- **UNIT-1**

Lecture- 1

Introduction to Number System,

Complex Number,

Open and Closed Interval, Number Line

Key Points

- Types of Numbers
- Real Numbers
- Complex Numbers
- Even and odd Numbers
- Prime Numbers
- Number line

TYPES OF NUMBERS

1) Natural Numbers(Counting Numbers)
2) Whole Numbers
3) Integers
4) Rational Numbers
5) Irrational Numbers

Natural Number

Natural Number

- A natural number is an integer greater than 0. Natural numbers begin at 1 and increment to infinity: 1, 2, 3, 4, 5, etc.

NOTE

Natural numbers will never include a minus symbol (-) because they cannot be negative.

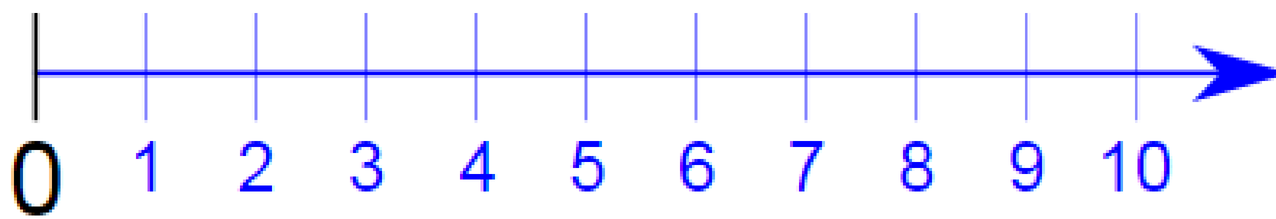
Definition of

Whole Number

Any of the numbers {0, 1, 2, 3, ...} etc.

There is no fractional or decimal part. And no negatives.

Example: 5, 49 and 980 are all whole numbers.



LET'S SEE COMBINDLY

Natural Numbers (N)

They are the numbers {1, 2, 3, 4, 5, ...}These numbers are called counting Numbers.

Whole Numbers (W).

This is the set of natural numbers, plus zero, i.e., {0, 1, 2, 3, 4, 5, ...}.

Integers (Z).

Integers

- **An integer** is a whole number (not a fractional number) that can be positive, negative, or zero.
- Ex {……..-4,-3,-2,-1,0,1,2,3,4…}

Rational Numbers

In mathematics, a **rational number** is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q.

Rational Numbers

In mathematics, a **rational number** is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q.

Since q may be equal to 1, every integer is a **rational number**. ½, ¾…………

Irrational Numbers

In mathematics, an irr**rational number** is any **number** that can not be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q. Since q may be equal to 1

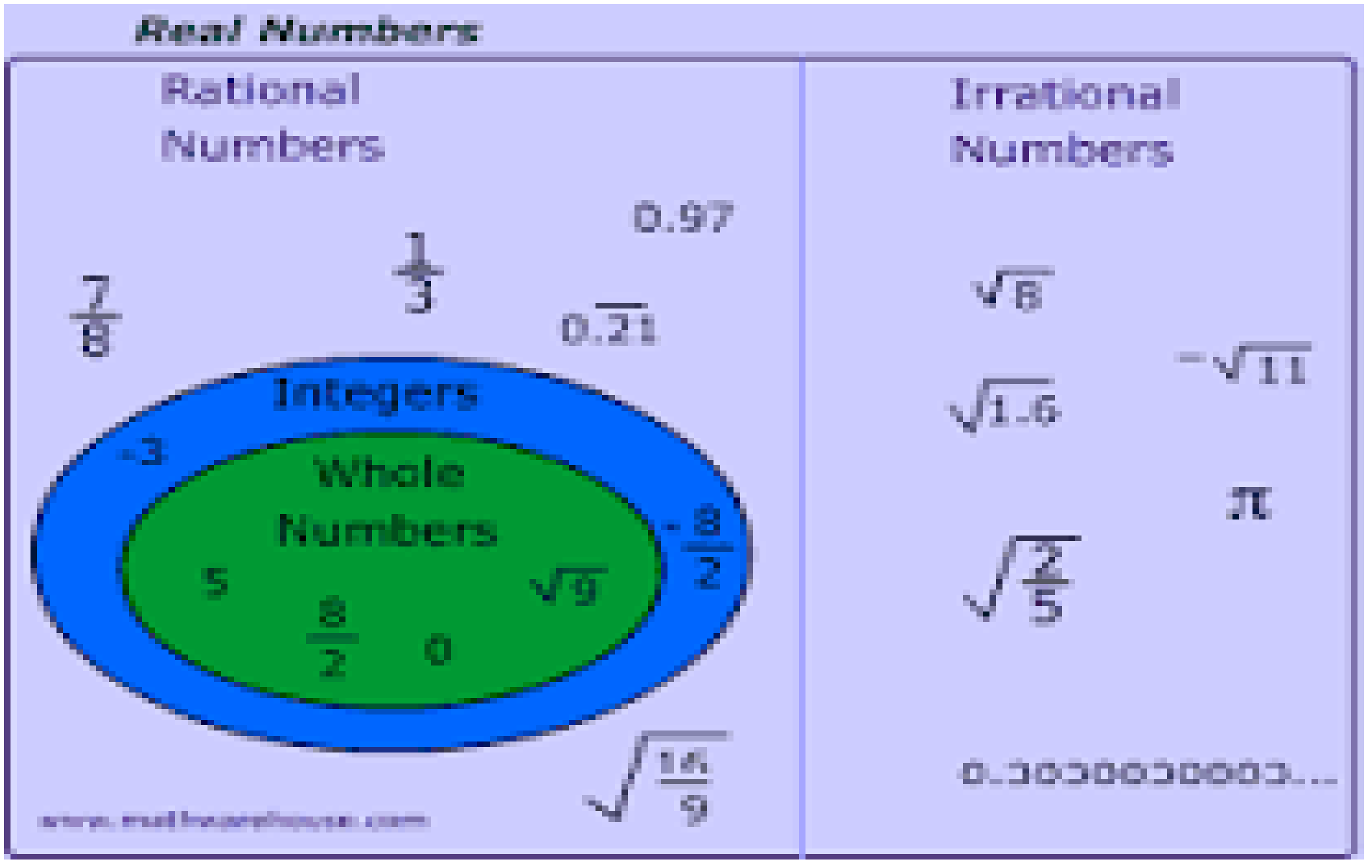
Examples

$$\sqrt{2} = 1.4142135623730950488016887242096...$$

$$\pi = 3.14159265358979323846264338327950...$$

Real Numbers

If we include irrational numbers in the set of rational numbers then it is called set of real numbers and denoted by *R*.


Real Numbers
Rational Numbers
Irrational Numbers
0.97
$\frac{1}{3}$
$\frac{7}{8}$
$0.\overline{21}$
Integers
-3
Whole Numbers
5
$\frac{8}{2}$
0
$\sqrt{9}$
$-\frac{8}{2}$
$\sqrt{\frac{16}{9}}$
$\sqrt{8}$
$-\sqrt{11}$
$\sqrt{1.6}$
π
$\sqrt{\frac{2}{5}}$

Even Numbers

Any integer that can be divided exactly by 2 is an even number .

Example: −24, 0, 6 and 38 are all even numbers.

.

Odd Numbers:

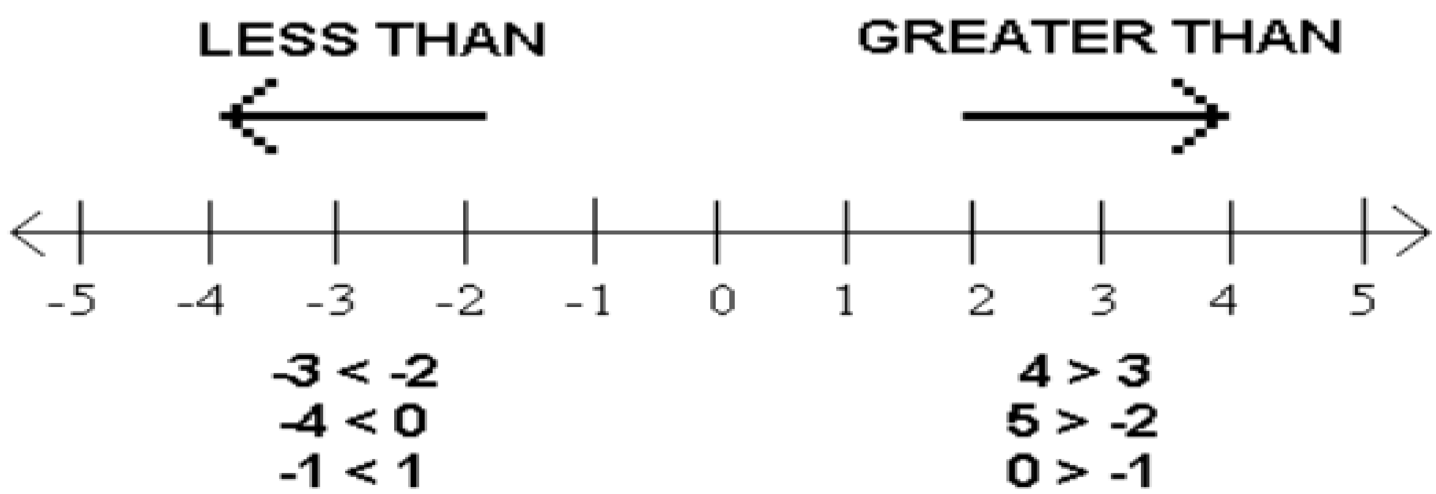
Any integer that can not be divided exactly by 2 is an even number .

Example: −3, 1, 7 and 35 are all odd numbers

Number System

Number Line

A line on which we represent the numbers is called number line.



INTERVALS AS SUBSETS OF *R*

Let $a, b \in R$ and $a < b$. Then, we define:

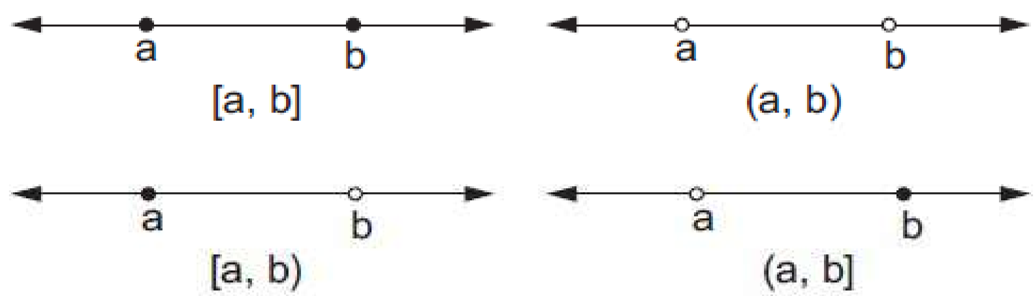
(i) *Closed Interval* $[a, b] = \{x \in R : a \leq x \leq b\}$.

(ii) *Open Interval* (a, b) or $]a, b[= \{x \in R : a < x < b\}$.

(iii) *Right Half Open Interval* $[a, b)$ or $[a, b[= \{x \in R : a \leq x < b\}$.

