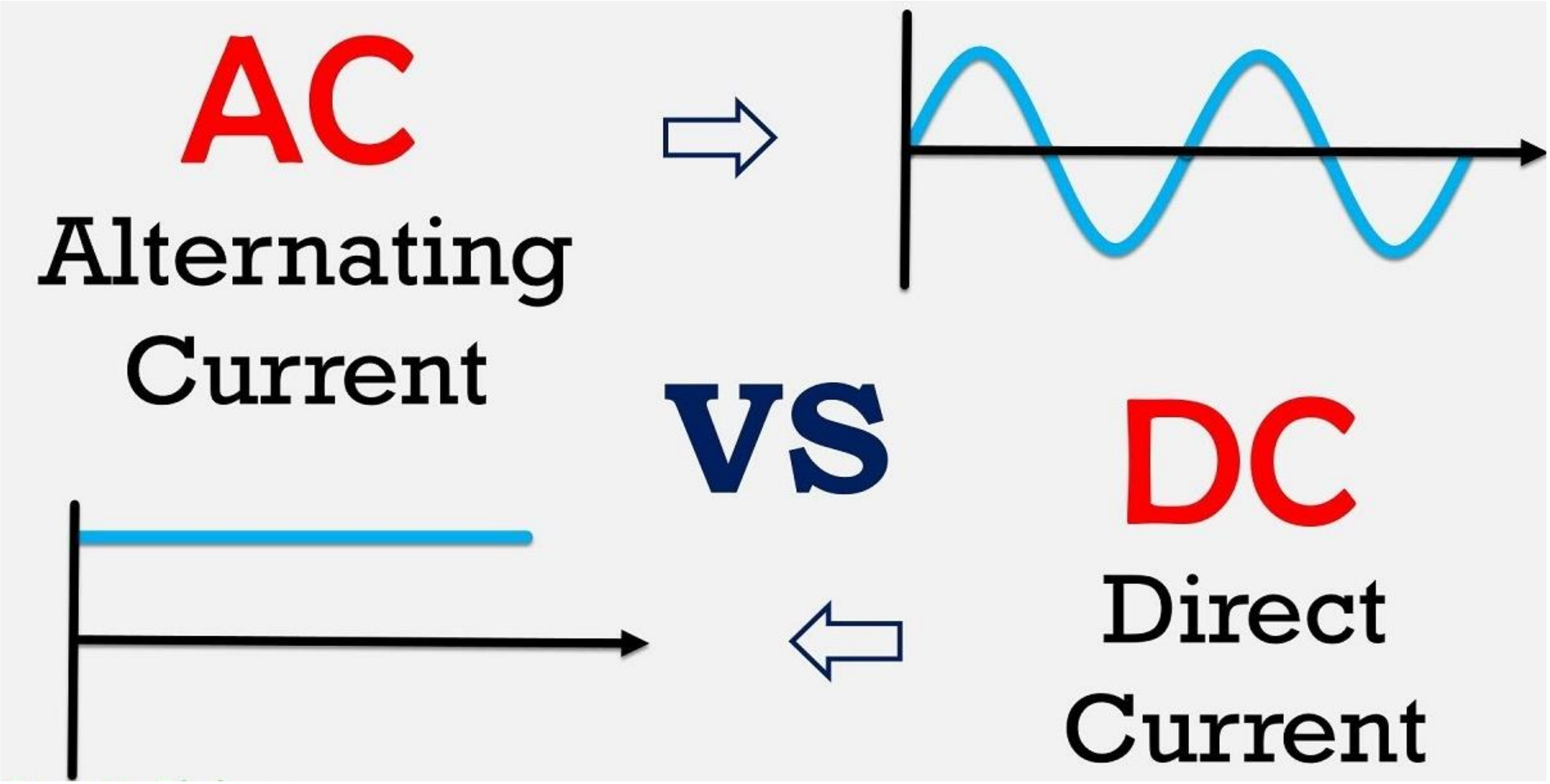
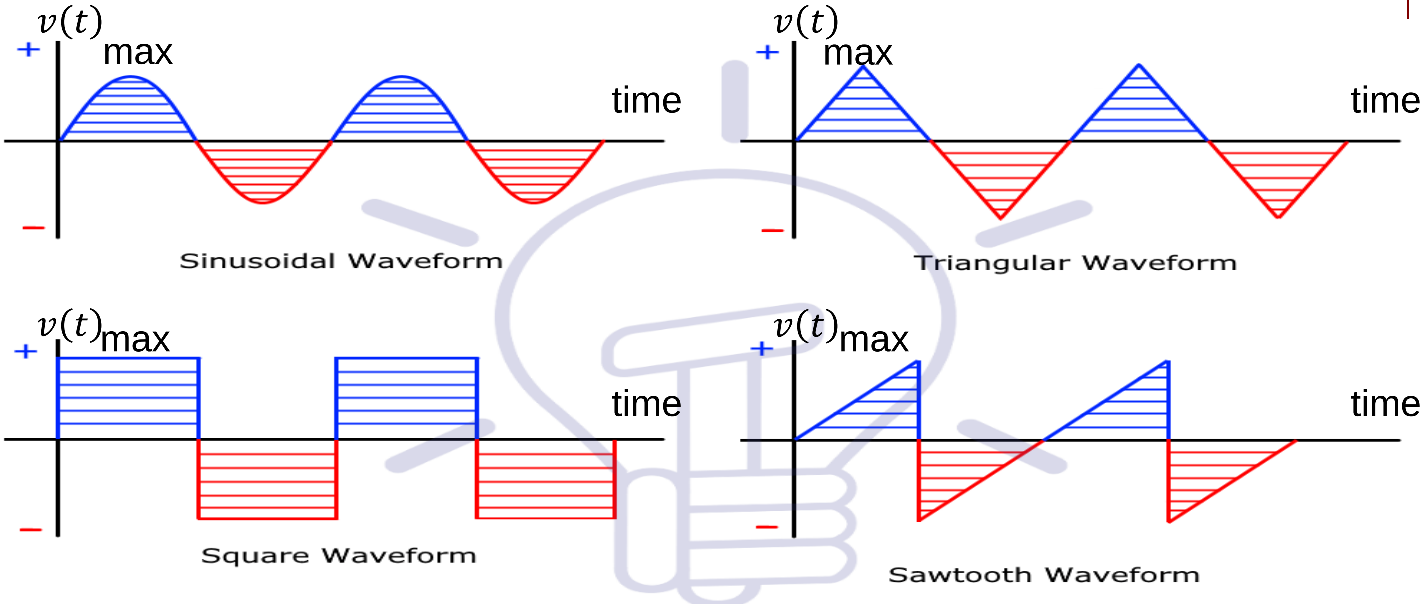
Steady State Analysis of AC Circuits

UNIT-II

Lecture 7


AC
Alternating
Current
VS
DC
Direct
Current

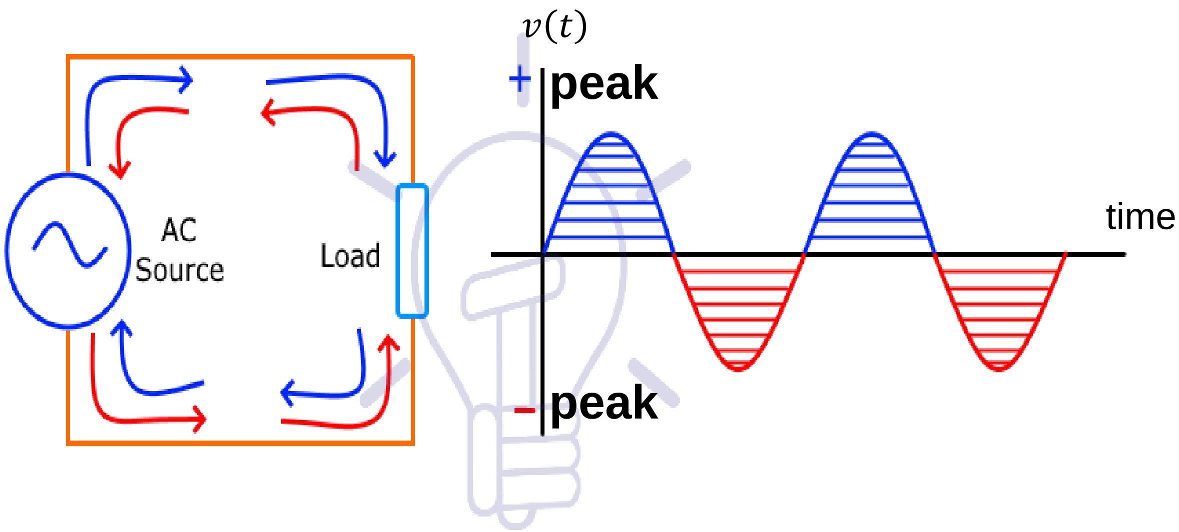
AC Quantity



Alternating Current Waveforms

- Changes periodically both in magnitude and direction with respect to time
- Alternates between two values (+ve and –ve maximum)
- Analogous example - Pendulum action.

Sinusoidal AC

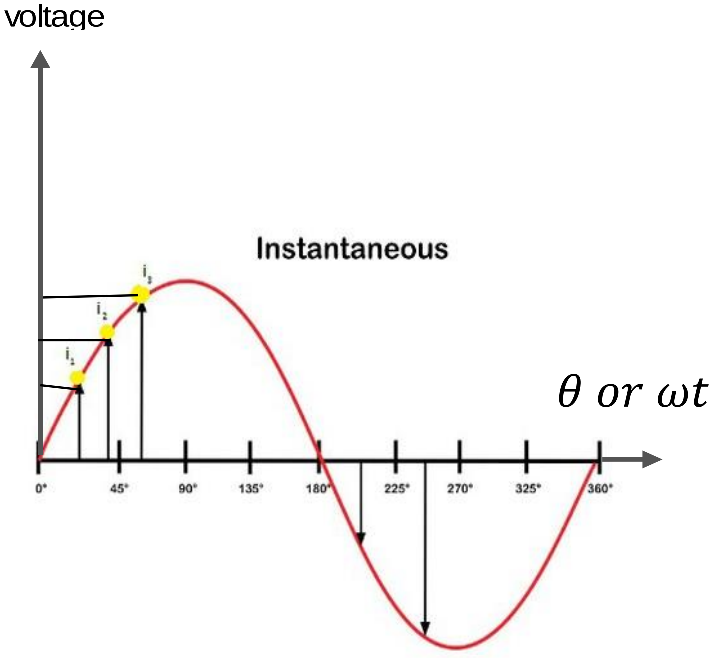


Alternating Current

- The nature of transition between +ve and –ve maximum is sinusoidal
- If A is at higher potential- +ve half cycle
- If B is at higher potential- -ve half cycle

Instantaneous Value

- The discrete value at any instant.
- The instantaneous value changes at every instant in a sinusoidal function
- Instantaneous value is given as
- $v(\theta) = V_0 \sin \theta$
- $v(\omega t) = V_0 \sin \omega t$

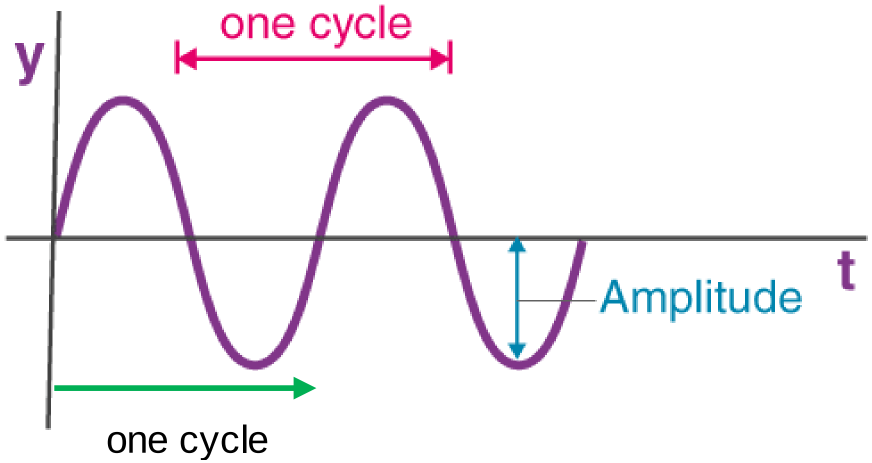


Waveform

- A graph of instantaneous values plotted against time or angular displacement.

