition of Equilibrium or Equilibrium Equation:

When external forces are acting on a stationary body, the body may start moving or may start rotating about any point. But if the body does not start moving and does not start rotating about any point, then the body is said to be in equilibrium

There are three conditions for equilibrium when system of force is subjected to rigid body

1. Algebraic sum of all the horizontal forces acting upon the body must be zero
2. Algebraic sum of all the vertical forces acting upon the body must be zero
3. Algebraic sum of moments acting on the body must be zero

Mathematically the condition for equilibrium is:

$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$

Equations of equilibrium:

(i)A non-concurrent coplanar force system will be in equilibrium, if the resultant of all the forces and moment is zero

Equation of equilibrium for non-concurrent forces are:

$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M = 0$

(ii)For concurrent force systems the lines of action of all forces meet at a point and hence the moment of those force is equal to zero

Equation of equilibrium for concurrent forces are:

$\Sigma F_x = 0, \Sigma F_y = 0.$

3.14 Types of supports:

Various types of support and reactions developed at these ends are following

1. Roller support
2. Hinge or pin support
3. Fixed support

1. **Roller support**: In roller support, the beam is supported on a roller. In this case, reaction is developed normal to the support end due to rollers are free to roll along the support. In roller support, one vertical reaction is developed w.r.t. to motion of roller so that displacement in vertical direction is prevented. Roller supported at A is shown in Fig.3.14

Fig.3.14

2. **Hinge or pin support**: In this case, the position of the end of the beam is fixed but the end is free to rotate. In hinge support, two reactions (one along horizontal and other along vertical) developed in two mutually perpendicular directions. In these types of support, displacements in horizontal and vertical are prevented but do not provide any resistance to moment. Hinge supported at A is shown in Fig. 3.15

Fig.3.15

3. **Fixed support**: At fixed support the end of the beam is neither permitted to move in any direction nor allowed to rotate. Displacements in horizontal, vertical and rotation are prevented In fixed support one vertical, one horizontal and one moment are acting at fixed support. Fixed supported at A is shown in Fig. 3.16

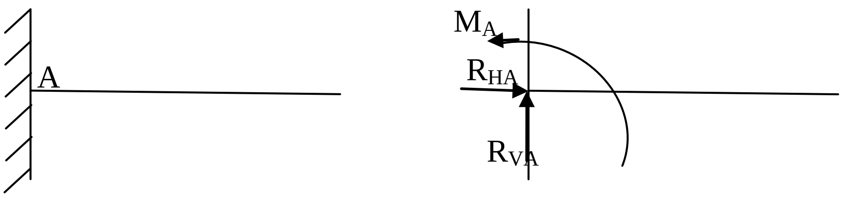


Fig.3.16

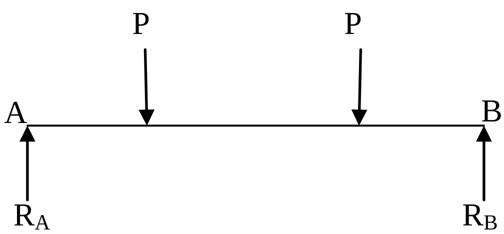
3.15 Beam and Types of Beam:

Beam: A beam may be defined as a structural element which has one dimension (Length) considerably larger than other two dimensions namely breadth and depth and is supported at few points. Load is always acting in a vertical direction. Due to applied loads reactions develop at supports. The nature of reaction depends upon the type of supports.

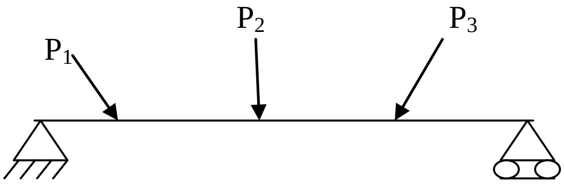
Types of beam:

1. Simply supported beam
2. Beam with one end hinged and other end is roller supported
3. Cantilever beam

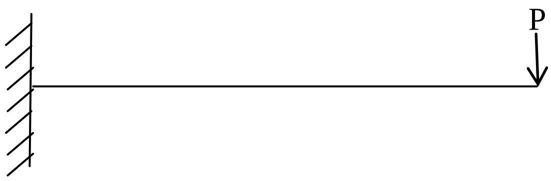
1. **Simply supported beam:** When both ends of the beam are simply supported, it is called simply supported beam. Such a beam can supported load in the direction normal to axis.



2. **Beam with one end hinged and other end is roller supported:** If one end of the beam is hinged and other end is on the rollers, the beam can resist load in any direction



3. **Cantilever beam**: If a beam is fixed at one end and is free at the other end, it is called cantilever beam.