(iv) *Left Half Open Interval* $(a, b]$ or $]a, b] = \{x \in R : a < x \leq b\}$.

On the real line, we represent these intervals as shown below:



OPEN & CLOSED INTERVALS

- OPEN & CLOSED INTERVALS

LENGTH OF AN INTERVAL The length of each of the intervals $[a, b]$, (a, b) $[a, b)$ and $(a, b]$ is $(b - a)$.

Examples on intervals

(i) $[-2, 3] = \{x \in R : -2 \leq x \leq 3\}$ (ii) $(-2, 3) = \{x \in R : -2 < x < 3\}$

(iii) $[-2, 3) = \{x \in R : -2 \leq x < 3\}$ (iv) $(-2, 3] = \{x \in R : -2 < x \leq 3\}$

POWER SET *The set of all subsets of a given set A is called the power set of A, denoted by* $P(A)$.

If $n(A) = m$ *then* $n[P(A)] = 2^m$.

- COMPLEX NUMBERS

Complex Numbers

$$a + bi$$

real imaginary

The **complex numbers** consist of all sums a + bi, where a and b are real numbers and i is the imaginary unit. The real part is a, and the imaginary part is bi.

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-Imaginary numbers were invented so that negative numbers would have square roots and certain equations would have solutions.

-These numbers were devised using an imaginary unit named i.

$$i = \sqrt{-1}$$

-The first four powers of i establish an important pattern and should be memorized.

Powers of i

$i^1 = i$ $\quad$ $i^2 = -1$ $\quad$ $i^3 = -i$ $\quad$ $i^4 = 1$

Thus, we have

$$\boxed{i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1.}$$

EXAMPLES (i) $i^{98} = i^{4\times 24+2} = (i^4)^{24} \times i^2 = i^2 = -1$. $[\because i^4 = 1]$

(ii) $i^{-98} = \dfrac{1}{i^{98}} = \dfrac{1}{i^{98}} \times \dfrac{i^2}{i^2} = \dfrac{i^2}{i^{(98+2)}} = \dfrac{-1}{1} = -1.$ $[\because\ i^{100} = 1 \text{ and } i^2 = -1]$

SOLVED EXAMPLES

EXAMPLE 1 *Evaluate:*

(i) i^{23} (ii) i^{998} (iii) i^{-998} (iv) i^{-71}

(v) $(\sqrt{-1})^{91}$ (vi) $(i^{37} \times i^{-61})$ (vii) i^{-1}

SOLUTION We have

(i) $i^{23} = i^{(4\times 5)+3} = (i^4)^5 \times i^3 = i^3 = -i.$ $[\because\ i^4 = 1]$

(ii) $i^{998} = i^{4 \times 249 + 2} = (i^4)^{249} \times i^2 = (1 \times i^2) = i^2 = -1.$ $[\because\ i^4 = 1]$

(iii) $i^{-998} = \frac{1}{i^{998}} \times \frac{i^2}{i^2} = \frac{i^2}{i^{1000}} = \frac{-1}{1} = -1. \quad [\because\ i^{1000} = (i^4)^{250} = 1]$

$$\text{(iv) } i^{-71} = \frac{1}{i^{71}} \times \frac{i}{i} = \frac{i}{i^{72}} = \frac{i}{(i^4)^{18}} = \frac{i}{1} = i. \quad [\because i^4 = 1]$$

v) $(\sqrt{-1})^{91} = i^{91} = i^{4\times 22+3} = (i^4)^{22} \times i^3 = 1 \times (-i) = -i, \quad [\because i^3 = i]$

$$\text{(vi) } i^{37} = i^{4\times 9+1} = (i^4)^9 \times i = 1 \times i = i.$$

$$i^{-61} = \frac{1}{i^{61}} \times \frac{i^3}{i^3} = \frac{i^3}{i^{64}} = \frac{-i}{(i^4)^{16}} = \frac{-i}{1} = -i.$$

$$\therefore \quad (i^{37} + i^{-61}) = i + (-i) = 0.$$

Evaluate:

(i) $\sqrt{-25} \times \sqrt{-49}$ (ii) $\sqrt{-36} \times \sqrt{16}$

SOLUTION We have

$$\text{(i) } \sqrt{-25} \times \sqrt{-49} = (5i) \times (7i) = (35 \times i^2) = 35 \times (-1) = -35.$$

(ii) $\sqrt{-36} \times \sqrt{16} = (6i) \times 4 = 24i.$

Evaluate $\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$.

SOLUTION We have

$$\sqrt{-16}+3\sqrt{-25}+\sqrt{-36}-\sqrt{-625}$$

$$=(4i+3\times 5i+6i-25i)=(4i+15i+6i-25i)=0.$$

CONJUGATE OF A COMPLEX NUMBER

Conjugate of a complex number $z = (a + ib)$ *is defined as,* $\bar{z} = (a - ib)$.

<u>EXAMPLES</u> (i) $\overline{(3+8i)} = (3-8i)$ (ii) $\overline{(-6-2i)} = (-6+2i)$ (iii) $-\bar{3} = -3$.

PRACTICE QUESTIONS

1. Evaluate: (i) i^{19} (ii) i^{62} (iii) i^{373}
2. Evaluate: (i) $(\sqrt{-1})^{192}$ (ii) $(\sqrt{-1})^{93}$ (iii) $(\sqrt{-1})^{30}$
3. Evaluate: (i) i^{-50} (ii) i^{-9} (iii) i^{-131}
4. Evaluate: (i) $\left(i^{41}+\frac{1}{i^{71}}\right)$ (ii) $\left(i^{53}+\frac{1}{i^{53}}\right)$

Prove that:

5. $1+i^2+i^4+i^6=0.$

6. $6i^{50}+5i^{33}-2i^{15}+6i^{48}=7i.$

7. $\frac{1}{i}-\frac{1}{i^2}+\frac{1}{i^3}-\frac{1}{i^4}=0.$

8. $(1+i^{10}+i^{20}+i^{30})$ is a real number.

9. $\left\{i^{21}-\left(\frac{1}{i}\right)^{46}\right\}^2=2i.$

10. $\left\{i^{18}+\left(\frac{1}{i}\right)^{25}\right\}^3=2(1-i).$

ANSWERS

1. (i) $-i$ (ii) -1 (iii) i
2. (i) 1 (ii) i (iii) -1
3. (i) -1 (ii) $-i$ (iii) i
4. (i) $2i$ (ii) 0

Lecture- 2

Fundamental Theorem of Algebra &
Solution of quadratic equations by
factorization Method

- **POLYNOMIAL**

Fundamental Theorem of Algebra

Fundamental Theorem of Algebra: [2011-12, 2013-14 2018-19]

Statement: A polynomial equation of degree n has at the most n roots.

Fundamental Theorem of Algebra

Example: Consider a polynomial $x^2 + 5x + 6$. Then, its corresponding polynomial equation is $x^2 + 5x + 6 = 0$.

The highest power of x is 2. Hence, the degree of equation is 2. According to Fundamental Theorem of Algebra, This equation has at most n roots.

Solution of quadratic equation in the complex number system

Quadratic Equation - A polynomial equation of a second degree is known as quadratic equation

General form of quadratic equation-

$$ax^2+bx+c=0$$

where $a, b, c \in R$ (set of real numbers) and $a \neq 0$

Method for solving quadratic equation

(i) Factorization Method

(ii) Quadratic formula (Sridharacharya Rule)

Example 1 : Check whether the following are quadratic equations:

(i) $(x-2)^2+1=2x-3$

(ii) $x(x+1)+8=(x+2)(x-2)$

(iii) $x(2x+3)=x^2+1$

(iv) $(x+2)^3=x^3-4$

(i) LHS $= (x-2)^2 + 1 = x^2 - 4x + 4 + 1 = x^2 - 4x + 5$

Therefore, $(x-2)^2 + 1 = 2x - 3$ can be rewritten as

$$x^2 - 4x + 5 = 2x - 3$$

i.e.,

$$x^2 - 6x + 8 = 0$$

It is of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is a quadratic equation.

(ii) Since $x(x + 1) + 8 = x^2 + x + 8$ and $(x + 2)(x - 2) = x^2 - 4$

Therefore, $x^2 + x + 8 = x^2 - 4$

i.e., $x + 12 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Therefore, the given equation is not a quadratic equation.

(iii) Here, LHS $= x(2x + 3) = 2x^2 + 3x$

So, $x(2x + 3) = x^2 + 1$ can be rewritten as

$$2x^2 + 3x = x^2 + 1$$

Therefore, we get $x^2 + 3x - 1 = 0$

It is of the form $ax^2 + bx + c = 0$.

So, the given equation is a quadratic equation.

(iv) Here, LHS $= (x + 2)^3 = x^3 + 6x^2 + 12x + 8$

Therefore, $(x + 2)^3 = x^3 - 4$ can be rewritten as

$$x^3 + 6x^2 + 12x + 8 = x^3 - 4$$

i.e., $6x^2 + 12x + 12 = 0$ or, $x^2 + 2x + 2 = 0$

It is of the form $ax^2 + bx + c = 0$.

So, the given equation is a quadratic equation.

Practice Questions(H.W.)

. Check whether the following are quadratic equations :

(i) $(x+1)^2=2(x-3)$

(ii) $x^2-2x=(-2)(3-x)$

(iii) $(x-2)(x+1)=(x-1)(x+3)$

(iv) $(x-3)(2x+1)=x(x+5)$

(i) Factorization Method

Suppose the quadratic equation

$$ax^2+bx+c=0, \quad \text{where } a \neq 0$$

1. Put all terms on one side of the equal sign, leaving zero on the other side
2. Factor.
3. Set each factor equal to zero
4. Solve each of these equations.
5. Check by inserting your answer in the original equation.

Example : Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Solution : We have

$$
\begin{aligned}
6x^2 - x - 2 &= 6x^2 + 3x - 4x - 2 \\
&= 3x\,(2x + 1) - 2\,(2x + 1) \\
&= (3x - 2)(2x + 1)
\end{aligned}
$$

The roots of $6x^2 - x - 2 = 0$ are the values of x for which $(3x - 2)(2x + 1) = 0$

Therefore, $3x - 2 = 0$ or $2x + 1 = 0$,

i.e., $$x = \frac{2}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

Therefore, the roots of $6x^2 - x - 2 = 0$ are $\frac{2}{3}$ and $-\frac{1}{2}$.

We verify the roots, by checking that $\frac{2}{3}$ and $-\frac{1}{2}$ satisfy $6x^2 - x - 2 = 0$.

Example : Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

Solution : $3x^2 - 2\sqrt{6}x + 2 = 3x^2 - \sqrt{6}x - \sqrt{6}x + 2$

$$= \sqrt{3}x\left(\sqrt{3}x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}x - \sqrt{2}\right)$$

$$= \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right)$$

So, the roots of the equation are the values of x for which

$$\left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$$

Now, $\sqrt{3}x - \sqrt{2} = 0$ for $x = \sqrt{\frac{2}{3}}$.

So, this root is repeated twice, one for each repeated factor $\sqrt{3}x - \sqrt{2}$.

Therefore, the roots of $3x^2 - 2\sqrt{6}x + 2 = 0$ are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$.