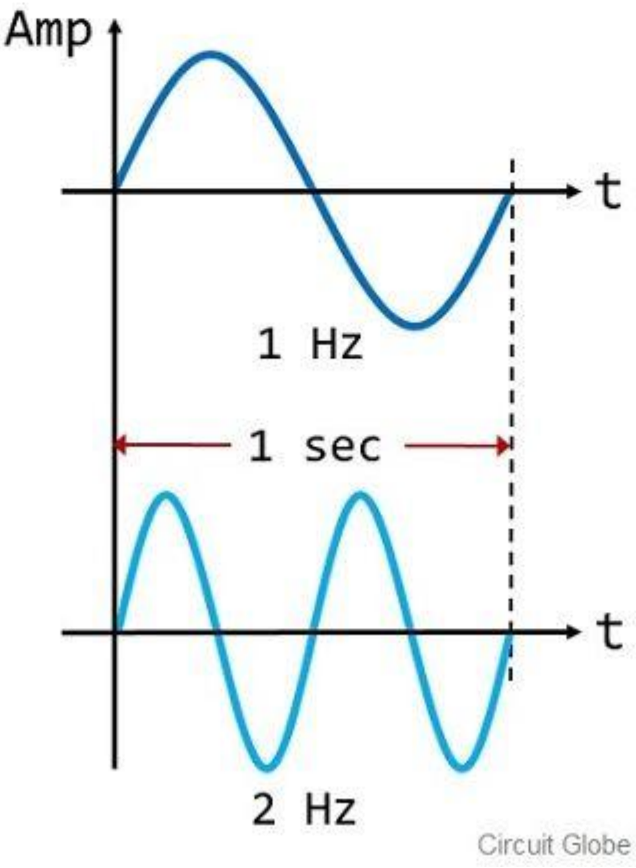
Cycle

- Set of values which repeat itself again and again is called one cycle



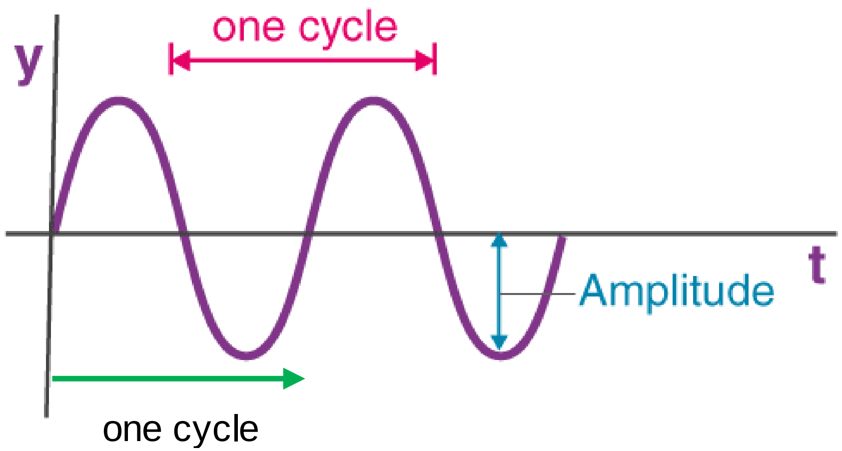
Frequency

- Number of cycles per second is called frequency
- Unit of measurement for frequency is Hertz or cycles/second
- 50 Hz frequency means the signal repeats 50 cycles in each second



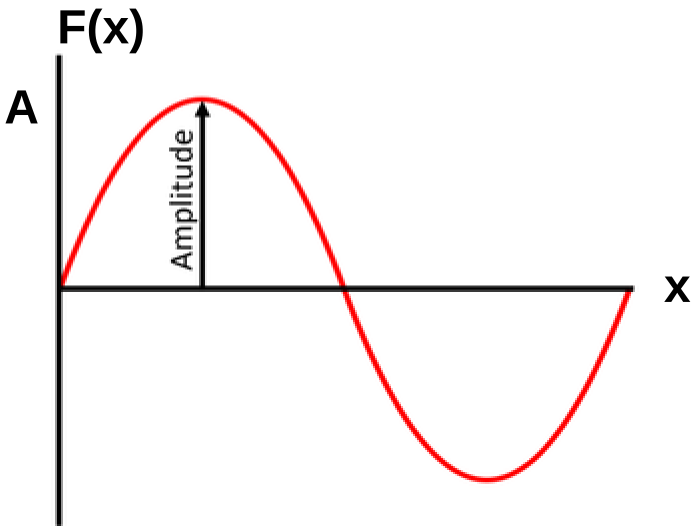
Time Period

- Total time consumed during one complete cycle is called time period
- For a 50 Hz frequency signal Time Period will be 0.02 seconds
- $T = 1/f$



Amplitude

- The displacement across y-axis either up to maximum positive or maximum negative is called amplitude.
- For a sinusoidal signal, if amplitude is A, then mathematically, A is multiplied with a sin function of amplitude 1.
- $f(x) = A * \sin x$

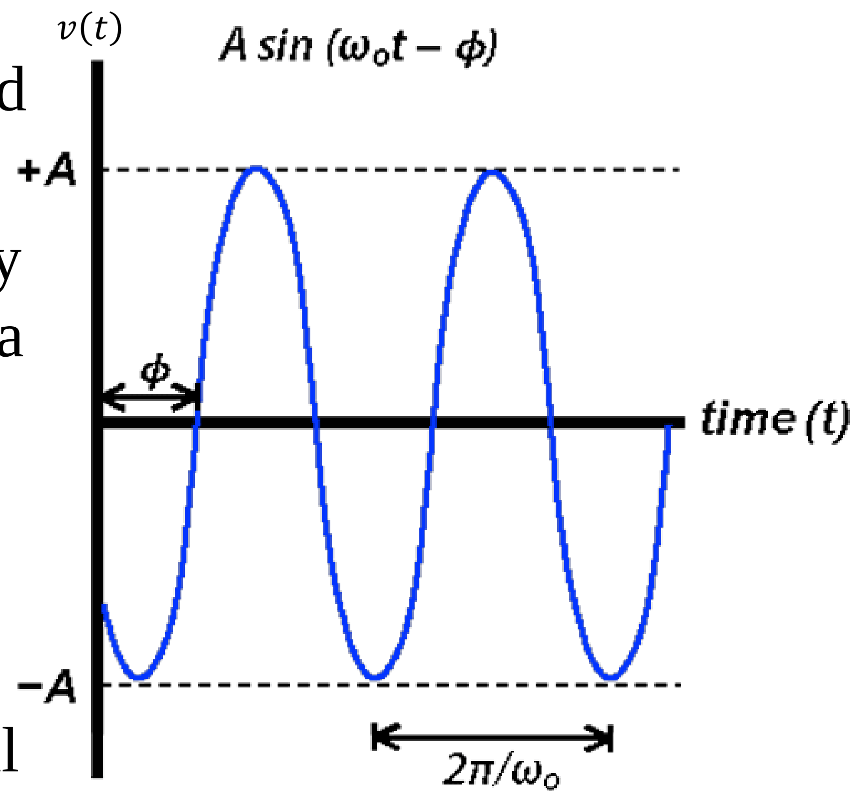


Angular Frequency

- Also called angular velocity
- The angular displacement per second
- Measured either in degrees per second or radians per second
- Total displacement in one rotation $= 2\pi$ radians
- Time period $= T$ seconds
- Then speed of rotating phasor is given
- $\omega = \frac{2\pi}{T} = 2\pi f$ radians per second

Phase

- Phase is defined as the fraction of time by which any signal is shifted on x-axis from origin.
- Phase gives the comparison of any signal in the context of time with a reference signal or origin.
- Something can be in phase with reference or if out of phase, it can be leading or lagging.
- When compared to origin, a signal is said to be with zero phase, +ve phase or –ve phase respectively.



Problem

- An alternating voltage is given by
  - $v = 141.3 \sin 314t$
- Find out
  - Frequency
  - Amplitude
  - Instantaneous value at t = 3 milliseconds
  - Time taken for the voltage to reach 100 V just after
    - Origin
    - First maxima

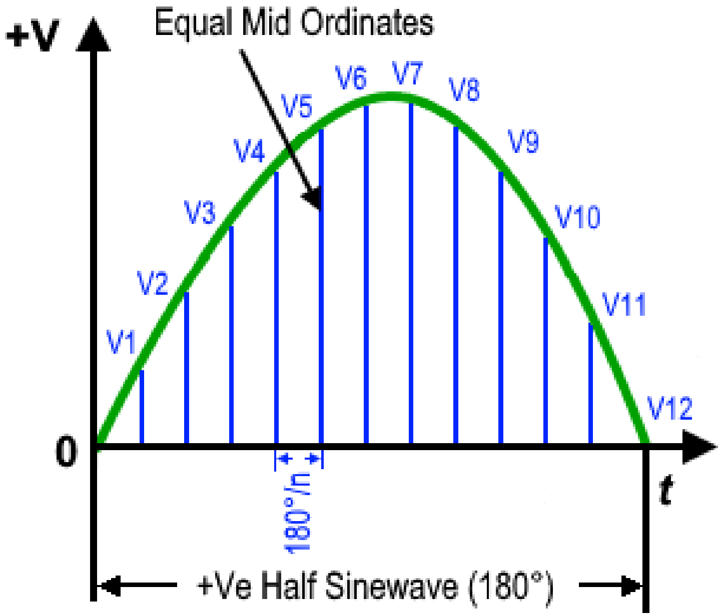
Solution

- Given
  - $v = 141.3 \sin 314t$
- Frequency
  - $\omega = 314\ rad/sec$
  - $f = {}^{\omega}/_{2\pi} = \frac{314}{2*3.14} = 50$ Hertz(Hz)
- Amplitude
  - From given data
  - $V_m = 141.3$ Volts
- Instant. value at t = 3ms can be known after putting value of t in the given equation
  - $v = 141.3 \sin(314 * 3 * 10^{-3})$
  - $v = 114.4$ Volts
- Time taken after origin to 100V
  - $100 = 141.3 \sin 314t$
  - $\sin 314t = \frac{100}{141.3} = 0.707714$
  - $t = \frac{\sin^{-1} 0.707714}{314} = \frac{0.7853\ radians}{314\ rad/sec}$
  - $t = 2.5$ms
- Time taken after first maxima to 100V
  - $100 = 141.3 \sin(314t + {}^{\pi}/_{2})$
  - $100 = 141.3 \cos 314t$
  - $t = \frac{1}{314} \cos^{-1}(\frac{100}{141.3}) = \frac{0.7853\ radians}{314\ rad/sec}$
  - $t = 2.5$ms

Lecture 8 & 9

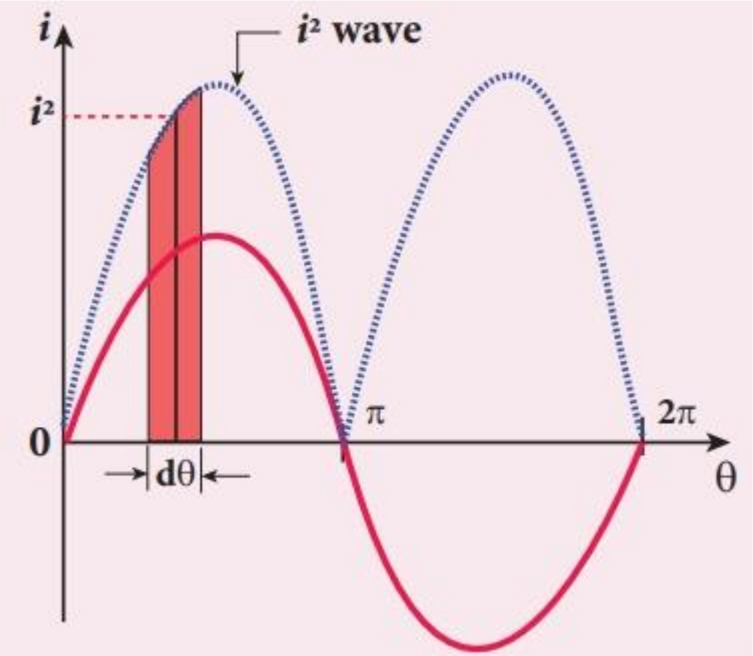
Average Value

- Average or Mean of all instantaneous values over a time period
- Represents actual DC content in the signal
  - $V_{avg} = \frac{1}{T}\int_0^T v(t)d(t)$
- For pure AC signals, $V_{avg} = 0$, that is DC content in a pure AC wave is zero for a single time period.
- Area under the curve per unit base length also gives the average value for the curve



RMS Value

- The signal is squared before taking mean of it to make every instantaneous value positive (i.e. –ve value to +ve) and a square root to counter the square.
- RMS value is that DC equivalent value of an AC quantity which dissipates the same power in a common resistance.
- Represents the equivalent DC value, hence also called effective DC value



- $I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)d(t)}$
- $I_{rms} = \sqrt{\frac{1}{base}\int_0^{2\pi} i^2(\theta)d(\theta)}$