3.16 Types of loads:

In a structure, generally self-load, load due to object, wind load are acting on the structure. The different types of loads are as following

1. Point or concentrated load
2. Uniformly distributed load (UDL)
3. Uniformly varying load (UVL)

1. **Point or concentrated load**: if a load is acting on a beam over a very small length compared to the span of the beam, it is approximated as a point load. It is generally represented by an arrow as shown in Fig.3.17(a)

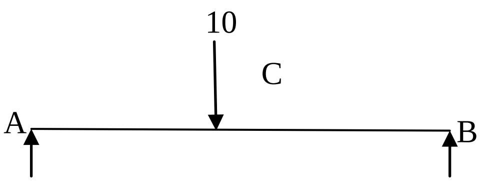


Fig.3.17(a) Point load

2. **Uniformly distributed load (UDL):** If the intensity of load remains constant over a considerable length, it is called uniformly distributed load. It is shown in Fig.3.17(b)

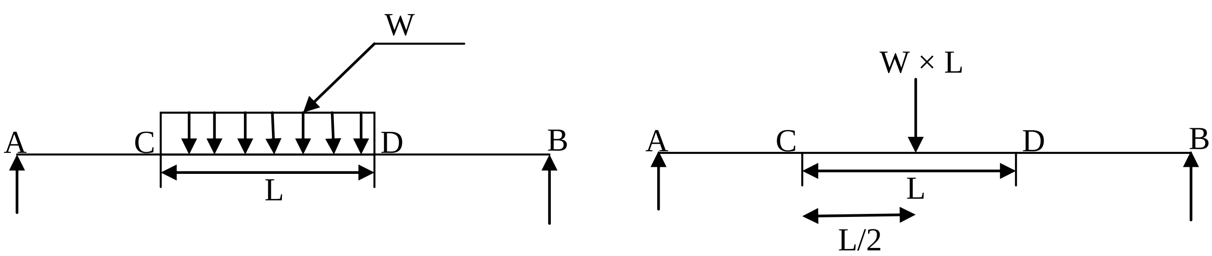


Fig.3.17(b) Uniformly distributed

Equivalent load = Load intensity x distance = W × L,
Equivalent load will be acting at the middle of the distance at which the uniformly distributed load is acting.

3. **Uniformly varying load (UVL):** if the intensity of the load is varying linearly over a considerable length it is called uniformly varying load. This load is also called triangular load. Area of the

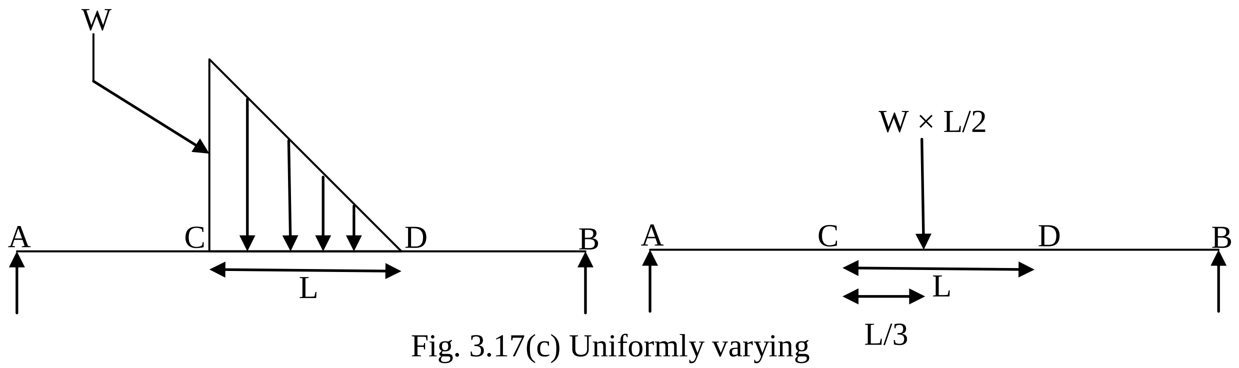


Fig. 3.17(c) Uniformly varying

triangle represents the total load and the centroid of the triangle represents the centre of gravity of the load. It is shown in Fig.3.17(c)

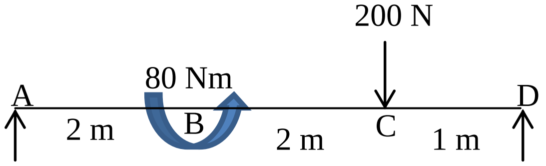
Total equivalent load = Area of triangle = $\frac{W \times L}{2}$

And equivalent load is acting at a distance of 1/3 from the big end and 2/3 from the small end