PRACTICE QUES

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Some Solved Questions

Q-1 Solve $x^2 - 5ix - 6 = 0$ by factorization method. [2019-20]

Some Solved Questions

Q-1 Solve $x^2 - 5ix - 6 = 0$ by factorization method. [2019-20]

Sol. $x^2 - 5ix - 6 = 0$

$x^2 - (2i + 3i)x - 6 = 0$ $\quad \because \left(\begin{array}{l} 2i + 3i = 5i \text{ and} \\ 2i \times 3i = -6 \end{array} \right)$

$x^2 - 2ix - 3ix + 6i^2 = 0$ $\quad (\because i^2 = -1)$

$x(x - 2i) - 3i(x - 2i) = 0$

$(x - 2i)(x - 3i) = 0$

$x = 2i$ and $x = 3i$

Solution set $= \{2i, 3i\}$

Q-2 Salve $x^2-5x+6=0$ by factorization method.

Q-2 Solve $x^2 - 5x + 6 = 0$ by factorization method.

Sol:

$$x^2 - 5x + 6 = 0$$

$$x^2 - (2+3)x + 6 = 0 \qquad \because (2+3 = 5 \text{ and } 2 \times 3 = 6)$$

$$x^2 - 2x - 3x + 6 = 0$$

$$x(x-2) - 3(x-2) = 0$$

$$(x-2)(x-3) = 0$$

$$x - 2 = 0, \quad x - 3 = 0$$

$$x = 2 \text{ and } x = 3$$

Solution set $= \{2, 3\}$

Q-3 Solve $x^2+3ix+10=0$

Q-3 Solve $x^2+3ix+10=0$

Sol. $x^2+3ix+10=0$

$x^2+(5i-2i)x+10=0$ $\quad \because (5i-2i=3i$ and $5i \times -2i = 10)$

$x^2+5ix-2ix+10=0$

$x^2+5ix-2ix-10i^2=0$ $\quad \because (i^2=-1)$

$x(x+5i)-2i(x+5i)=0$

$(x+5i)\ (x-2i)=0$

$(x+5i)=0, \quad (x-2i)=0$

$x=-5i$ and $x=2i$

Solution Set $=\{-5i, 2i\}$

Quadratic Formula

Let the given quadratic equation be

$$ax^2+bx+c=0, \text{ where } a \neq 0 \longrightarrow (1)$$

It is defined as follows (Roots of the equation (1))

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

The expression $D=(b^2-4ac)$ is called Discriminant. (This discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic equation)

If $D=0$ $\Rightarrow$ Repeated real number roots

If $D>0$ $\Rightarrow$ Two distinct real number roots

If $D<0$ $\Rightarrow$ Complex roots

Some Solved Questions

Q-1 Solve the equation: [2011-12]

$$x^2+3x+5=0$$

Some Solved Questions

Q-1 Solve the equation: [2011-12]

$$x^2+3x+5=0$$

Sol. Comparing the given equation with general form quadratic equation $ax^2+bx+c=0$

$$a=1,\ b=3,\ c=5$$

$$x = \frac{-3 \pm \sqrt{9-20}}{2}$$

$$x = \frac{-3 \pm \sqrt{-11}}{2} \qquad \because \sqrt{-1} = i$$

$$x = \frac{-3 \pm \sqrt{11}\, i}{2}$$

$$x = \frac{-3}{2} + \frac{i\sqrt{11}}{2}, \quad x = \frac{-3}{2} - \frac{i\sqrt{11}}{2}$$

Solution set $= \left\{ \frac{-3}{2} + \frac{i\sqrt{11}}{2},\ \frac{-3}{2} - \frac{i\sqrt{11}}{2} \right\}$

Q-2 For what value of K, [2018-19]

$(4-k)x^2 + (2k+4)x + (8k+1) = 0$ is a perfect square

Q-2 For what value of K, [2018-19]

$(4-k)x^2+(2k+4)x+(8k+1)=0$ is a perfect square

Sol. A polynomial (ax^2+bx+c) is a perfect square if $b^2-4ac=0$

So we have

$$a=(4-k),\ b=(2k+4),\ c=(8k+1)$$

$$b^2-4ac=0$$

$$(2k+4)^2-4(4-k)(8k+1)=0$$

$$4k^2+16+16k-4(32k+4-8k^2-k)=0$$

$$4k^2+\cancel{16}+16k-128k-\cancel{16}+32k^2+4k=0$$

$$36k^2-108k=0$$

$$36k(k-3)=0$$

$$k=0,\ k-3=0 \qquad 36\neq 0$$

$$\boxed{k=0,\ k=3}$$

Q-3 Solve: $x^2 - 1 = 0$ [2019-20]

Q-4 Solve: $(x+1)(x-2) + x = 0$ [2020-21]

Q-3 Solve: $x^2 - 1 = 0$ [2019-20]

Sol. We have, $x^2 = 1$

$$x = \pm\sqrt{1}$$

$$x = \pm 1$$

Solution Set $= \{-1, 1\}$

Q-4 Solve: $(x+1)(x-2) + x = 0$ [2020-21]

Sol. We have

$$(x+1)(x-2) + x = 0$$

$$x^2 - 2x + x - 2 + x = 0$$

$$x^2 - 2 = 0$$

$$x = \pm\sqrt{2}$$

Solution Set $= \{-\sqrt{2}, \sqrt{2}\}$

Q-5 Solve: $\sqrt{2}x^2 + x + \sqrt{2} = 0$

Q-6 Solve: $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

Q-5 Solve: $\sqrt{2}x^2+x+\sqrt{2}=0$

Sol. $a=\sqrt{2}, b=1, c=\sqrt{2}$, Roots are given by.

$$x=\frac{-1\pm\sqrt{1-8}}{2\sqrt{2}}$$

$$x=\frac{-1}{2\sqrt{2}}\pm\frac{i\sqrt{7}}{2}$$

Solution Set $=\left\{\frac{-1}{2\sqrt{2}}+\frac{\sqrt{7}}{2}i, -\frac{1}{2\sqrt{2}}-\frac{\sqrt{7}}{2}i\right\}$

Q-6 Solve: $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

Sol. $a=\sqrt{3}, b=-\sqrt{2}, c=3\sqrt{3}$

Roots are given by.

$$x=\frac{\sqrt{2}\pm\sqrt{2-36}}{2\sqrt{3}}$$

$$x=\frac{\sqrt{2}\pm\sqrt{34}i}{2\sqrt{3}}$$

Solution Set $=\left\{\frac{\sqrt{2}+\sqrt{34}i}{2\sqrt{3}}, \frac{\sqrt{2}-\sqrt{34}i}{2\sqrt{3}}\right\}$

Q-7 Solve: $x^2+5=0$

Sol. $x^2=-5$

$x=\pm\sqrt{5}\,i$

Solution Set $=\{-\sqrt{5}\,i,\ \sqrt{5}\,i\}$

Q-8 Solve: $3x^2-4x+\frac{20}{3}=0$

Sol. $a=3,\ b=-4,\ c=20/3$

Roots are given by

$$x=\frac{4\pm\sqrt{16-80}}{6}$$

$$x=\frac{2}{3}\pm\frac{\sqrt{-64}}{6}=\frac{2}{3}\pm\frac{4}{3}i$$

Solution Set $=\left\{\frac{2}{3}+\frac{4}{3}i,\ \frac{2}{3}-\frac{4}{3}i\right\}$

PRACTICE QUESTIONS

Solve:

1. $x^2+2=0$
2. $x^2+5=0$
3. $2x^2+1=0$
4. $x^2+x+1=0$
5. $x^2-x+2=0$
6. $x^2+2x+2=0$
7. $2x^2-4x+3=0$
8. $x^2+3x+5=0$
9. $\sqrt{5}x^2+x+\sqrt{5}=0$
10. $25x^2-30x+11=0$
11. $8x^2+2x+1=0$
12. $27x^2+10x+1=0$

ANSWERS

1. $\{i\sqrt{2}, -i\sqrt{2}\}$ 2. $\{i\sqrt{5}, -i\sqrt{5}\}$ 3. $\left\{\frac{i}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right\}$

4. $\left\{\frac{-1}{2}+\frac{\sqrt{3}}{2}i, \frac{-1}{2}-\frac{\sqrt{3}}{2}i\right\}$ 5. $\left\{\frac{1}{2}+\frac{\sqrt{7}}{2}i, \frac{1}{2}-\frac{\sqrt{7}}{2}i\right\}$ 6. $\{-1+i, -1-i\}$

7. $\left\{1+\frac{1}{\sqrt{2}}i, 1-\frac{1}{\sqrt{2}}i\right\}$ 8. $\left\{\frac{-3}{2}+\frac{\sqrt{11}}{2}i, \frac{-3}{2}-\frac{\sqrt{11}}{2}i\right\}$

9. $\left\{\frac{-1}{2\sqrt{5}}+\frac{\sqrt{19}}{2\sqrt{5}}i, \frac{-1}{2\sqrt{5}}-\frac{\sqrt{19}}{2\sqrt{5}}i\right\}$ 10. $\left\{\frac{3}{5}+\frac{\sqrt{2}}{5}i, \frac{3}{5}-\frac{\sqrt{2}}{5}i\right\}$

11. $\left\{\frac{-1}{8}+\frac{\sqrt{7}}{8}i, \frac{-1}{8}-\frac{\sqrt{7}}{8}i\right\}$

12. $\left\{\frac{-5}{27}+\frac{\sqrt{2}}{27}i, \frac{-5}{27}-\frac{\sqrt{2}}{27}i\right\}$

- THANK YOU

Lecture- 3

Fundamental Theorem of Algebra & Solution of quadratic equations by Discriminant Rule

Degree of a Polynomial

Example: Consider a polynomial $x^2 + 5x + 6$. Then, its corresponding polynomial equation is $x^2 + 5x + 6 = 0$.

The highest power of x is 2. Hence, the degree of equation is 2. According to Fundamental Theorem of Algebra, This equation has at most n roots.

Fundamental Theorem of Algebra

Fundamental Theorem of Algebra: [2011-12, 2013-14 2018-19]

Statement: A polynomial equation of degree n has at the most n roots.

Solution of quadratic equation in the complex number system

Quadratic Equation - A polynomial equation of a second degree is known as quadratic equation

General form of quadratic equation-

$$ax^2+bx+c=0$$

where $a, b, c \in R$ (set of real numbers) and $a \neq 0$

Method for solving quadratic equation

(i) Factorization Method

(ii) Quadratic formula (Sridharacharya Rule)

Quadratic Formula

Let the given quadratic equation be

$$ax^2+bx+c=0, \text{ where } a\neq 0 \longrightarrow (1)$$

It is defined as follows (Roots of the equation (1))

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

The expression $D=(b^2-4ac)$ is called Discriminant. (This discriminant determines the nature of roots of the quadratic equation based on the coefficients of the quadratic equation)

If $D=0$ $\Rightarrow$ Repeated real number roots

If $D>0$ $\Rightarrow$ Two distinct real number roots

If $D<0$ $\Rightarrow$ Complex roots

EXAMPLE : *Solve:* $x^2+3=0.$

SOLUTION We have

$$x^2+3=0 \Rightarrow x^2=-3 \Rightarrow x=\pm\sqrt{-3}=\pm i\sqrt{3}.$$

$\therefore$ solution set $=\{i\sqrt{3}, -i\sqrt{3}\}.$

EXAMPLE . *Solve:* $x^2 + 3x + 9 = 0.$

SOLUTION The given equation is $x^2 + 3x + 9 = 0$.