Form Factor

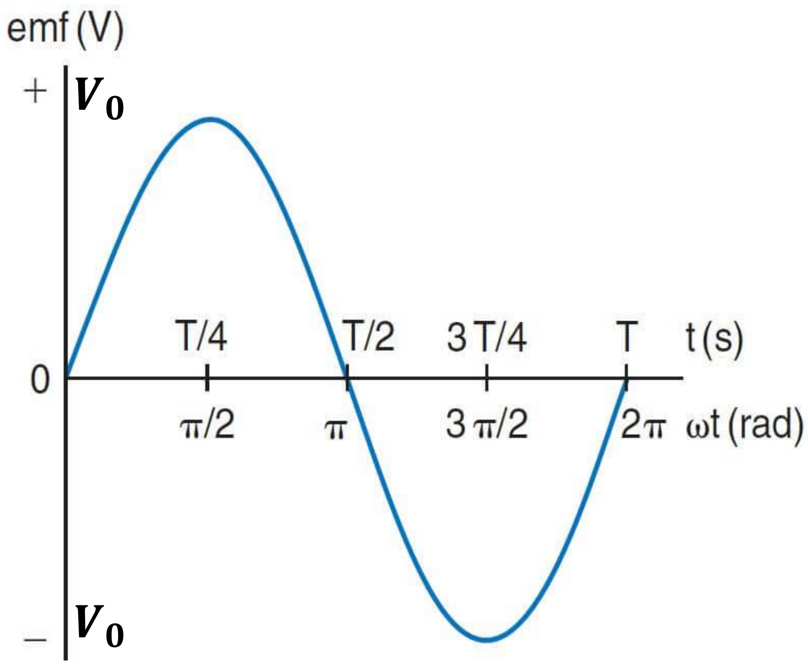
- It is defined as the ratio of RMS value to the average value.
- It describes the shape of any waveform.
- It is expressed as,
  - $K_f = \frac{V_{rms}}{V_{avg}}$

Peak Factor

- It is defined as the ratio of maximum value to the RMS value.
- It gives an idea for the peak value per unit of RMS value.
- It is expressed as,
  - $K_p = \frac{V_{max}}{V_{rms}}$

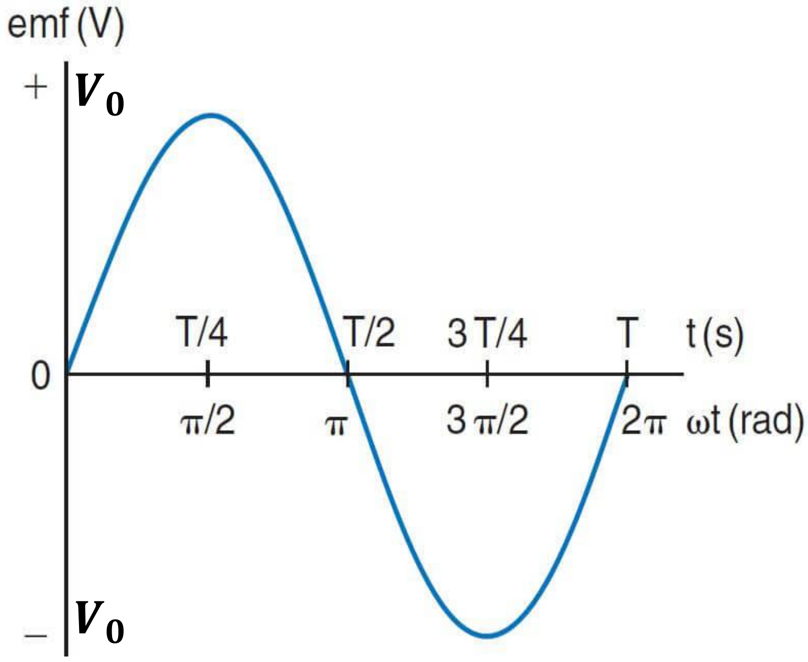
Pure Sinusoidal Wave: Average

- For a pure sinusoidal wave,
- Function, $v(\omega t) = V_0 \sin \omega t$
- For the average of complete cycle
- Base = 2π,
- Limits = $0 - 2\pi$
- $V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\omega t) d(\omega t)$
- $V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} V_0 \sin \omega t \, d(\omega t)$



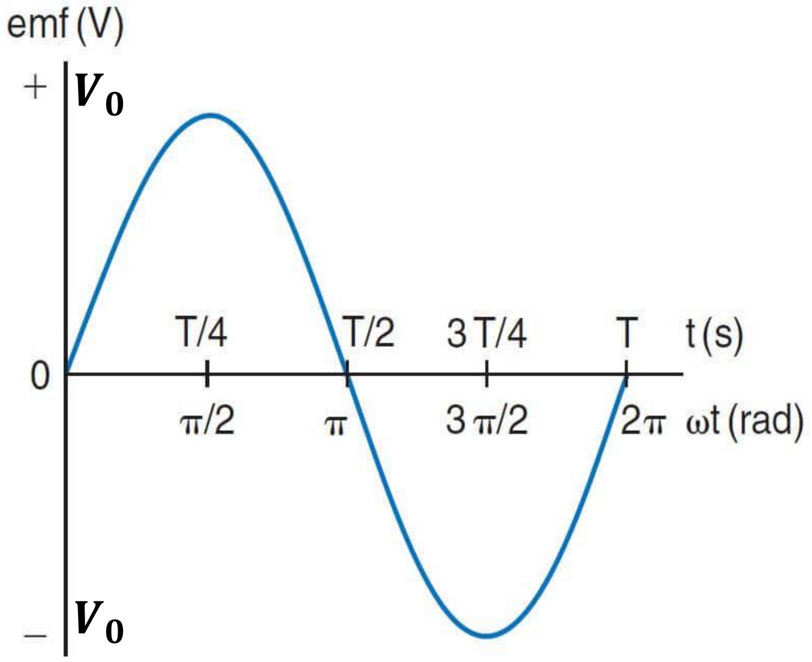
Pure Sinusoidal Wave: Average cont…

- $V_{avg} = \frac{V_0}{2\pi}\int_0^{2\pi} \sin\omega t \, d(\omega t)$
- As,
- $\int_0^{2\pi} \sin\omega t \, d(\omega t) = [-\cos\omega t]_0^{2\pi} = 0$
- Hence,
- $V_{avg} = 0$



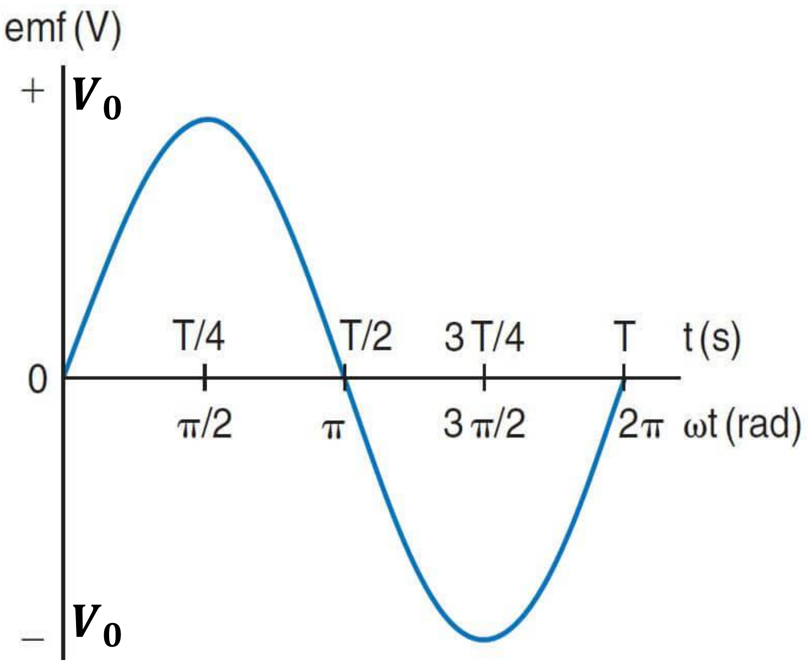
Pure Sinusoidal Wave: Average cont…

- For a pure sinusoidal wave,
- Function, $v(\omega t) = V_0 \sin \omega t$
- For the average of half cycle
- Base = π,
- Limits = $0 - \pi$
- $V_{avg} = \frac{1}{\pi} \int_0^{\pi} v(\omega t) d(\omega t)$
- $V_{avg} = \frac{1}{\pi} \int_0^{\pi} V_0 \sin \omega t \, d(\omega t)$



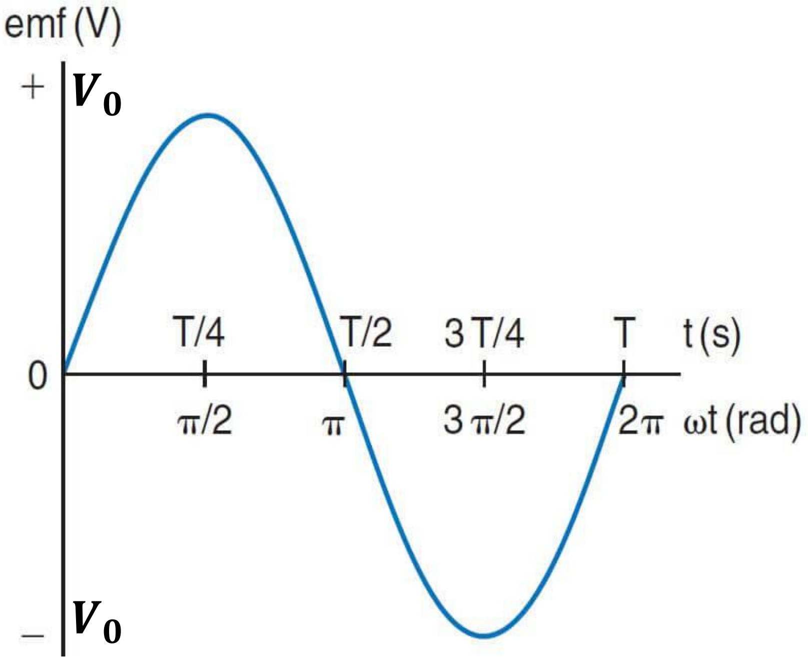
Pure Sinusoidal Wave: Average cont…

- $V_{avg} = \frac{V_0}{\pi}\int_0^{\pi} \sin\omega t \, d(\omega t)$
- As,
- $\int_0^{\pi} \sin\omega t \, d(\omega t) = [-\cos\omega t]_0^{\pi} = 2$
- Hence,
- $V_{avg} = \frac{2V_0}{\pi}$



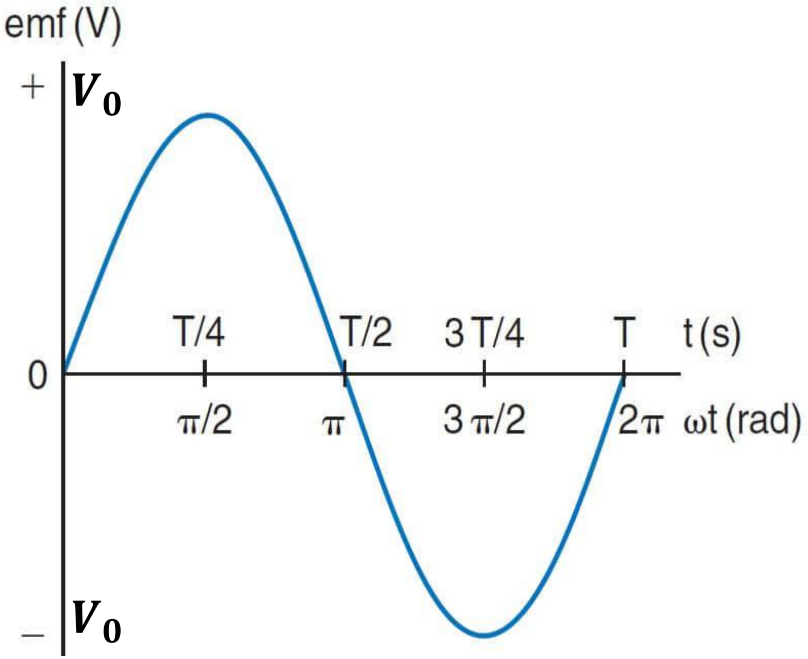
Pure Sinusoidal Wave: RMS

- For a pure sinusoidal wave,
- Function, $v(\omega t) = V_0 \sin \omega t$
- For the RMS of complete cycle
- Base = 2π,
- Limits = $0 - 2\pi$
- $V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\omega t)d(\omega t)}$
- $V_{rms} = \sqrt{\frac{V_0^2}{2\pi}\int_0^{2\pi} sin^2(\omega t)d(\omega t)}$