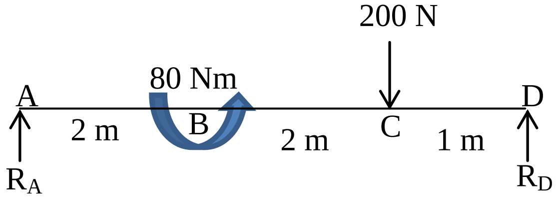
Ex: water pressure acting on the side wall of the water tank

3.16.1 Numerical Problem based on support reactions

1. Determine the support reaction at end A and D for a given beam as shown in Fig



Solution:



Applying condition of Equilibrium, we get

$\Sigma F_Y = 0$, $R_A + R_D = 200N$ ……….(1),

$\Sigma M_A = 0$, Tacking clockwise moment is positive,

$-80 + 200 \times 4 - R_D \times 5 = 0$,

$5R_D = 800 - 80 = 720$, $R_D = \frac{720}{5} = 144N$,

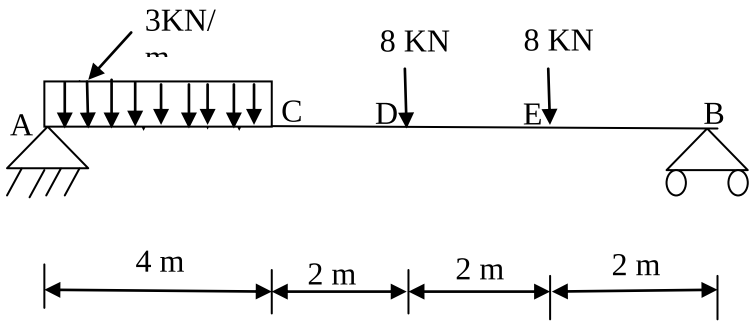
Putting the value of R_D in equation 1, we get

$R_A + 144 = 200N$, $R_A = 56N$

Reaction at end A = R_A = 56N,

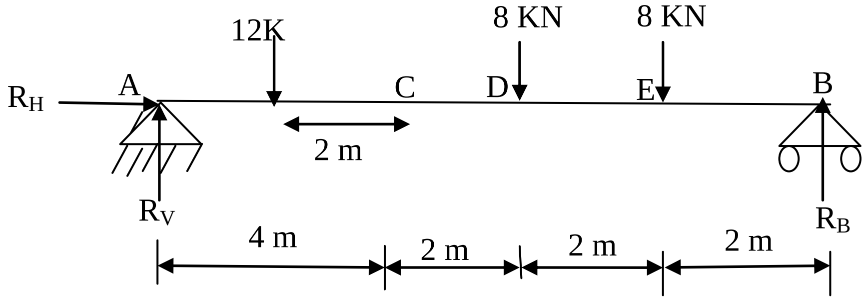
Reaction at end D = R_D = 144N

2. A beam AB of span 10 m is loaded as shown in Fig. Determine the reactions at A and B.



Solution:

Free body diagram of beam AB is drawn as shown in Fig



Applying condition of equilibrium, we get

$\Sigma F_Y = 0, \Sigma F_X = 0, \Sigma M_A = 0,$

$\Sigma F_Y = R_{VA} - 12 - 8 - 8 + R_B = 0,$

$R_{VA} + R_B = 28KN........(1)$

$\Sigma F_X = R_{HA} = 0$

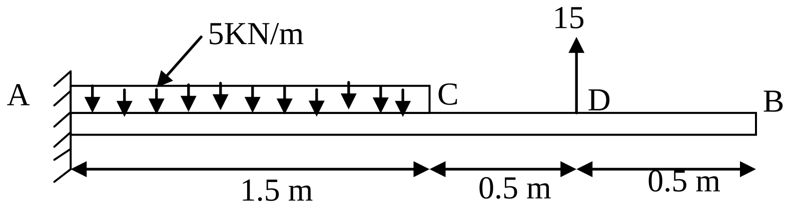
$\Sigma M_A = 12 \times 2 + 8 \times 6 + 8 \times 8 - R_B \times 10 = 0, R_B = \frac{136}{10} = 13.6KN,$