This is of the form $ax^2 + bx + c = 0$, where $a = 1, b = 3$ and $c = 9$.

$$\therefore \quad (b^2 - 4ac) = (3^2 - 4 \times 1 \times 9) = (9 - 36) = -27 < 0.$$

So, the given equation has complex roots.

These roots are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{-27}}{2 \times 1} \qquad [\because\ b^2 - 4ac = -27]$$

$$= \frac{-3 \pm i\sqrt{27}}{2} = \frac{-3 \pm i3\sqrt{3}}{2}.$$

$$\therefore \quad \text{solution set} = \left\{\frac{-3 + i3\sqrt{3}}{2}, \frac{-3 - i3\sqrt{3}}{2}\right\}$$

$$= \left\{\frac{-3}{2} + \frac{3\sqrt{3}}{2}i, \frac{-3}{2} - \frac{3\sqrt{3}}{2}i\right\}.$$

EXAMPLE 3 *Solve:* $9x^2 + 10x + 3 = 0.$

SOLUTION The given equation is $9x^2 + 10x + 3 = 0.$

This is of the form $ax^2 + bx + c = 0$, where $a = 9, b = 10$ and $c = 3.$

$\therefore \quad (b^2 - 4ac) = \{(10)^2 - 4 \times 9 \times 3\} = (100 - 108) = -8 < 0.$

So, the given equation has complex roots.

These roots are given by

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-10 \pm \sqrt{-8}}{2 \times 9} \qquad [\because (b^2-4ac) = -8]$$

$$= \frac{-10 \pm i2\sqrt{2}}{18} = \frac{-5 \pm i\sqrt{2}}{9}.$$

$$\therefore \quad \text{solution set} = \left\{\frac{-5+i\sqrt{2}}{9}, \frac{-5-i\sqrt{2}}{9}\right\} = \left\{\frac{-5}{9}+\frac{\sqrt{2}}{9}i, \frac{-5}{9}-\frac{\sqrt{2}}{9}i\right\}.$$

EXAMPLE Solve: $3x^2+8ix+3=0$.

SOLUTION The given equation is

$$3x^2+8ix+3=0.$$

This is of the form $ax^2+bx+c=0$, where $a=3$, $b=8i$ and $c=3$.

$\therefore \quad (b^2-4ac)=\{(8i)^2-4\times3\times3\}=(-64-36)=-100<0.$

So, the given equation has complex roots.

These roots are given by

$$\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-8i\pm\sqrt{-100}}{2\times3} \qquad [\because\ b^2-4ac=-100]$$

$$=\frac{-8i\pm10i}{6}=\frac{-4i\pm5i}{3}.$$

Thus, the roots of the given equation are

$$\frac{-4i+5i}{3}=\frac{i}{3} \text{ and } \frac{-4i-5i}{3}=\frac{-9i}{3}=-3i.$$

$\therefore \quad$ solution set $=\left\{\frac{i}{3},-3i\right\}$.

Q-7 Solve: $x^2+5=0$

Sol. $x^2 = -5$

$x = \pm\sqrt{5}\,i$

Solution Set $= \{-\sqrt{5}i, \sqrt{5}i\}$

Q-8 Solve: $3x^2-4x+\frac{20}{3}=0$

Sol. $a=3$, $b=-4$, $c=20/3$

Roots are given by

$$x = \frac{4 \pm \sqrt{16-80}}{6}$$

$$x = \frac{2}{3} \pm \frac{\sqrt{-64}}{6} = \frac{2}{3} \pm \frac{4}{3}i$$

Solution Set $= \left\{\frac{2}{3}+\frac{4}{3}i, \frac{2}{3}-\frac{4}{3}i\right\}$

PRACTICE QUESTIONS

Solve:

1. $x^2+2=0$
2. $x^2+5=0$
3. $2x^2+1=0$
4. $x^2+x+1=0$
5. $x^2-x+2=0$
6. $x^2+2x+2=0$
7. $2x^2-4x+3=0$
8. $x^2+3x+5=0$
9. $\sqrt{5}x^2+x+\sqrt{5}=0$
10. $25x^2-30x+11=0$
11. $8x^2+2x+1=0$
12. $27x^2+10x+1=0$

ANSWERS

1. $\{i\sqrt{2}, -i\sqrt{2}\}$
2. $\{i\sqrt{5}, -i\sqrt{5}\}$
3. $\left\{\frac{i}{\sqrt{2}}, \frac{-i}{\sqrt{2}}\right\}$
4. $\left\{\frac{-1}{2}+\frac{\sqrt{3}}{2}i, \frac{-1}{2}-\frac{\sqrt{3}}{2}i\right\}$
5. $\left\{\frac{1}{2}+\frac{\sqrt{7}}{2}i, \frac{1}{2}-\frac{\sqrt{7}}{2}i\right\}$
6. $\{-1+i, -1-i\}$
7. $\left\{1+\frac{1}{\sqrt{2}}i, 1-\frac{1}{\sqrt{2}}i\right\}$
8. $\left\{\frac{-3}{2}+\frac{\sqrt{11}}{2}i, \frac{-3}{2}-\frac{\sqrt{11}}{2}i\right\}$
9. $\left\{\frac{-1}{2\sqrt{5}}+\frac{\sqrt{19}}{2\sqrt{5}}i, \frac{-1}{2\sqrt{5}}-\frac{\sqrt{19}}{2\sqrt{5}}i\right\}$
10. $\left\{\frac{3}{5}+\frac{\sqrt{2}}{5}i, \frac{3}{5}-\frac{\sqrt{2}}{5}i\right\}$

11. $\left\{\dfrac{-1}{8}+\dfrac{\sqrt{7}}{8}i, \dfrac{-1}{8}-\dfrac{\sqrt{7}}{8}i\right\}$

12. $\left\{\dfrac{-5}{27}+\dfrac{\sqrt{2}}{27}i, \dfrac{-5}{27}-\dfrac{\sqrt{2}}{27}i\right\}$

- THANK YOU

Lecture- 5

Introduction of linear equation and Linear inequalities

OPEN & CLOSED INTERVALS

- OPEN & CLOSED INTERVALS

INTERVALS AS SUBSETS OF *R*

Let $a, b \in R$ and $a < b$. Then, we define:

(i) *Closed Interval* $[a, b] = \{x \in R : a \leq x \leq b\}$.

(ii) *Open Interval* (a, b) or $]a, b[= \{x \in R : a < x < b\}$.

(iii) *Right Half Open Interval* $[a, b)$ or $[a, b[= \{x \in R : a \leq x < b\}$.

(iv) *Left Half Open Interval* $(a, b]$ or $]a, b] = \{x \in R : a < x \leq b\}$.

On the real line, we represent these intervals as shown below:

[a, b]

(a, b)

[a, b)

(a, b]

Examples on intervals

(i) $[-2,3]=\{x\in R:-2\leq x\leq 3\}$

(ii) $(-2,3)=\{x\in R:-2<x<3\}$

(iii) $[-2,3)=\{x\in R:-2\leq x<3\}$

(iv) $(-2,3]=\{x\in R:-2<x\leq 3\}$

Linear Equation & Inequation (Inequality) in One Variable

Linear Equations in One variable: An equation which is expressed in the form of $ax+b=0$, where $a, b \in R$ $a \neq 0$

for example- $2x+3=0$, $2x+3=8$

Linear Inequations (Inequalities) in one variable:

[2019-20
2020-21]

Inequalities of the form

i) $ax+b<0$ (ii) $ax+b \leq c$ (iii) $ax+b>c$ (iv) $ax+b \geq c$

where a, b, c are real numbers, and x is a variable

Rules for solving an equation

1. Adding the same number or expression to each side of inequation does not change the inequality.

2. Subtracting the same number or expression from each side of an inequation does not change the inequality.

3. Multiplying (or dividing) each side of an inequation by the same positive number does not change the inequality.

4. Multiplying (or dividing) each side of an inequation by the same negative number reverses the inequality.

Some Solved Questions

Q-1 Solve: $5x < 24$ when (i) $x \in N$, (ii) $x \in Z$

Sol. $5x < 24$

$\Rightarrow \frac{5x}{5} < \frac{24}{5}$ [Dividing both sides by 5]

$\Rightarrow \boxed{x < \frac{24}{5}}$

So (i) Solution Set $= \{x \in N : x < 4.8\}$
$= \{1, 2, 3, 4\}$

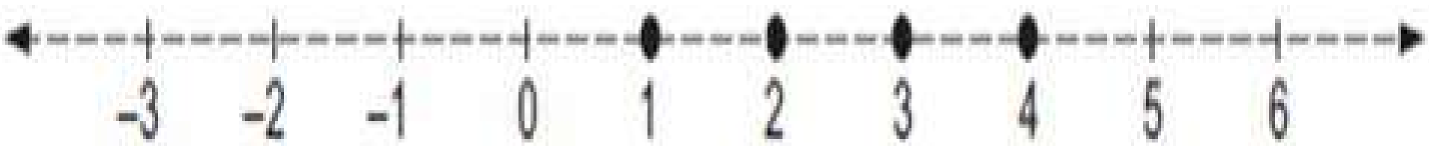
(ii) Solution set $= \{x \in Z : x < 4.8\}$
$= \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$

$$\Rightarrow x < 4.8.$$

$$\text{Solution set} = \{x \in N : x < 4.8\}$$

$$= \{1, 2, 3, 4\}.$$

On the number line, we may represent it as shown below.



The darkened circles indicate the natural numbers contained in the set.

EXAMPLE Write down the solution set of the inequation $x < 6$, when the replacement set is (i) N, (ii) W, (iii) Z.

SOLUTION (i) Solution set $= \{x \in N : x < 6\} = \{1, 2, 3, 4, 5\}$.

(ii) Solution set $= \{x \in W : x < 6\} = \{0, 1, 2, 3, 4, 5\}$.

(iii) Solution set $= \{x \in Z : x < 6\} = \{5, 4, 3, 2, 1, 0, -1, -2, -3, \ldots\}$.