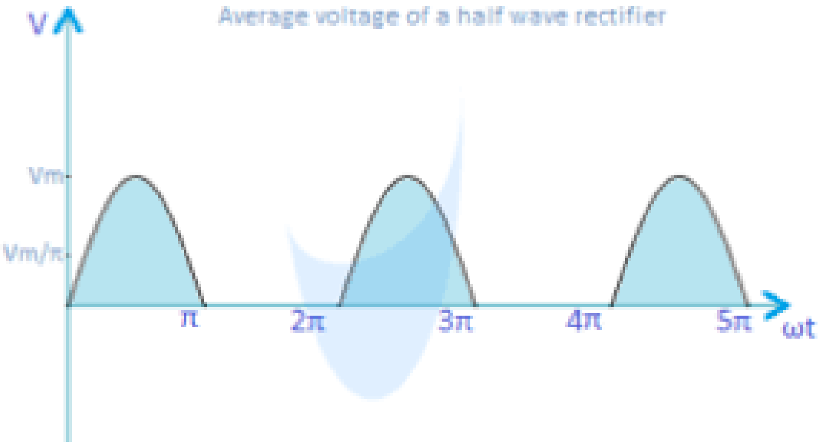
Pure Sinusoidal Wave: RMS cont…

- $V_{rms} = \sqrt{\frac{V_0^2}{2\pi}\int_0^{2\pi} sin^2(\omega t)d(\omega t)}$
- As,
- $\int_0^{2\pi} sin^2(\omega t)d(\omega t) = \int_0^{2\pi} \frac{1}{2}(1 - \cos 2\omega t)d(\omega t)$
- $= \frac{1}{2}[\omega t - \frac{\sin 2\omega t}{2}]_0^{2\pi} = \pi$
- Hence,
- $V_{rms} = \frac{V_0}{\sqrt{2}}$



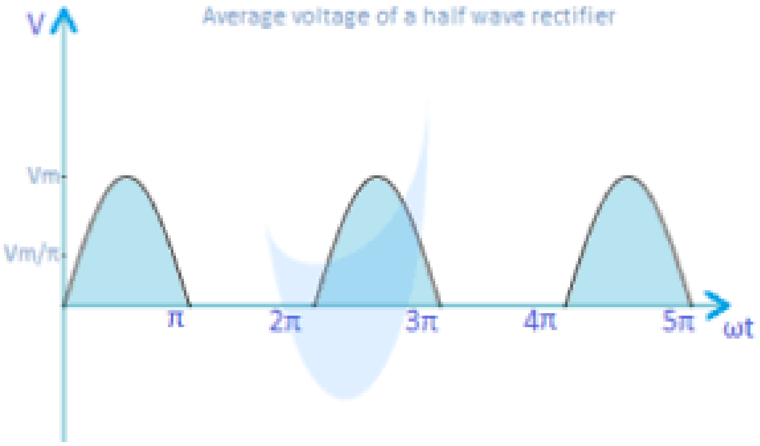
Half Wave Rectifier: Average

- Function,
- $v(\omega t) = \begin{matrix} V_0 \sin \omega t \\ 0 \end{matrix} \begin{cases} 0-\pi \\ \pi - 2\pi \end{cases}$
- Base = 2π,
- $V_{avg} = \frac{1}{2\pi}\int_0^{\pi} v(\omega t) d(\omega t)$
- $V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_0 \sin \omega t \, d(\omega t)$
  - $\int_0^{\pi} \sin \omega t \, d(\omega t) = [-\cos \omega t]_0^{\pi} = 2$
- $V_{avg} = \frac{V_0}{\pi}$



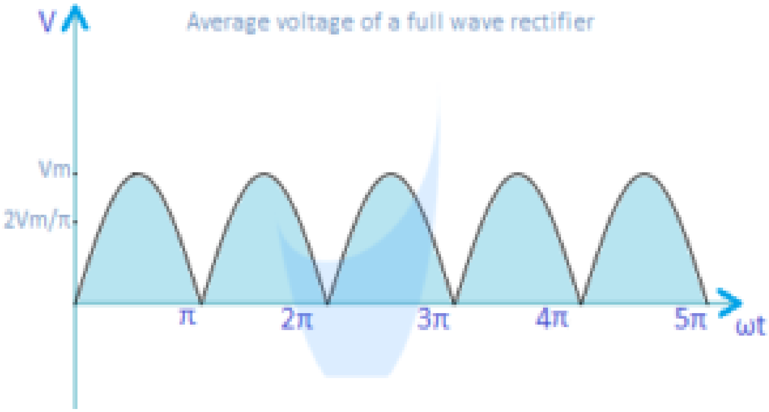
Half Wave Rectifier: RMS

- Function,
- $v(\omega t) = \begin{matrix} V_0 \sin \omega t \\ 0 \end{matrix} \begin{cases} 0 - \pi \\ \pi - 2\pi \end{cases}$
- Base = 2π,
- $V_{rms} = \sqrt{\frac{V_0^2}{2\pi} \int_0^{\pi} sin^2(\omega t) d(\omega t)}$
  - $\int_0^{\pi} sin^2(\omega t) d(\omega t) = \int_0^{\pi} \frac{1}{2}(1 - \cos 2\omega t) d(\omega t)$
  - $= \frac{1}{2}[\omega t - \frac{\sin 2\omega t}{2}]_0^{\pi} = \pi$
- $V_{rms} = \frac{V_0}{2}$



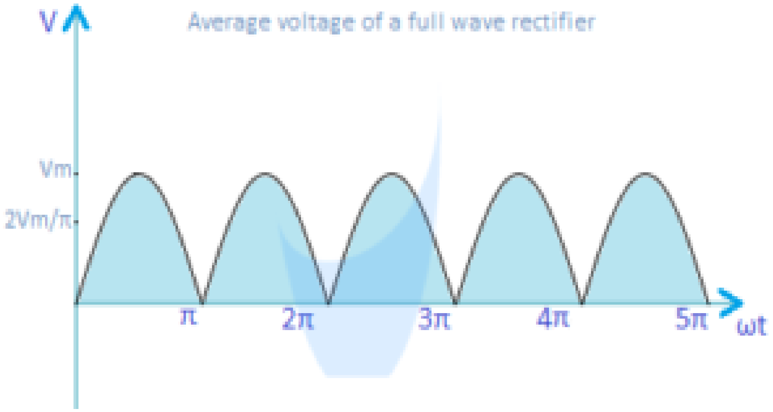
Full Wave Rectifier: Average

- Function,
- $v(\omega t) = V_0 \sin \omega t \; \{0 - \pi$
- Base = π,
- $V_{avg} = \frac{1}{\pi} \int_0^{\pi} v(\omega t) d(\omega t)$
- $V_{avg} = \frac{1}{\pi} \int_0^{\pi} V_0 \sin \omega t \, d(\omega t)$
    - $\int_0^{\pi} \sin \omega t \, d(\omega t) = [-\cos \omega t]_0^{\pi} = 2$
- $V_{avg} = \frac{2V_0}{\pi}$



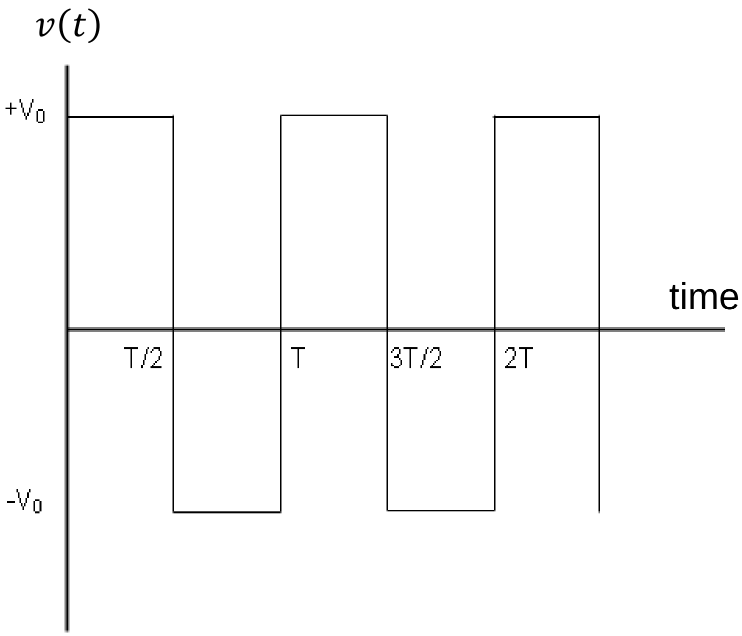
Full Wave Rectifier: RMS

- Function,
- $v(\omega t) = V_0 \sin \omega t \, \{0 - \pi$
- Base = π,
- $V_{rms} = \sqrt{\frac{V_0^2}{\pi} \int_0^{\pi} sin^2(\omega t) d(\omega t)}$
  - $\int_0^{\pi} sin^2(\omega t) d(\omega t) = \int_0^{\pi} \frac{1}{2} (1 - \cos 2\omega t) d(\omega t)$
  - $= \frac{1}{2} [\omega t - \frac{\sin 2\omega t}{2}]_0^{\pi} = \pi$
- $V_{rms} = \frac{V_0}{\sqrt{2}}$



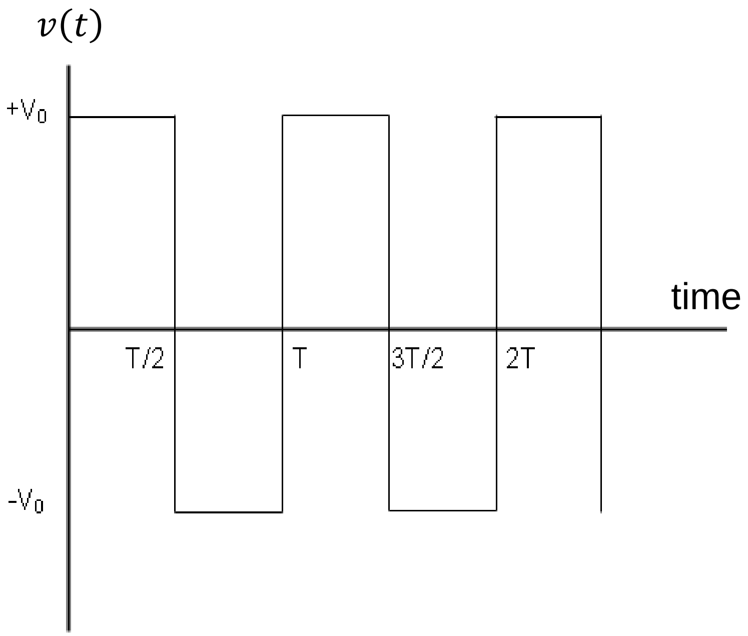
Square Wave: Average

- Function,
- $v(\omega t) = \begin{cases} +V_0 & 0 - T/2 \\ -V_0 & T/2 - T \end{cases}$
- Base =Time period= T,
- $V_{avg} = \frac{1}{T}\int_0^T v(t)d(t)$
- Average for full cycle is zero.
- For half cycle,
  - $V_{avg} = \frac{V_0}{T/2}[\int_0^{T/2} d(t)]$
- $V_{avg} = V_0$



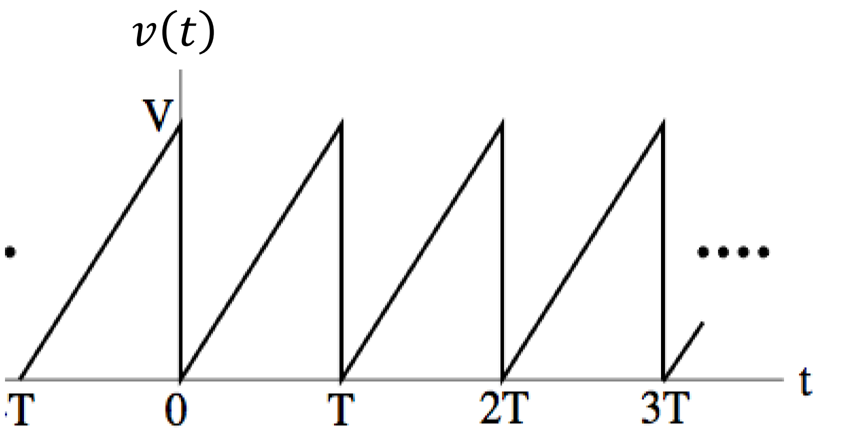
Square Wave: RMS

- Function,
- $v(\omega t) = \begin{matrix} +V_0 \\ -V_0 \end{matrix} \begin{cases} 0 - T/2 \\ T/2 - T \end{cases}$
- Base =Time period= T,
- $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
  - $V_{rms} = \sqrt{\frac{{V_0}^2}{T}[\int_0^{T/2} d(t) + \int_{T/2}^{T} d(t)]}$
- $V_{rms} = V_0$