Putting the value of R_B in equation (1), we get

$R_{VA} = 28 - R_B = 28 - 13.6 = 14.4KN,$

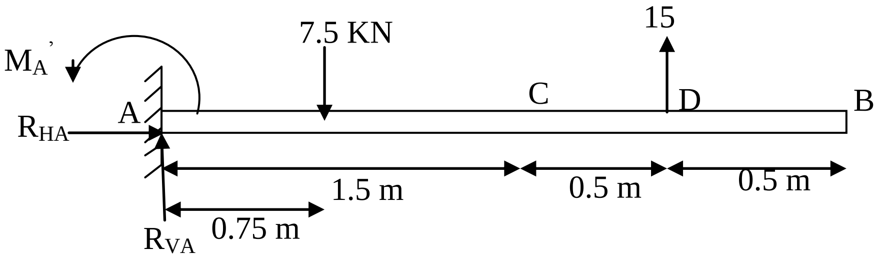
$R_A = \sqrt{(R_{VA})^2 + (R_{HA})^2} = R_{VA} = 14.4KN, R_B = 13.6KN$

3. What force and moment is transmitted to the supporting wall at A in the given cantilever beam as shown in Fig



Solution:

Free body diagram of Beam AB is as shown in Fig.



Applying condition of equilibrium, we get

$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_A = 0$,

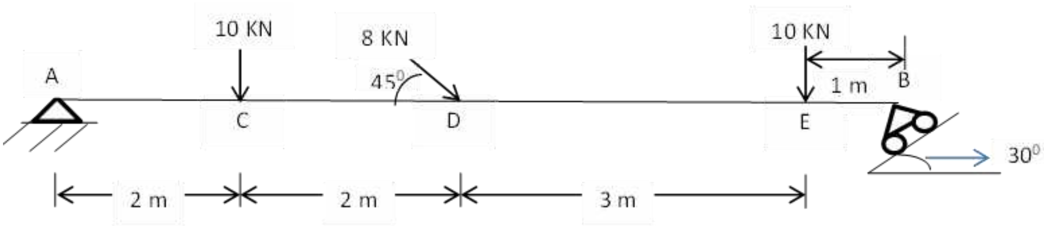
$\Sigma F_x = 0$, $R_{HA} = 0$,

$\Sigma F_y = 0$, $R_{VA} - 7.5 + 15 = 0$, $R_{VA} = -7.5KN$,

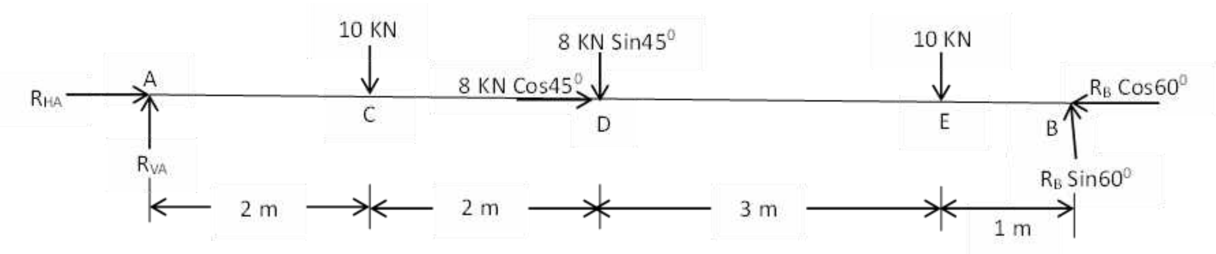
$\Sigma M_A = 0, -M_A^{'} + 7.5 \times 0.75 - 15 \times 2 = 0$, $M_A^{'} = -24.375$ Nm(Anti-clockwise)

Negative sign means, R_{VA} is acting vertically downward (i.e. opposite to the assumed direction).Negative sign means, moment at A will be in opposite direction to assumed direction (i.e. $M_A^{'}$ is acting along clockwise direction).

4. A beam 8 m long is hinged at A and supported on roller over a smooth surface inclined at an angle 30^0 to the horizontal at B. The beam is loaded as shown in Fig. Determine the support reactions [AKTU 2023-24]



Solution:



Reaction at Support A is R_{VA} and R_{HA} respectively along vertical and horizontal direction. After resolving the 8 KN at end D, the vertical and horizontal components are 8 Sin 45^0 and 8 Cos 45^0 . Similarly Roller support at B makes an angle of 60^0 with horizontal direction. After resolving the support reaction at B, the vertical and horizontal components are R_B Sin60^0 and R_B Cos 60^0 respectively.

Applying the condition of equilibrium condition for non-concurrent coplanar force system, we get

$\Sigma F_X = 0$, R_{HA} + 8 Cos 45^0- R_B Cos 60^0 = 0, 0.5 R_B – R_{HA} = 5.768 KN …….(1),

$\Sigma F_Y = 0$, R_{VA} -10 – 8 Sin 45^0 -10 + R_B Sin 60^0 = 0,

R_{VA} + 0.866 R_B = 25.768 KN ……….(2)

Taking moment about A and clockwise moment is taken as positive.