Q-2 Solve: $5x - 3 < 3x + 1$ [2011-12]

Sol. $5x - 3 < 3x + 1$

$\Rightarrow 5x < 3x + 4$ [Adding +3 to both sides]

$\Rightarrow 2x < 4$ [Adding −3x to both sides]

$\boxed{x < 2}$ [Dividing both sides by 2]

Q-3 Solve the linear inequality [2015-16]

$4x + 3 < 5x + 7$

Sol. $\Rightarrow 4x < 5x + 4$ [Adding −3 to both sides]

$\Rightarrow -x < 4$ [Adding −5x to both sides]

$\boxed{x > -4}$

Q-4 Solve: $5x-3<3x-1$, where x is a real number [2020-21]

Sol. $5x-3<3x-1$

$\Rightarrow 5x<3x+2$ [Adding +3 to both sides]

$\Rightarrow 2x<2$ [Adding −3x to both sides]

$\boxed{x<1}$ [Dividing both sides by 2]

Q-5 Solve: $x-4\geq 10$ [2020-21]

Sol. $x-4\geq 10$

$\boxed{x\geq 14}$ [Adding +4 on both sides]

Q-6 Solve: $\frac{(2x+4)}{(x-1)}\geq 5$ [2020-21]

Sol. $\frac{(2x+4)}{(x-1)}\geq 5$

$\Rightarrow \frac{(2x+4)}{(x-1)}-5\geq 0$ [Adding −5 to both sides]

$$\Rightarrow \frac{(2x+4)-5(x-1)}{(x-1)} \geq 0$$

$$\Rightarrow \frac{(-3x+9)}{(x-1)} \geq 0$$

$\because \left[\frac{a}{b} \geq 0 \Rightarrow \text{Either } a \geq 0, b > 0 \text{ or } a \leq 0, b < 0\right.$

$\therefore$ Either $\{(-3x+9) \geq 0$ and $(x-1) > 0\}$ or $\{(-3x+9) \leq 0$ and $(x-1) < 0\}$

$\Rightarrow \{-3x \geq -9$ and $x > 1\}$ or $\{-3x \leq -9$ and $x < 1\}$

$\Rightarrow \{-x \geq -3$ and $x > 1\}$ or $\{-x \leq -3$ and $x < 1\}$

$\Rightarrow \{x \leq 3$ and $x > 1\}$ or $\{x \geq 3$ and $x < 1\}$

$\Rightarrow \{1 < x \leq 3\}$ or ϕ

$\because [x \geq 3$ and $x < 1$ is not possible$]$
So $\phi =$ null set

$\Rightarrow \quad 1 < x \leq 3$

$\Rightarrow \quad x \in (1, 3]$

$$\boxed{\text{Solution Set } = (1, 3]}$$

Q-7 Solve: $\frac{x-3}{x+4} < 0$

Sol. We know that

$\frac{a}{b} < 0$ when $(a > 0 \,\&\, b < 0)$ or $(a < 0 \,\&\, b > 0)$

$$\frac{x-3}{x+4} < 0$$

Either $(x-3 > 0 \,\&\, x+4 < 0)$ or $(x-3 < 0$ and $x+4 > 0)$

Either $(x > 3$ and $x < -4)$ or $(x < 3$ and $x > -4)$

ϕ or $(-4 < x < 3)$

$-4 < x < 3$

$$\boxed{\text{Solution set} = (-4, 3)}$$

EXAMPLE

Solve $12 + 1\frac{5}{6}x \leq 5 + 3x$ when (i) $x \in N$, (ii) $x \in R$.

Draw the graph of the solution set in each case.

SOLUTION $12 + 1\frac{5}{6}x \le 5 + 3x$

$$\Rightarrow \quad 12 + \frac{11}{6}x \le 5 + 3x$$

$$\Rightarrow \quad 72 + 11x \le 30 + 18x \qquad \text{[multiplying both sides by 6]}$$

$$\Rightarrow \quad 11x \le 18x - 42 \qquad \text{[adding } -72 \text{ to both sides]}$$

$$\Rightarrow \quad -7x \le -42 \qquad \text{[adding } -18x \text{ to both sides]}$$

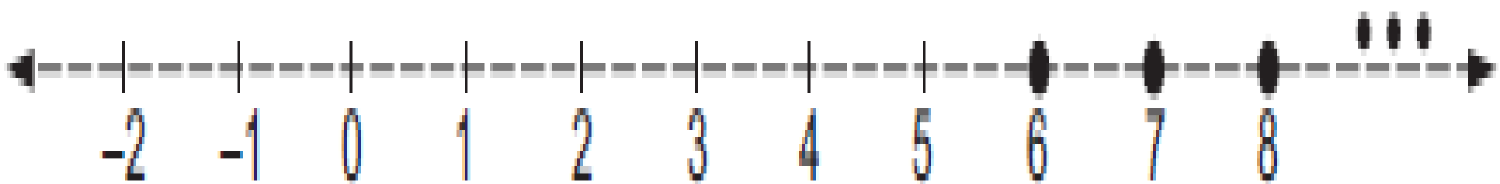
$$\Rightarrow \quad x \ge 6 \qquad \text{[dividing both sides by } -7\text{].}$$

$\Rightarrow \quad x \geq 6$

(i) Solution set $= \{x \in N : x \geq 6\}$

$= \{6, 7, 8, 9, \ldots\}.$

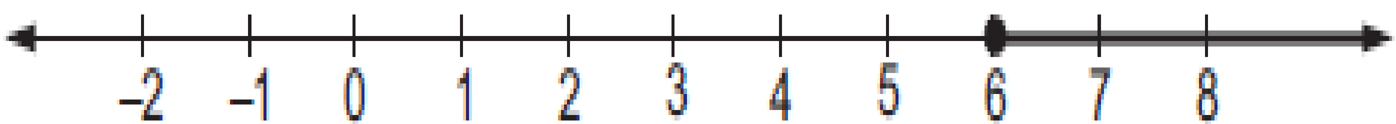
The graph of this set is the number line, shown below.



The darkened circles indicate the natural numbers contained in the set. Three dots above the right part of the line show that the natural numbers are continued indefinitely.

(ii) Solution set = $\{x \in R : x \geq 6\} = [6, \infty[$.

The graph of this set is shown below.



This graph consists of 6 and all real numbers greater than 6.

PRACTICE QUESTIONS

1. Fill in the blanks with correct inequality sign ($>, <, \geq, \leq$).

(i) $5x < 20 \Rightarrow x \ldots\ldots 4$

(ii) $-3x > 9 \Rightarrow x \ldots\ldots -3$

(iii) $4x > -16 \Rightarrow x \ldots\ldots -4$

(iv) $-6x \leq -18 \Rightarrow x \ldots\ldots 3$

(v) $x > -3 \Rightarrow -2x \ldots\ldots 6$

(vi) $a < b$ and $c < 0 \Rightarrow \dfrac{a}{c} \ldots\ldots \dfrac{b}{c}$

(vii) $p - q = -3 \Rightarrow p \ldots\ldots q$

(viii) $u - v = 2 \Rightarrow u \ldots\ldots v$

ANSWERS

1. (i) $<$ (ii) $<$ (iii) $>$ (iv) $\geq$ (v) $<$ (vi) $>$ (vii) $<$ (viii) $>$

- THANK YOU

Lecture- 6 & 7

Solution of linear inequalities in one variable representation on the number line

Some Solved Examples

Q. ① Solve the system of inequalities: [2011-12]

$$3x-7<5+x$$
$$11-5x \le 1$$

And represent the solutions on the number line.

Sol. We have

$3x-7<5+x$

$2x-7<5$ [Adding $-x$ to both sides]

$2x<12$ [Adding $+7$ to both sides]

$x<6$ [Dividing both sides by 2] → ①

Now, $11-5x \le 1$

[Adding -11 to both sides]

$-5x \le -10$

[Dividing both sides by 5]

$-x \le -2$

$x \ge 2$ → ②

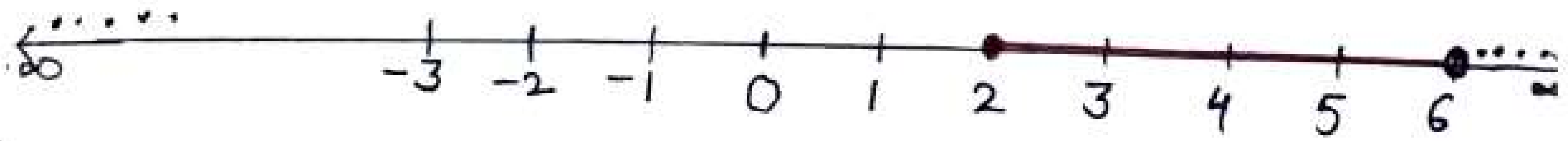
from ① & ②, Solution set for the given system is

$= \{x<6\} \cap \{x \ge 2\}$

$$= (-\infty, 6) \cap [2, \infty)$$

$$= [2, 6)$$

The solution set on the numberline may be represente as shown below.



Q-2- Solve: $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$. Show that the graph of the solution on the numberline. [2011-12]

Sol. We have, $\frac{3x-4}{2} \geq \frac{x+1}{4} - 1$

$$\Rightarrow \quad 4(3x-4) \geq 2(x+1) - 8$$

$\Rightarrow \quad 12x - 16 \geq 2x - 6$ [Adding +16 to both sides]

$\Rightarrow \quad 12x \geq 2x + 10$ [Adding $-2x$ to both sides]

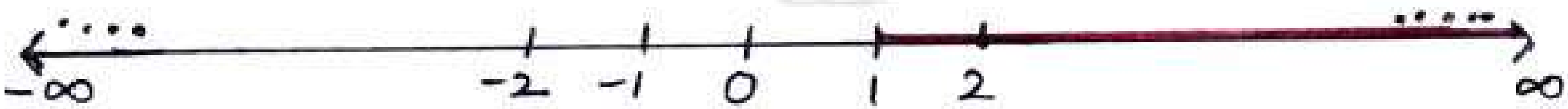
$\Rightarrow \quad 10x \geq 10$ [Dividing both sides by 10]

$\Rightarrow \quad x \geq 1$

$\Rightarrow \quad x \in [1, \infty)$

$$\boxed{\text{Solution set} = \{[1, \infty)\}}$$

On number line, we may represent it as shown below.



Q-3. Solve the inequality and represent it on number line

$$5(2x-7)-3(2x+3)\le 0,\quad 2x+19\le 6x+47$$

[2018-19]

Sol. We have,

$$5(2x-7)-3(2x+3)\le 0$$
$$10x-35-6x-9\le 0$$
$$4x-44\le 0$$
$$4x\le 44$$
$$x\le 11 \qquad \text{[Dividing both sides by 4]}$$
$$x\in(-\infty, 11] \text{ - - - - - - - - } \rightarrow (1)$$

Now, $2x+19\le 6x+47$

$$2x\le 6x+28 \qquad \text{[Adding -19 to both sides]}$$
$$-4x\le 28 \qquad \text{[Adding } -6x \text{ to both sides]}$$
$$-x\le 7 \qquad \text{[Dividing both sides by 4]}$$

$x \geq -7$

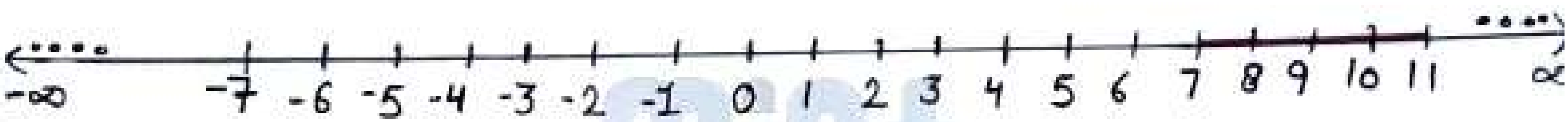
$x \in [-7, \infty)$ ---------- → (2)

Solution set for the given system is

$= \{(-\infty, 11] \cap [-7, \infty)\}$

$= \{[-7, 11]\}$

The solution set on the number line may be represented as shown below.