Sawtooth Wave: AVERAGE

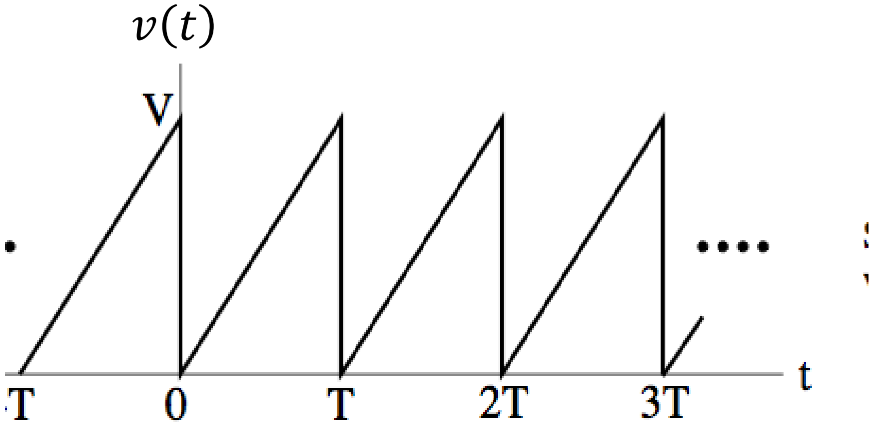
- Function for straight line is
  - $y = mx + C$
    - Where y is a function of x of slope m. C is the point on y-axis where x is zero.
  - $m = \frac{dy}{dx}$, and here $C = 0$
- Hence,
  - $v(t) = \frac{V}{T} t$ for 0-T
- Base =Time period= T,
  - $V_{avg} = \frac{1}{T} \int_0^T v(t) d(t)$
  - $V_{avg} = \frac{1}{T} \int_0^T \frac{V}{T} t d(t)$



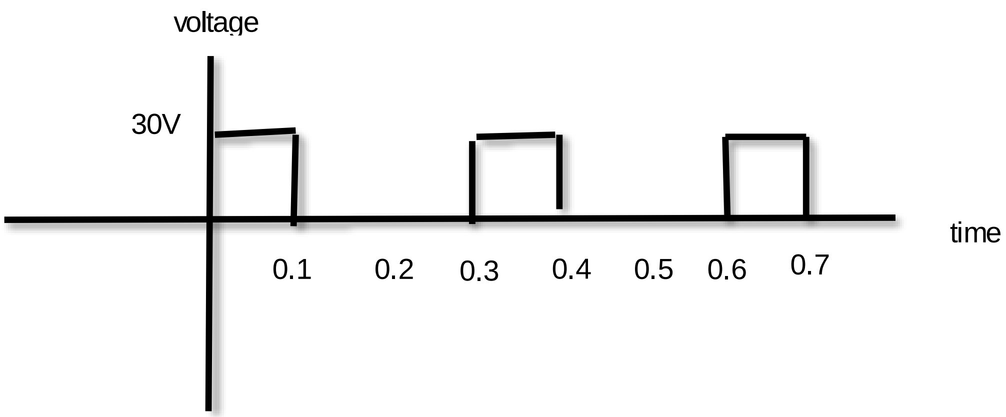
- $V_{avg} = \frac{V}{T^2} [\frac{t^2}{2}]_0^T$
- $V_{avg} = \frac{V}{2}$

Triangular Wave: RMS

- $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
- $V_{rms} = \sqrt{\frac{1}{T}\int_0^T (\frac{V}{T}t)^2 d(t)}$
- $V_{rms} = \sqrt{\frac{V^2}{T^3}[\frac{t^3}{3}]_0^T}$
- $V_{rms} = \frac{V}{\sqrt{3}}$



Problem

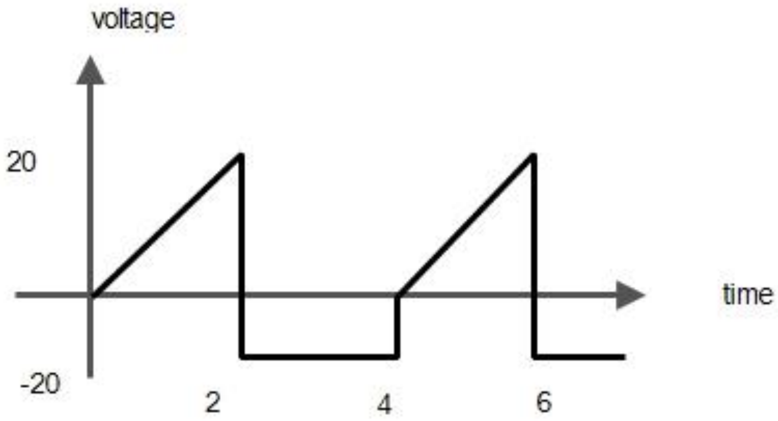


- Find
  - V_{avg}
  - V_{rms}
  - Form factor and Peak factor

Solution

- Given
  - T = 0.3 sec
  - $v(t) = \begin{cases} 30V, & 0 < t < 0.1 \\ 0, & 0.1 < x < 0.3 \end{cases}$
  - Average value is given as
    - $V_{avg} = \frac{1}{T}\int_0^T v(t)d(t)$
    - $V_{avg} = \frac{1}{0.3}[\int_0^{0.1} 30.dt + \int_{0.1}^{0.3} 0.dt]$
    - $V_{avg} = \frac{30}{0.3}[t]_0^{0.1} = 100 * 0.1$
    - $V_{avg} = 10\text{Volts}$
- RMS value is given as
  - $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
  - $V_{rms} = \sqrt{\frac{1}{0.3}[\int_0^{0.1} 30^2 d(t) + \int_{0.1}^{0.3} 0dt]}$
  - $V_{rms} = \sqrt{\frac{900}{0.3}[t]_0^{0.1}} = \sqrt{300} = 17.32\text{V}$
- Form factor
  - $K_f = \frac{17.32}{10} = 1.732$
- Peak factor
  - $K_p = \frac{30}{17.32} = 1.732$

Problem

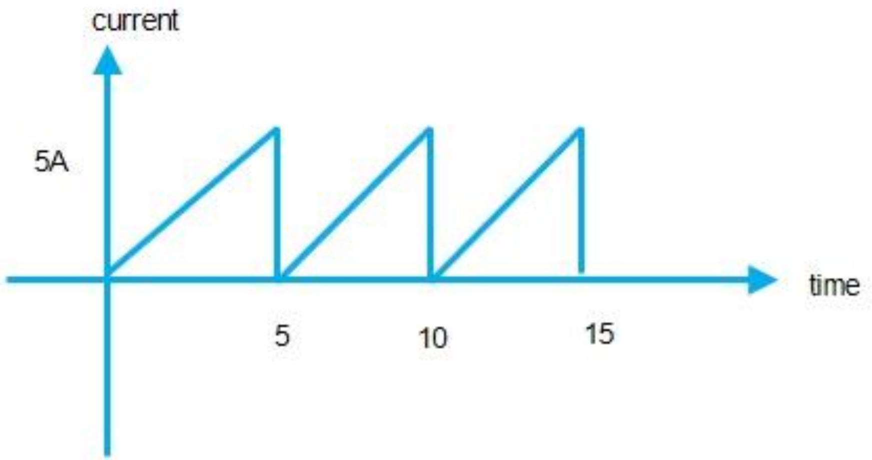


- Find
  - V_{avg}
  - V_{rms}
  - Form factor and Peak factor

Solution

- Given
  - T = 4 sec
  - For t = 0-2 sec, function is given as
  - $v(t) = mt + C$
  - $m = \frac{10}{1} = 10$, and $C = 0$
  - $v(t) = \begin{cases} 10t, & 0 < t < 2 \\ -20, & 2 < x < 4 \end{cases}$
  - Average value is given as
    - $V_{avg} = \frac{1}{T}\int_0^T v(t)d(t)$
    - $= \frac{1}{4}[\int_0^2 10t(dt) + \int_2^4 (-20)d(t)]$
    - $V_{avg} = \frac{10}{4}[\frac{t^2}{2}]_0^2 - \frac{20}{4}[t]_2^4 = 5 - 10$
    - $V_{avg} = -5$Volts
- RMS value is given as
  - $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
  - $= \sqrt{\frac{1}{4}[\int_0^2 (10t)^2 d(t) + \int_2^4 (-20)^2 d(t)]}$
  - $V_{rms} = \sqrt{\frac{100}{4}[\frac{t^3}{3}]_0^2 + \frac{400}{4}[t]_2^4}$
  - $V_{rms} = \sqrt{266.66} = 16.32$V
- Form factor
  - $K_f = \frac{16.32}{5} = 3.26$
- Peak factor
  - $K_p = \frac{20}{16.32} = 1.22$

Problem

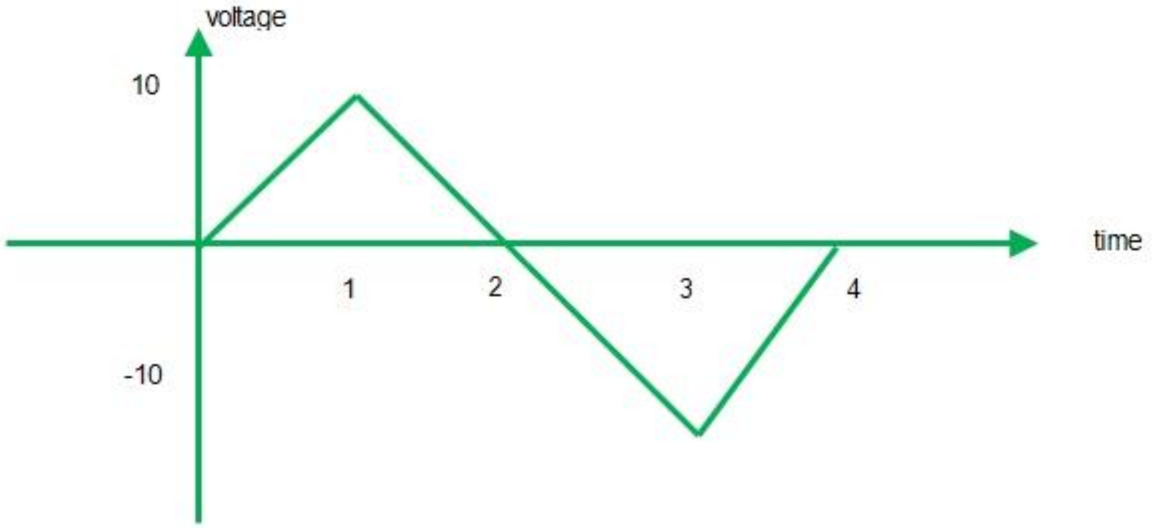


- Find
  - I_{avg}
  - I_{rms}
  - Form factor and Peak factor

Solution

- Given
  - T = 5 sec
  - $i(t) = mt + C$
  - Here,
    - $m = \frac{5}{5} = 1$, and $C = 0$
  - Hence,
  - $i(t) = t, 0 < t < 5$
  - Average value is given as
    - $I_{avg} = \frac{1}{T}\int_0^T v(t)d(t)$
    - $I_{avg} = \frac{1}{5}\int_0^5 tdt$
    - $I_{avg} = \frac{1}{5}[\frac{t^2}{2}]_0^5$
    - $I_{avg} = 2.5$Volts
- RMS value is given as
  - $I_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
  - $I_{rms} = \sqrt{\frac{1}{5}[\int_0^5 (t)^2 d(t)}$
  - $I_{rms} = \sqrt{\frac{1}{5}[\frac{t^3}{3}]_0^5} = \sqrt{\frac{125}{5*3}}$
  - $I_{rms} = \sqrt{8.33} = 2.88$V
- Form factor
  - $K_f = \frac{2.88}{2.5} = 1.152$
- Peak factor
  - $K_p = \frac{5}{2.88} = 1.733$

Problem

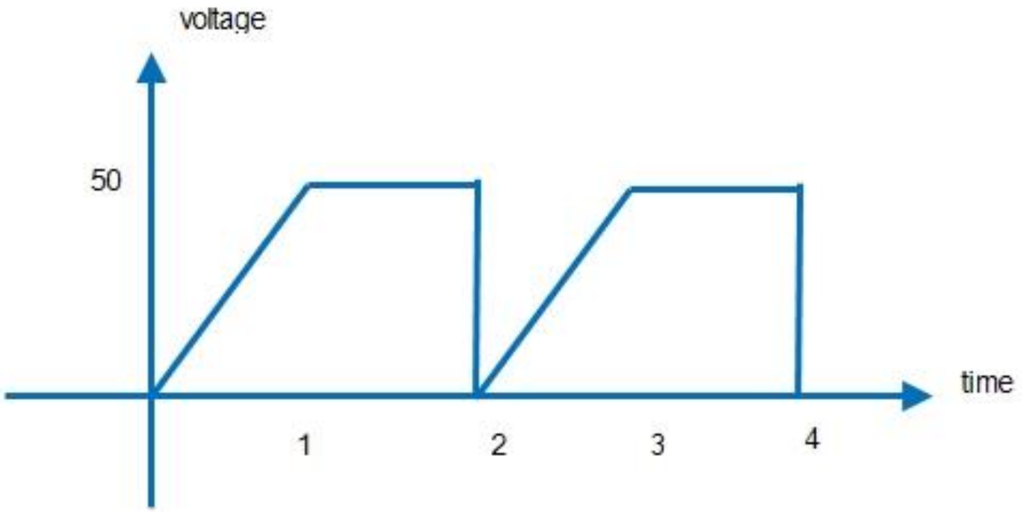


- Find
  - V_{avg}
  - V_{rms}
  - Form factor and Peak factor

Solution

- Average value for complete cycle is zero, for half cycle is given as
  - $V_{avg} = \frac{1}{2}\int_0^2 v(t)d(t)$
- Given
  - $v(t) = \begin{cases} 10t, & 0 < t < 1 \\ -10t + 20, & 1 < t < 2 \end{cases}$
  - $V_{avg} = \frac{1}{2}[\int_0^1 10tdt + \int_1^2(-10t + 20)dt]$
  - $V_{avg} = \frac{10}{2}[\frac{t^2}{2}]_0^1 + \frac{1}{2}[-10\frac{t^2}{2} + 20t]_1^2$
  - $V_{avg} = 5V$
- RMS value is given as
  - $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
  - $V_{rms} = \sqrt{\frac{2}{T}\int_0^{T/2} v^2(t)d(t)}$
- $= \sqrt{\frac{2}{4}[\int_0^1(10t)^2d(t) + \int_1^2(-10t + 20)^2d(t)]}$
- $V_{rms} = \sqrt{\frac{100}{2}[\frac{t^3}{3}]_0^1 + \frac{1}{2}[100\frac{t^3}{3} - 400t + 400]_2^4}$
  - $V_{rms} = \sqrt{33.33} = 5.773V$
- Form factor
  - $K_f = \frac{5.773}{5} = 1.154$
- Peak factor
  - $K_p = \frac{10}{5.773} = 1.732$