$\Sigma M_A = 0$, 10 x 2 + 8 Sin45^0 x 4 + 10 x 7 – (R_B Sin 60^0) x 8 = 0,

R_B = 16.25 KN,

Putting the value of R_B in Equation (1), We get

R_{HA} = 2.357 KN,

Putting the value of R_B in Equation (2), We get

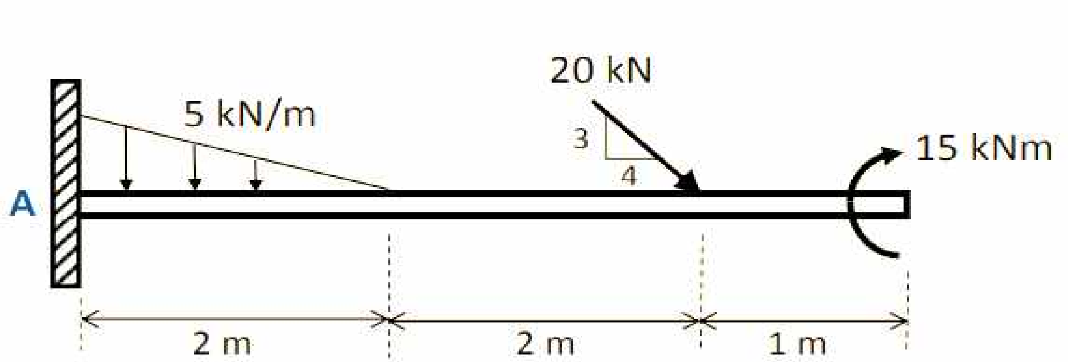
R_{VA} = 11.695 KN,

Support reaction at A = R_A = $\sqrt{(R_{VA})^2 + (R_{HA})^2}$,

$R_A = \sqrt{(11.695)^2 + (2.357)^2} = 11.93 KN$,

Support reaction at B = R_B = 16.25 KN

5. A cantilever beam is loaded as shown. Determine all reactions at support A.



Solution

Sinα = $\frac{3}{5}$ cosα = $\frac{4}{5}$

$\sum$Fx = 0

R_{AX} +20 cosα = 0

R_{AX} + 20× $\frac{4}{5}$ = 0

R_{AX} = 16 KN

$\sum$Fy = 0

R_{AY} -20 Sinα - $\frac{1}{2}$ × 5 × 2 = 0

R_{AY} -20 × $\frac{3}{5}$ - $\frac{1}{2}$ × 5 × 2 = 0

R_{AY} = 17 KN

$\sum$ M_A = 0

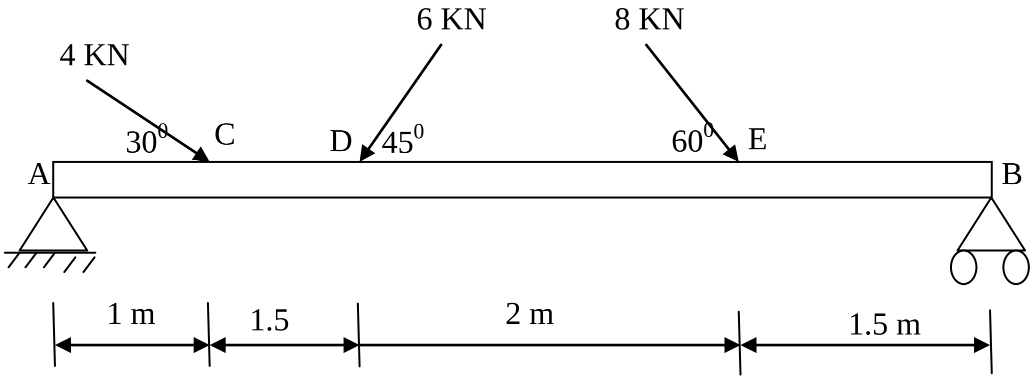
- M_A + $\frac{1}{2}$ × 5 × 2 × $\frac{1}{3}$ × 2 + 20 Sinα ×4 +15 = 0

M_A = 66.33 kNm

Long Types Question Problems

1. A beam AB of span 6 m is hinged at A and supported on rollers at the end B and carries load as shown in Fig. Determine the reactions at A and B.

[Ans. R_{AV} = 5.87 KN, R_{AH} = 3.222 KN, R_B = 7.3 KN]



Q. A beam has been loaded and supported as shown in figure given below. Determine the reactions at the support point A and B. (**Ans:** R_{ah} = 21.65 KN, R_{av} = 5.125 KN and R_b = 27.875 KN) (AKTU: Even Sem 2024-25)