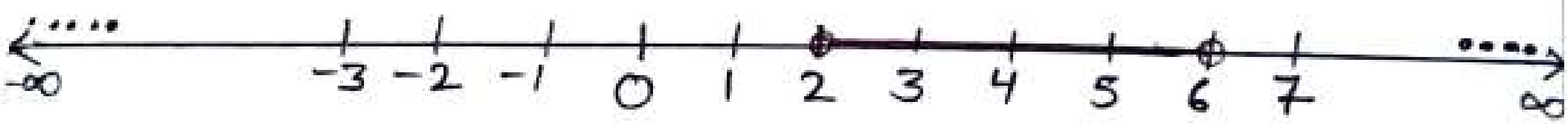
Q-4 Solve: $|4-x|<2$ and draw the graph of the solution set.

Sol. We know that, $|x|<a \Leftrightarrow -a<x<a$

$$\therefore \quad |4-x|<2$$
$$\Leftrightarrow -2<4-x<2$$
$$\Leftrightarrow -2<4-x \text{ and } 4-x<2$$
$$\Leftrightarrow -2-4<-x \text{ and } -x<2-4$$
$$\Leftrightarrow -6<-x \text{ and } -x<-2$$
$$\Leftrightarrow 6>x \text{ and } x>2$$
$$\Leftrightarrow 2<x<6$$

Solution set $= \{x \in R : 2<x<6\} = (2,6)$

We may represent it on the number line as shown below.



Q5. Solve: $|3-4x| \geq 9$, $x \in R$ and draw the graph of the solution set.

Sol. We have, $|x| \geq a \Leftrightarrow x \geq a$ or $x \leq -a$

$\therefore |3-4x| \geq 9 \Leftrightarrow 3-4x \geq 9$ or $3-4x \leq -9$

$\Leftrightarrow -4x \geq 9-3$ or $-4x \leq -9-3$

$\Leftrightarrow -4x \geq 6$ or $-4x \leq -12$

$\Leftrightarrow x \leq -\frac{3}{2}$ or $x \geq 3$

$\Leftrightarrow x \in \left(-\infty, \frac{3}{2}\right) \cup [3, \infty)$

Solution set $= \left\{\left(-\infty, \frac{3}{2}\right) \cup [3, \infty)\right\}$

We may represent it on the number line, as shown below.

Q-6 Solve: $|x-1|+|x-2| \geq 4, x \in R$ and draw the graph of the solution set.

Sol. We have, $x-1=0$ & $x-2=0$, $\Rightarrow x=1,2$ as critical points. These points divide the whole real line into three parts namely $(-\infty,1)$, $[1,2)$ and $[2,\infty)$

So there are arises three cases,

Case I : $-\infty < x < 1$

Case II : $1 \leq x < 2$

Case III : $2 \leq x < \infty$

Case I : When $-\infty < x < 1$

In this case, $x-1<0$ and $x-2<0$

So,

$$|x-1|+|x-2| = -(x-1)-(x-2) \quad \text{when } -\infty<x<1$$

$$= -2x+3$$

So we have, $|x-1|+|x-2| \geq 4$

$$\Rightarrow -2x+3 \geq 4 \Rightarrow -2x \geq 4-3 \Rightarrow -2x \geq 1$$

$$\Rightarrow x \leq -\frac{1}{2}$$

$$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right]$$

Solution set $= \left\{ \left(-\infty, -\frac{1}{2}\right] \right\}$ when $-\infty<x<1$ ——(1)

Case II: When $1 \le x < 2$

In this case, $x-1 \geq 0$ and $x-2 < 0$

$$|x-1| + |x-2| = (x-1) - (x-2) = -\cancel{x} + 2 + \cancel{x} - 1$$
$$= 1$$

So we have, $|x-1| + |x-2| \geq 4$

$$\cancel{x} - 1 - \cancel{x} + 2 \geq 4$$

$$+1 \geq 4 \text{ (which is absurd)}$$

So the given inequation has no solution in $[1,2)$. $\longrightarrow (2)$

Case III :

When $2 \leq x < \infty$

In this case, $x-2 \geq 0$ & $x-1 \geq 0$

So $|x-1| + |x-2| \geq 4$ $\quad$ $|x-1| = x-1$ &

$|x-2| = x-2$

$x-1+x-2 \geq 4$

$2x-3 \geq 4$

$2x \geq 7$

$x \geq 7/2$ $\quad$ Also $x \geq 2$

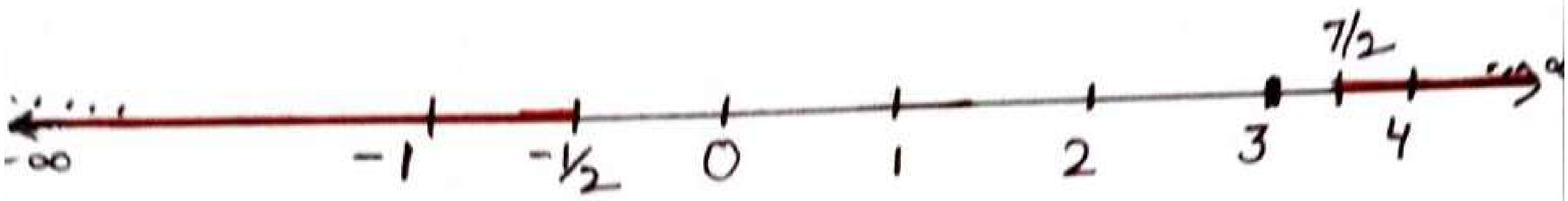
So Solution set in this case $\left\{\left[\frac{7}{2},\infty\right)\cap(2,\infty)\right\}$

$= \left\{\left[\frac{7}{2},\infty\right)\right\}$ ---→ (3)

Hence from all the above cases, we have

$$\text{Solution set} = \left(-\infty, \frac{-1}{2}\right] \cup \left[\frac{7}{2}, \infty\right)$$

The solution set on the number line may be represented as shown below.



Q-7 Find all pairs of consecutive odd positive integers both of which are smaller than 10 such their sum is more than 11.

Sol. Let the required consecutive odd positive integers be x and $x+2$ then

$\Rightarrow x+2<10$ and $x+(x+2)>11$

$\Rightarrow x<8$ and $2x+2>11$

$\Rightarrow x<8$ and $2x>9$

$\Rightarrow x<8$ and $x>9/2$

$\Rightarrow \frac{9}{2}<x<8$

$$\Rightarrow \quad \frac{9}{2} < x < 8$$

$$\Rightarrow \quad 4.5 < x < 8$$

So x can take the odd integral values 5 and 7.

Hence the required pairs of odd integers are $(5,7)$ and $(7,9)$.

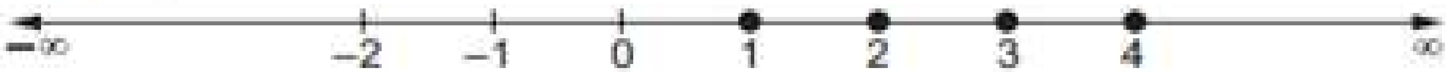
PRACTICE QUESTIONS

Solve each of the following inequations and represent the solution set on the number line.

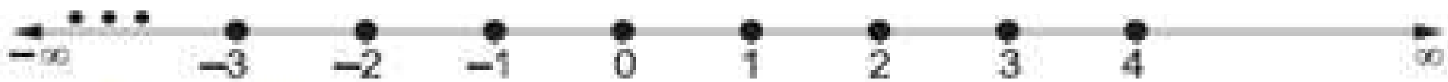
2. $6x \le 25$, where (i) $x \in N$, (ii) $x \in Z$.
3. $-2x > 5$, where (i) $x \in Z$, (ii) $x \in R$.
4. $3x + 8 > 2$, where (i) $x \in Z$, (ii) $x \in R$.
5. $5x + 2 < 17$, where (i) $x \in Z$, (ii) $x \in R$.

ANSWERS

2. (i) $\{1, 2, 3, 4\}$



(ii) $\{\ldots, -3, -2, -1, 0, 1, 2, 3, 4\}$

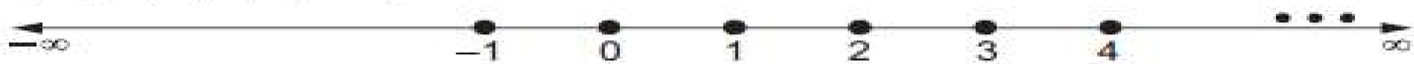


3. (i) $\{-3, -4, -5, -6, \ldots\}$

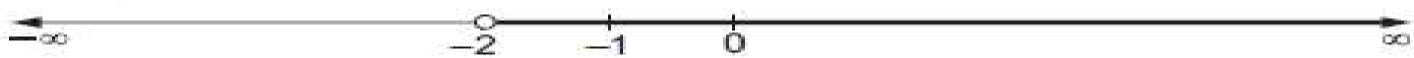
(ii) $(-\infty, -2.5)$

ANSWERS

4. (i) $\{-1, 0, 1, 2, 3, 4, \ldots\}$

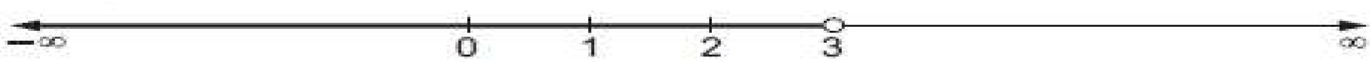


(ii) $(-2, \infty)$



5. (i) $\{2, 1, 0, -1, -2, \ldots\}$

(ii) $(-\infty, 3)$



- THANK YOU

Lecture- 8

Graphical Solution of Linear inequalities in two Variables

Graphical Solution of linear inequalities in two variables :

Linear Inequations in two variables -

Form : (i) $ax+by+c>0$ (ii) $ax+by+c\geq 0$

(iii) $ax+by+c<0$ (iv) $ax+by+c\leq 0$

$$\left.\begin{array}{ll} \text{(i) } ax+by+c>0 & \text{(ii) } ax+by+c\geq 0 \\ \text{(iii) } ax+by+c<0 & \text{(iv) } ax+by+c\leq 0 \end{array}\right\} \rightarrow (1)$$

Solution set: The set of all ordered pairs (x, y) which satisfy the given inequation is called the solution set of the inequation.

Graph of a linear inequation: Let an inequation of any types ① are given then to solve we proceed according to steps given below.

Step 1 Consider the equation, $ax+by+c=0$

Draw the graph of this equation, which is a line.

In case of strict inequation $>$ or $<$ draw the line dotted, other wise make it thick.

This line divides the plane into two equal parts.

Step 2 Choose a point [if possible (0,0)], not lying on this line. If this point satisfies the given inequation then shade the part of the plane containing this point, otherwise shade the other part.

The shaded portion represent the solution set of the given inequation.
(Dotted line is not part of the solution while thick line is a part of it)

Q-1 Solve: $3x+4y \le 12$ graphically.

Sol. Consider the equation.