Problem



- Find
  - V_{avg}
  - V_{rms}
  - Form factor and Peak factor

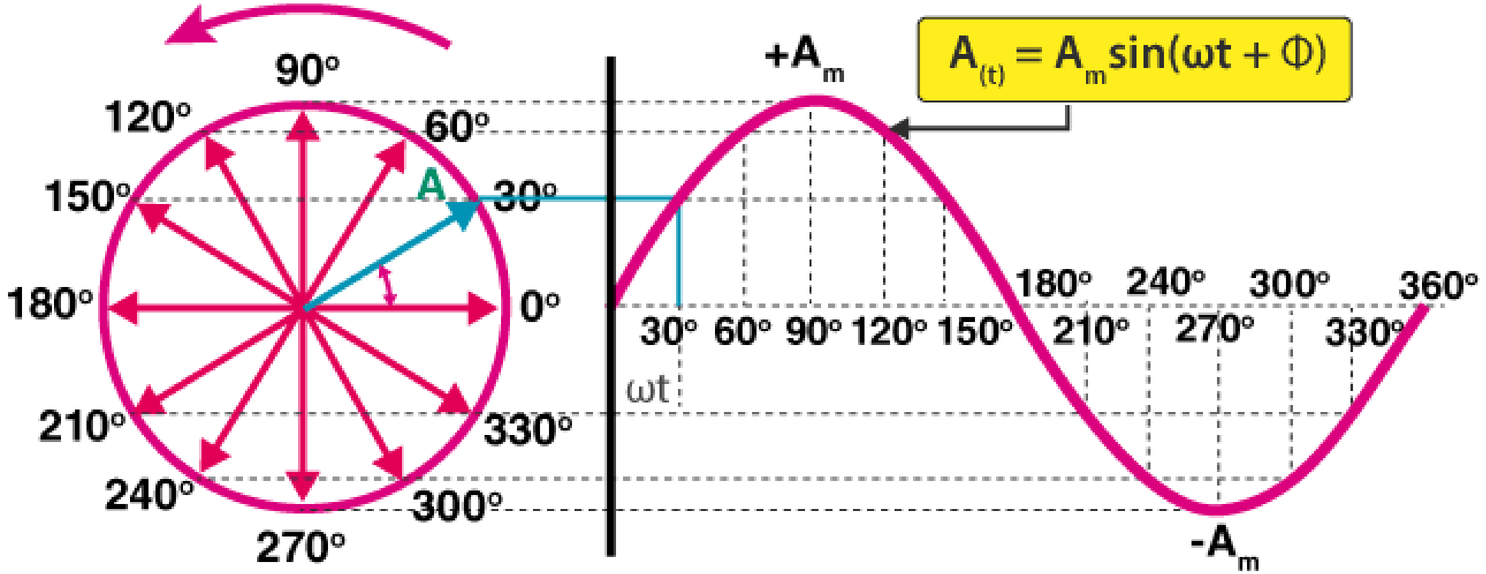
Solution

- Given
  - T = 2 sec
  - $v(t) = \begin{cases} 50t, & 0 < t < 1 \\ 50, & 1 < x < 2 \end{cases}$
  - Average value is given as
    - $V_{avg} = \frac{1}{T}\int_0^T v(t)d(t)$
    - $= \frac{1}{2}[\int_0^1 50t(dt) + \int_1^2 50d(t)]$
    - $V_{avg} = \frac{50}{2}[\frac{t^2}{2}]_0^1 + \frac{50}{2}[t]_1^2 = 12.5 + 25$
    - $V_{avg} = 37.5\text{Volts}$

- RMS value is given as
  - $V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)d(t)}$
  - $= \sqrt{\frac{1}{2}[\int_0^1 (50t)^2 d(t) + \int_1^2 50^2 d(t)]}$
  - $V_{rms} = \sqrt{\frac{2500}{2}[\frac{t^3}{3}]_0^1 + \frac{2500}{2}[t]_1^2}$
  - $V_{rms} = \sqrt{1664.64} = 40.8\text{V}$
- Form factor
  - $K_f = \frac{40.8}{37.5} = 1.088$
- Peak factor
  - $K_p = \frac{50}{40.8} = 1.225$

Lecture 10

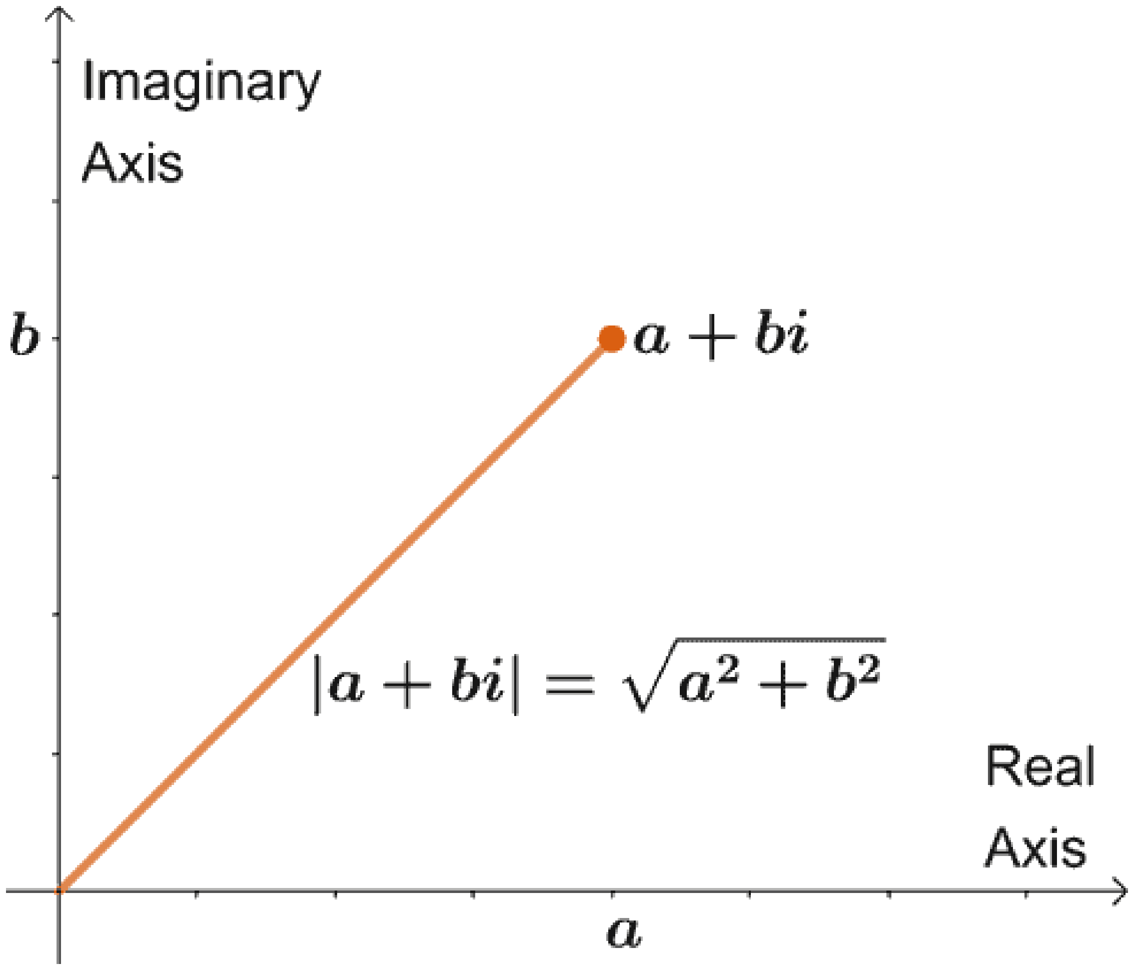
Phasor Theory: Basics



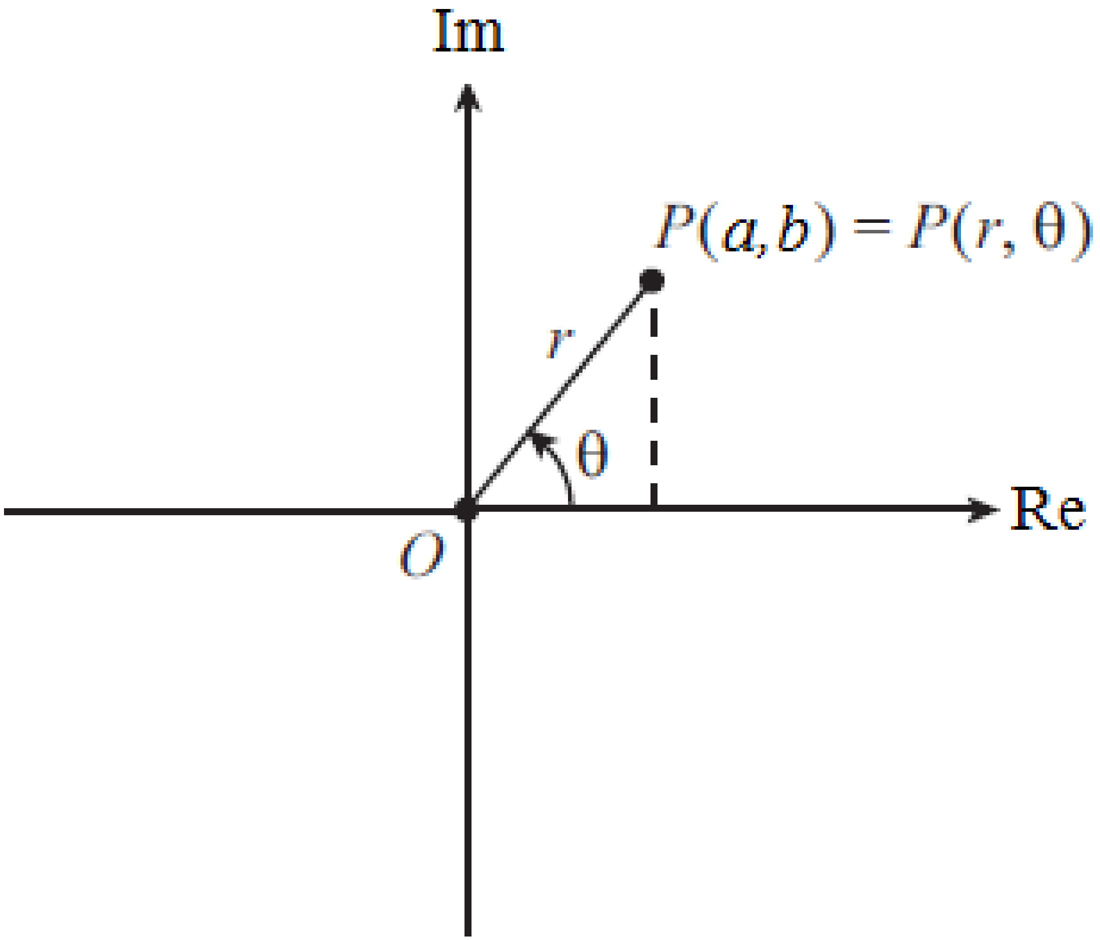
Rotating Phasor

Sinusoidal waveform in the time domain

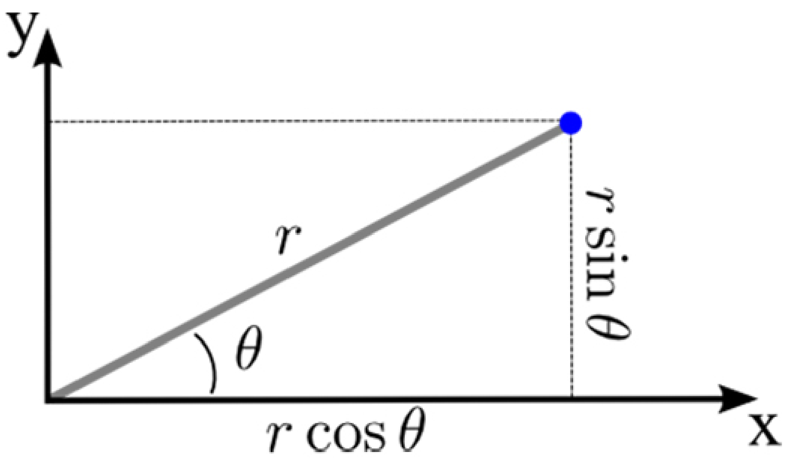
Rectangular Form



Polar Form

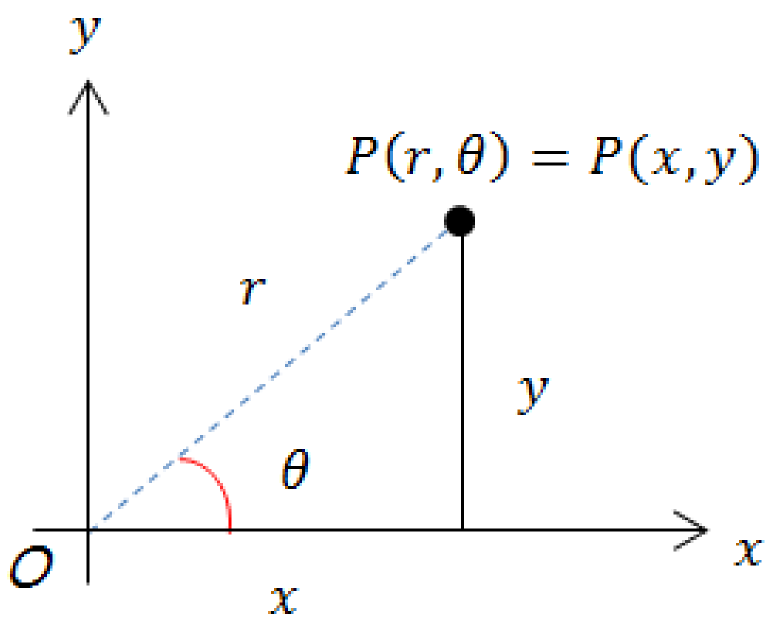


Polar to Rectangular



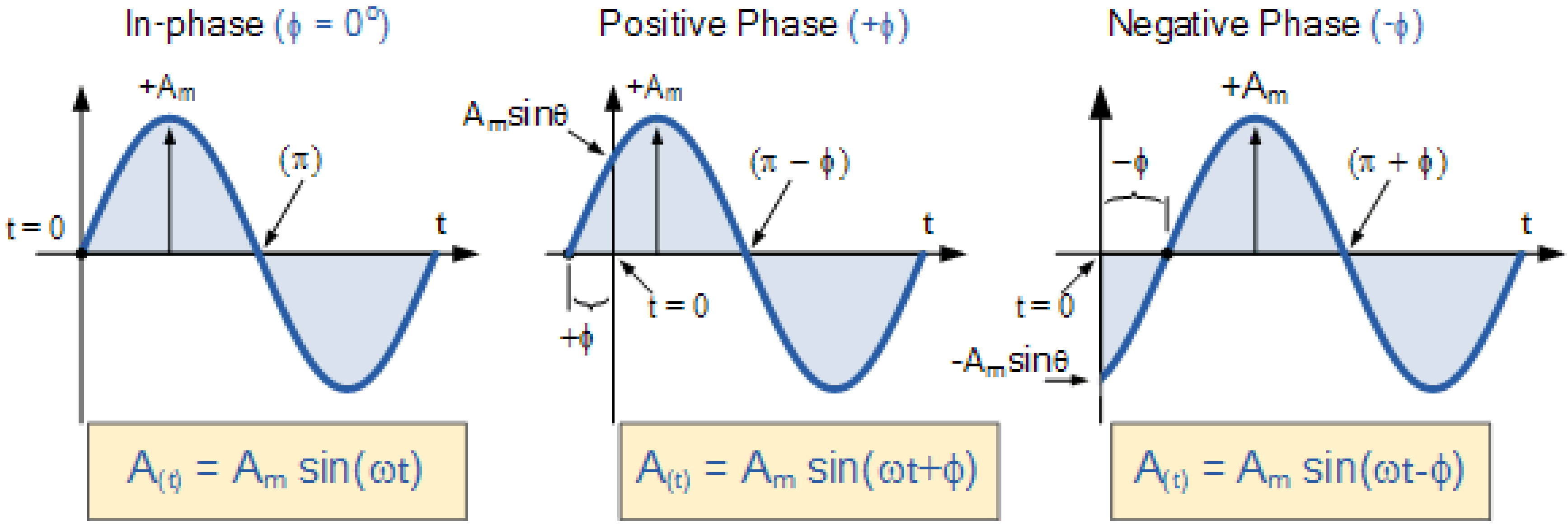
- A phasor is given in polar form $r\angle\theta$
- In rectangular form (x, y) is given as
- $x = r\cos\theta$
- $y = r\sin\theta$

Rectangular to Polar

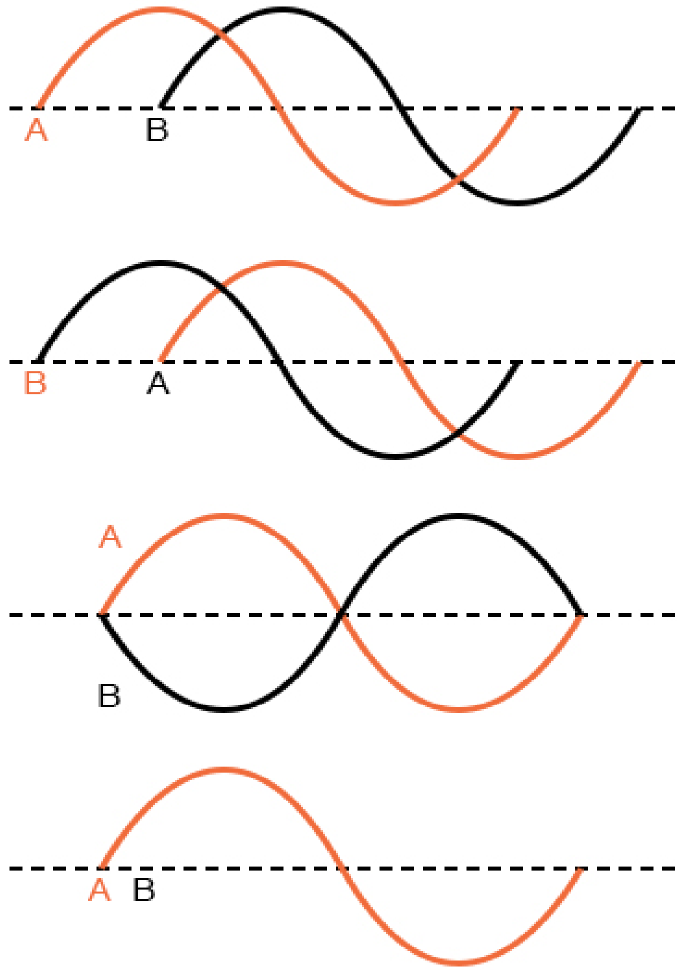


- A phasor is given in rectangular form (x, y)
- In polar form $r\angle\theta$ is given as
- $r = \sqrt{x^2 + y^2}$
- $\theta = \tan^{-1}(\frac{y}{x})$

Zero, Positive, Negative Phase



Phase Difference



Phase shift = 90 degrees
A is ahead of B
(A "leads" B)

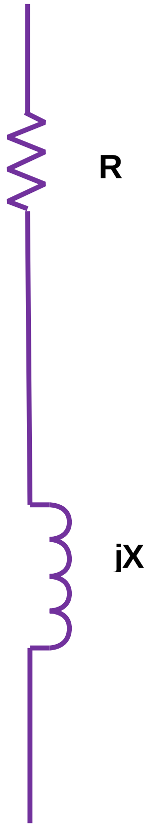
Phase shift = 90 degrees
B is ahead of A
(B "leads" A)

Phase shift = 180 degrees
A and B waveforms are
mirror-images of each other

Phase shift = 0 degrees
A and B waveforms are in
perfect step with each other

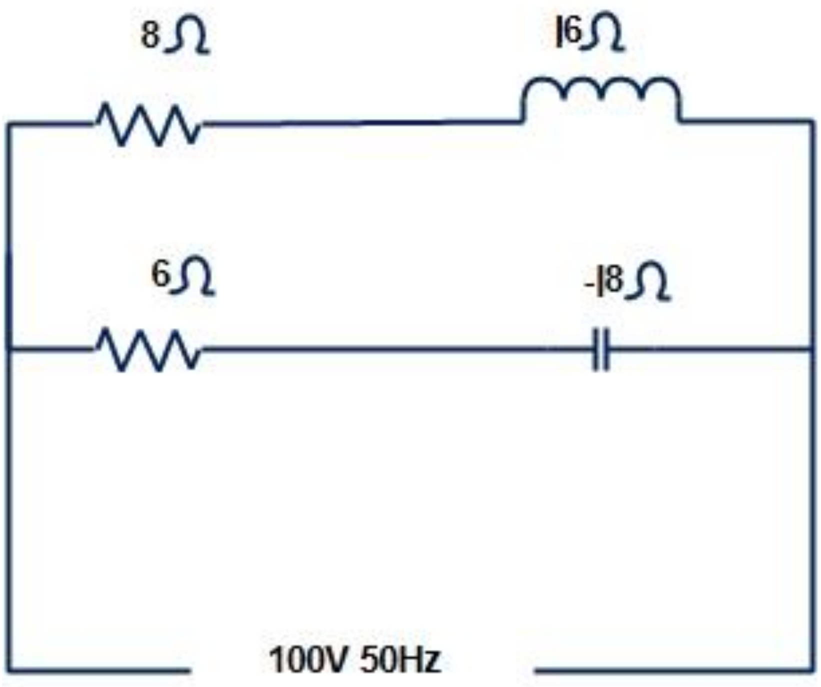
Practice

- Let
  - $R = 8\Omega$
  - $X = j6\Omega$
- Write polar and rectangular form for this RL combination
- Rectangular form
  - $Z = 8 + j6\Omega$
- Polar form
  - $Z = 10\angle 36.86$



Practice

- Find the equivalent impedance
  - $Z_1 = 8 + j6 = 10\angle 37°$
  - $Z_2 = 6 - j8 = 10\angle - 53°$
- Now, total impedance is,
  - $Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{10\angle 37° * 10\angle -53°}{(8+j6)+(6-j8)}$
  - $Z = \frac{100\angle 0}{14 - j2} = 7.071\angle - 8.1°\Omega$

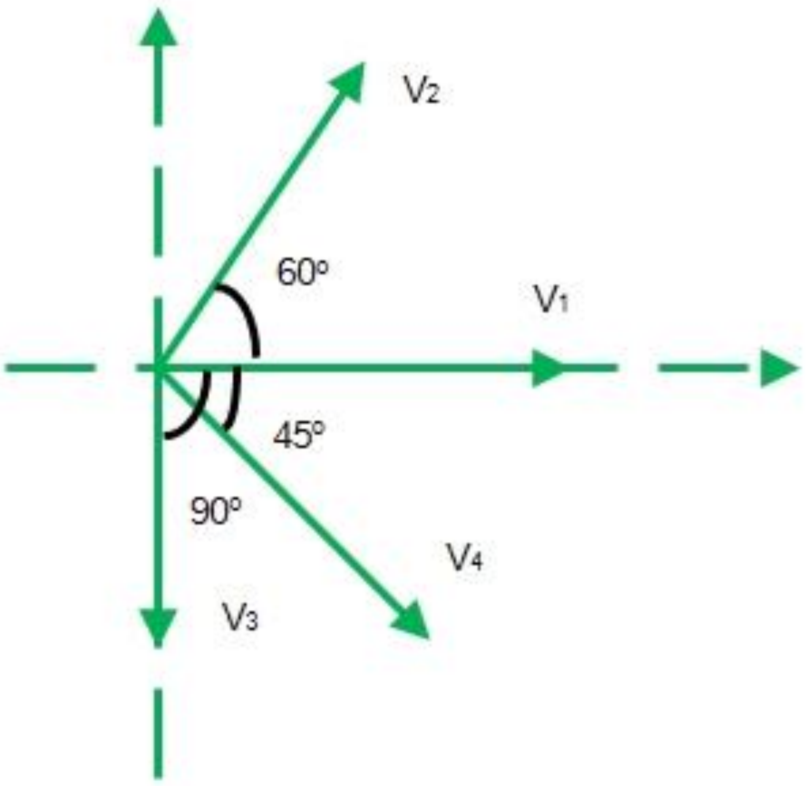


Problem

- Draw the phasor diagram for the following voltages and find the RMS value of the resultant voltage.
  - $v_1 = 100 \sin 500t$
  - $v_2 = 200 \sin(500t + {}^{\pi}/_{3})$
  - $v_3 = -50 \cos 500t$
  - $v_4 = 150 \sin(500t - {}^{\pi}/_{4})$