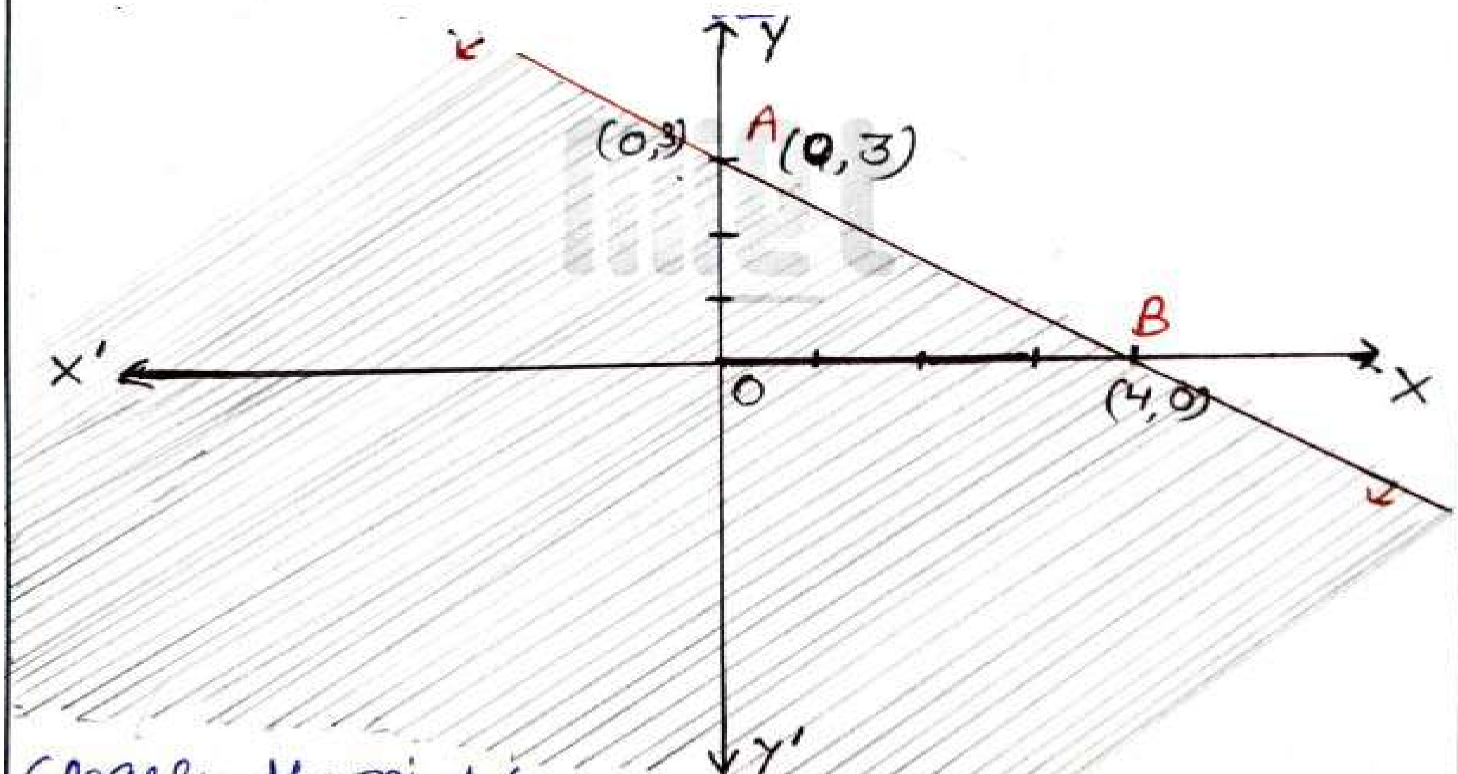
$$3x+4y=12 \longrightarrow (1)$$

The values of (x, y) satisfying the equation (1)

x	4	0
y	0	3

Plot the points $A(4,0)$ & $B(0,3)$ on a graph paper, then the line AB represents $3x+4y=12$.

This line divides the plane of the paper into two equal parts.



Clearly the point (0,0) satisfies $3x+4y \le 12$
So The shaded part of the plan together with all points on the line AB constitutes the solution set of the inequation $3x+4y \le 12$.

Q-2: Draw the graph of the solution set of the inequation $2x - y \geq 1$

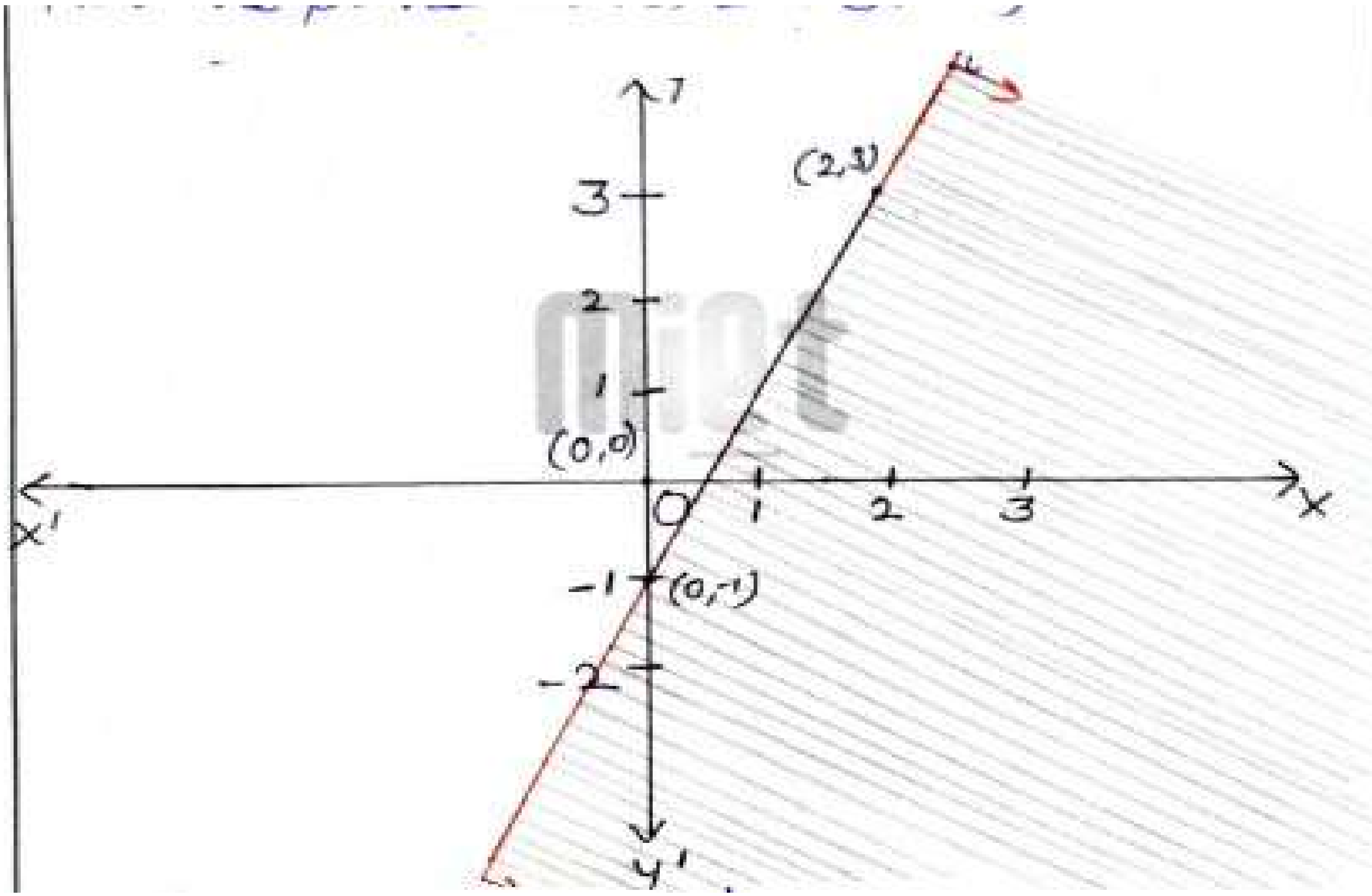
Sol. Consider the equation $2x - y = 1$

The values of (x, y) satisfying $2x - y = 1$ are

x	2	0
y	3	-1

Plot the points $A(2,3)$ and $B(0,-1)$

Then the line AB represents $2x - y = 1$


Y
3
2
1
(0,0)
(2,3)
X'
O
1
2
3
X
-1
(0,-1)
-2
Y'

Clearly the point (0,0) does not lie on $(2x-y=1)$
So the shaded part of the plane together with all points on the line AB constitutes the solution set of the inequation $2x-y \geq 1$

Q-3 Solve the following system of inequalities by graphically: [2015-16]

$2x+y \geq 6$ ---------- ①

$3x+4y \leq 12$ ---------- ②

Sol. First, we draw the graph of $2x+y=6$

(Plot points $A(0,6)$ and $B(3,0)$ satisfies $2x+y=6$)

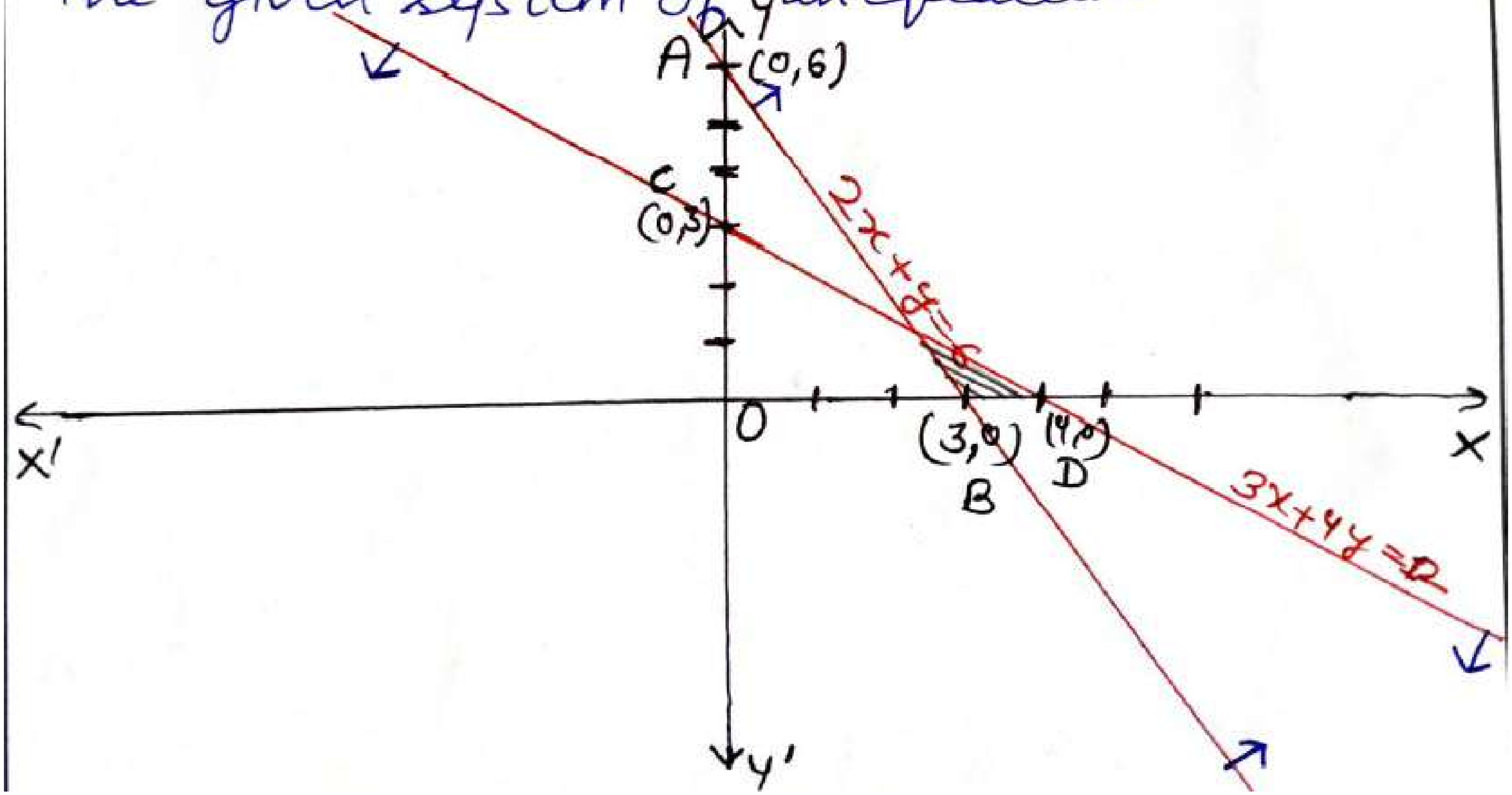
Then line AB represents $2x+1y=6$ and $(0,0)$ does not satisfies $2x+y \geq 6$

Secondly, we draw the graph of $3x+4y=12$

(Plot points $C(0,3)$ and $D(4,0)$ satisfies $3x+4y=12$)

Then line CD represents $3x+4y=12$ and $(0,0)$ satisfies $3x+4y \leq 12$

The intersection of these lines is the shaded part which is represents the solution of the given system of inequalities.



Q-2 Exhibit graphically the solution set of linear inequalities : [2019-20]

$3x + 4y \geq 12$ ——— (1)

$x \geq 0$, ——— (2)

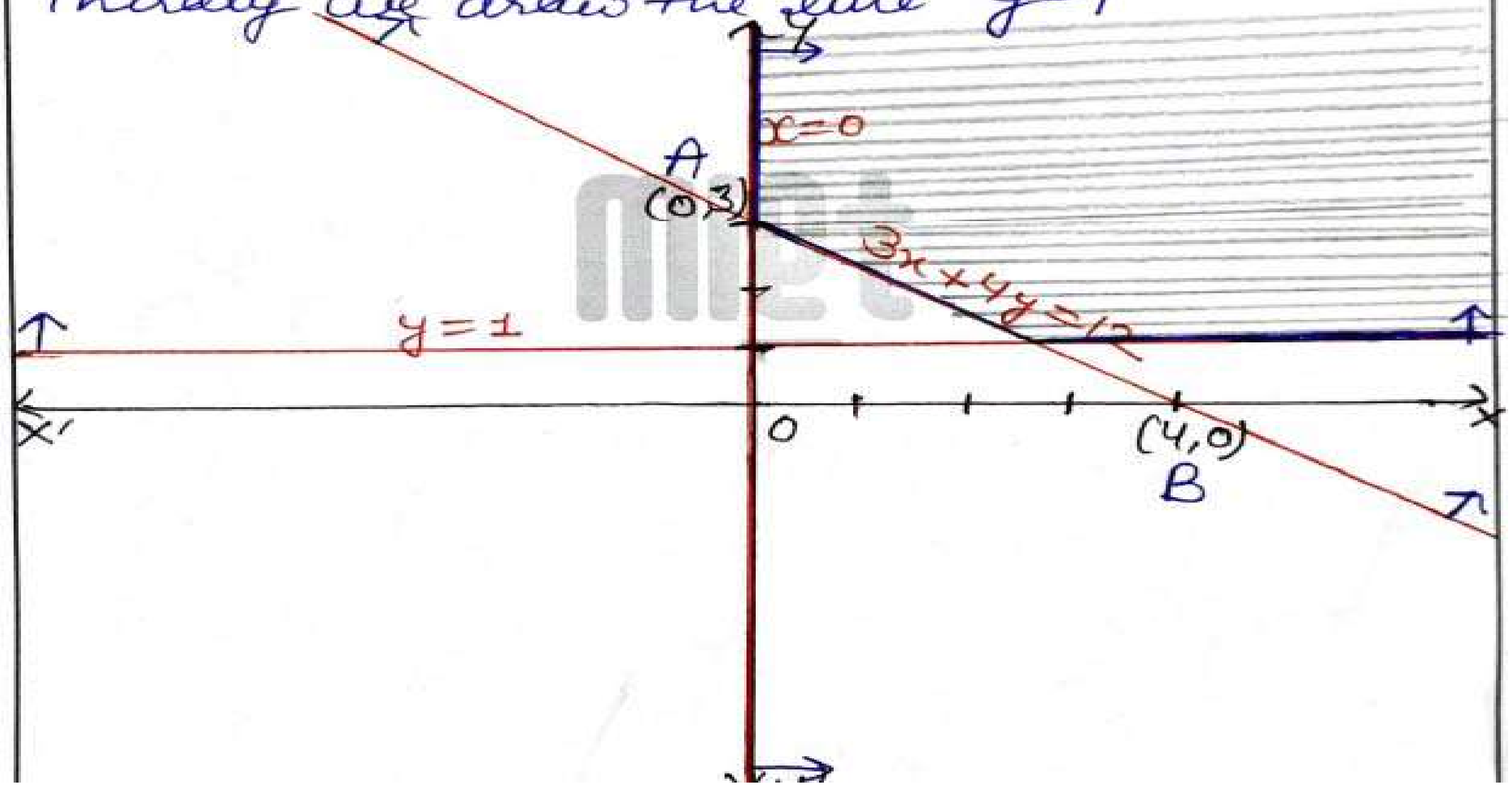
$y \geq 1$ ——— (3)

Sol. First we draw the graph of $3x+4y=12$

(Plot the points $A(0,3)$ & $B(4,0)$ satisfy $3x+4y=12$)

Clearly the point $(0,0)$ does not satisfy (1)

Clearly the point (0,0) does not satisfy (1)
Secondly, we draw the graph of $x=0$
Thirdly we draw the line $y=1$



The intersection of all of these planes is the shaded part represents the solution of the given system of inequalities.

Q-3 Solve the inequality graphically: [2019-20]

$$|y-x| \leq 3$$

Sol. We know that, $|x| \leq a \Leftrightarrow -a \leq x \leq a$

So $|y-x| \leq 3 \Leftrightarrow -3 \leq y-x \leq 3$

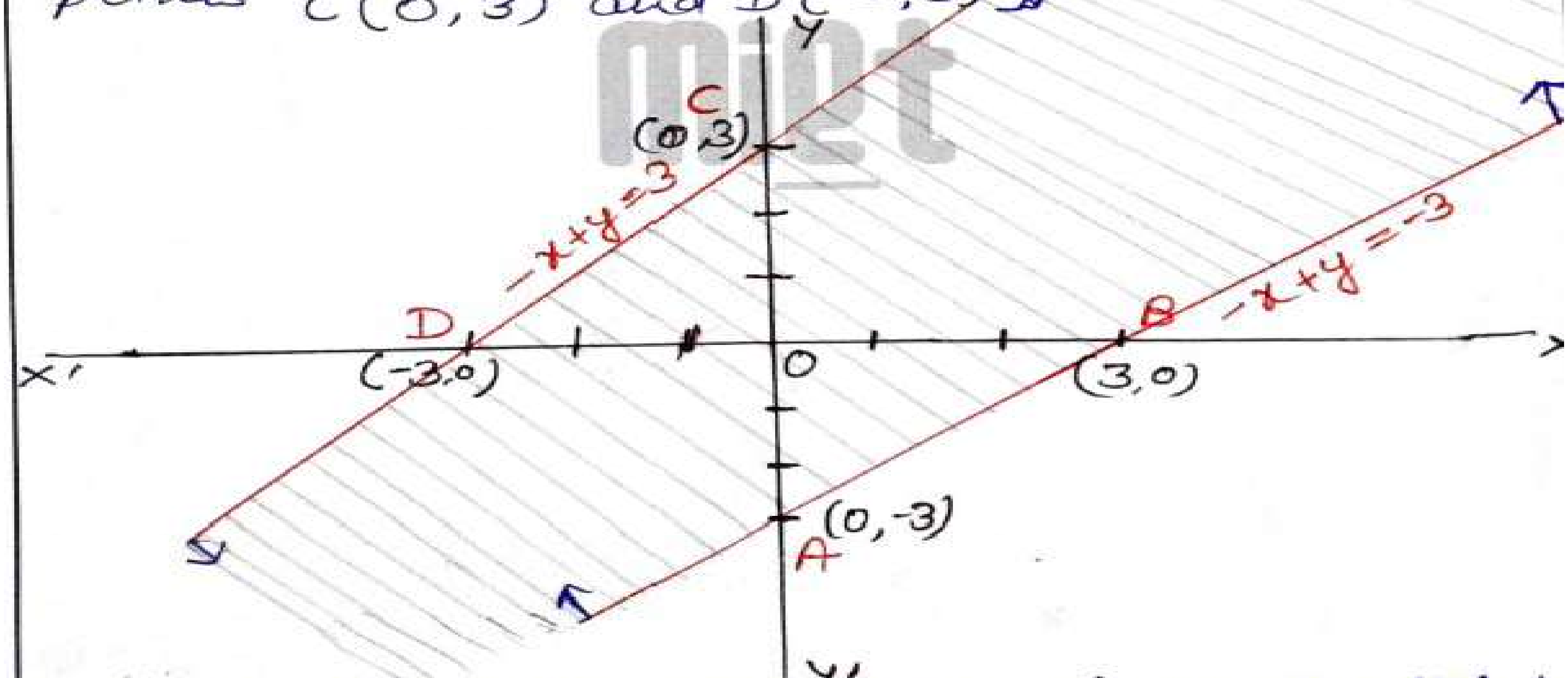
$\Leftrightarrow -3 \leq y-x$ and $y-x \leq 3$

So we have

$$-x+y \geq -3 \quad \text{------} \quad (1)$$

$$-x+y \leq 3 \quad \text{------} \quad (2)$$

Firstly, Draw the graph, $-x+y=-3$ [Plot the points $A(0,-3)$ and $B(3,0)$]
Secondly, Draw the graph, $-x+y=3$ [Plot the points $C(0,3)$ and $D(-3,0)$]



The shaded part of the plane forms the solution set of the inequations.

PRACTICE QUESTIONS

1. $x+y \le 9, y > x, x \ge 0$

2. $2x-y > 1, x-2y < -1$

3. $5x+4y \le 20, x \ge 1, y \ge 2$

4. $3x+4y \le 60, x+3y \le 30, x \ge 0, y \ge 0$

5. $2x+y \ge 4, x+y \le 3, 2x-3y \le 6$

6. $x+2y \le 10, x+y \ge 1, x-y \le 0, x \ge 0, y \ge 0$

THANK YOU