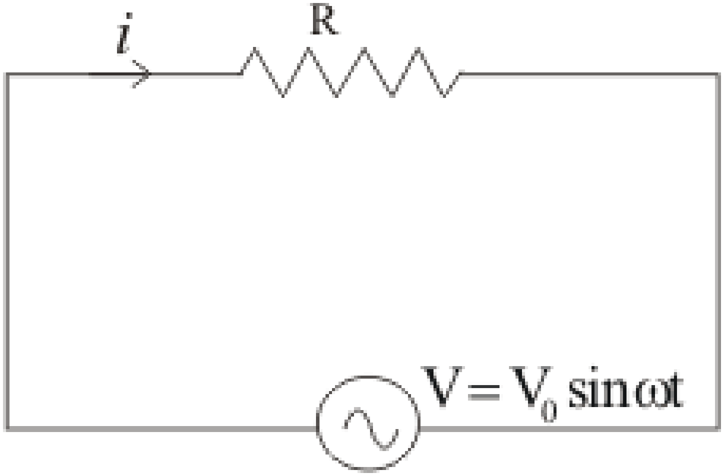
Solution

- In phasor form all voltages are given as
  - $v_1 = 100 \sin 500t = \frac{100}{\sqrt{2}} \angle 0$
  - $v_2 = 200 \sin(500t + {}^{\pi}/_{3}) = \frac{200}{\sqrt{2}} \angle 60$
  - $v_3 = -50 \cos 500t = -50 \sin(90 - \omega t) = 50 \sin(\omega t - 90) = \frac{50}{\sqrt{2}} \angle -90$
  - $v_4 = 150 \sin(500t - {}^{\pi}/_{4}) = \frac{150}{\sqrt{2}} \angle -45$
- Resultant voltage is given as
  - $\vec{v} = \overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} + \overrightarrow{v_4} =$
  - $\frac{100}{\sqrt{2}} \angle 0 + \frac{200}{\sqrt{2}} \angle 60 + \frac{50}{\sqrt{2}} \angle -90 + \frac{150}{\sqrt{2}} \angle -45$
  - $\vec{v} = 70.71 + j0 + 70.71 + j122.47 + 0 - j35.35 + 75 - j75$
  - $\vec{v} = 216.4212 + j12.1184$
  - $\vec{v} = 216.76 \angle 3.205°$
  - $v = 306.54 \sin(500t + 3.205°)$



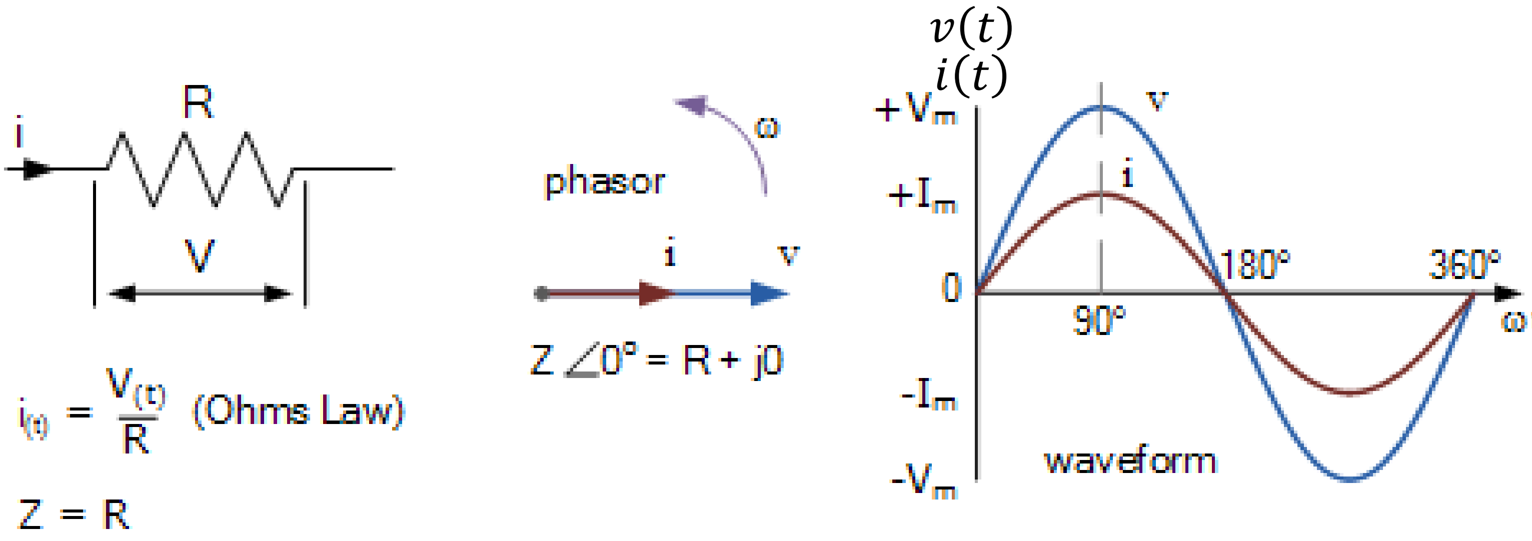
Lecture 11

Pure R Circuit



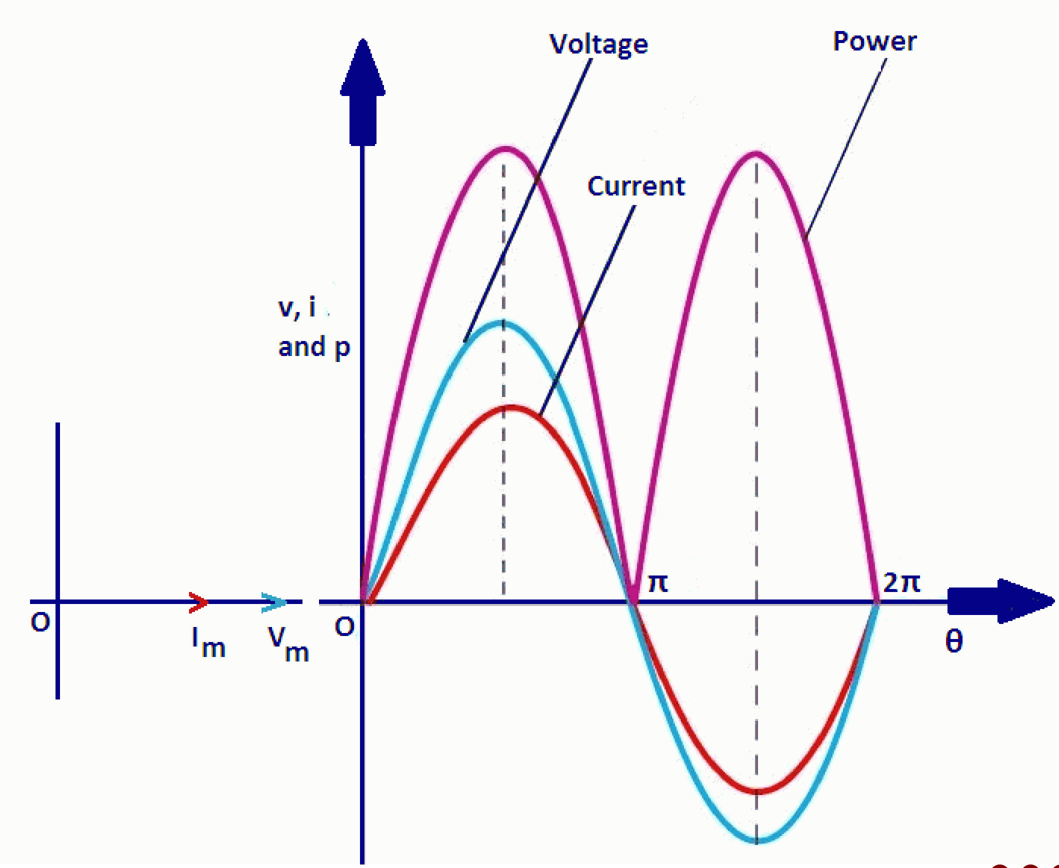
- $v = V_0 \sin \omega t$
- $Z = R$
- $i = \frac{v}{R} = \frac{V_0}{R} \sin \omega t = I_0 \sin \omega t$

Pure R Circuit: $v - i$ Relationship



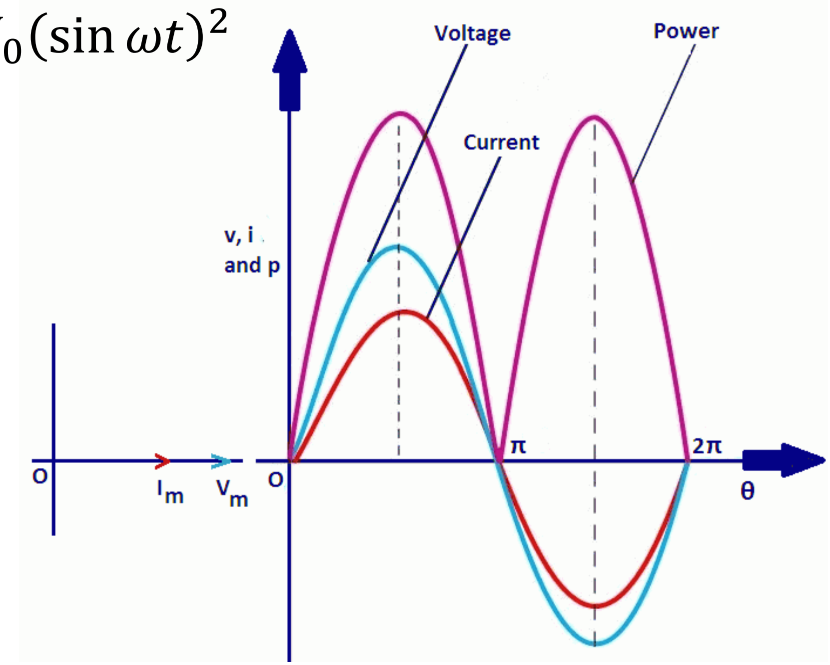
Pure R Circuit: Power-Graphical Approach

- From $0 - \pi$
  - $v = +ve$
  - $i = +ve$
  - $p = v * i = +ve$
- From $\pi - 2\pi$
  - $v = -ve$
  - $i = -ve$
  - $p = v * i = +ve$



Pure R Circuit: Power-Mathematical Approach

- Power at any instant is given as
- $p(\omega t) = v(\omega t) * i(\omega t)$
- $p(\omega t) = V_0 \sin \omega t * I_0 \sin \omega t = V_0 I_0 (\sin \omega t)^2$
- $p(\omega t) = V_0 I_0 (1 - \cos 2\omega t)/2$
- Average Power is given as
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} p(\omega t) d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2\pi} \int_0^{\pi} (1 - \cos 2\omega t)\, d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2\pi} [\omega t - \sin 2\omega t]_0^{\pi}$
- $P_{avg} = \frac{V_0 I_0}{2}$
- $P_{avg} = V_{rms} I_{rms}$

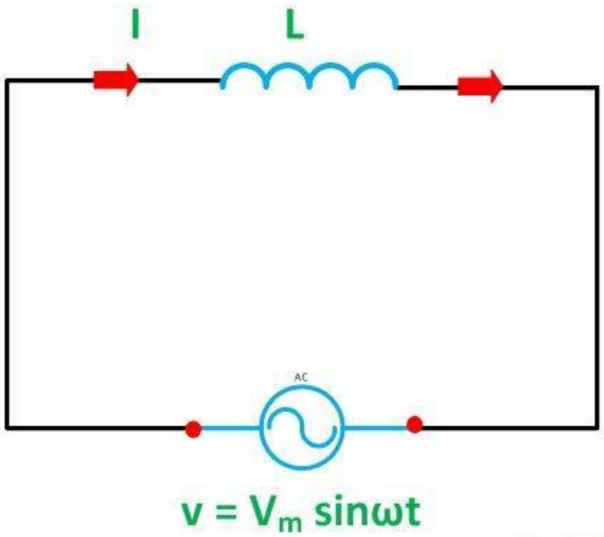


Pure R Circuit: Conclusions

- $Z = R$
- $v\ and\ i$ are in same phase
- $\emptyset = 0$
- $\cos\emptyset = 1$
- Power transfer takes place always from source to load ($p = +ve$, for all t)
- $frequency(power) = 2frequency(voltage, current)$

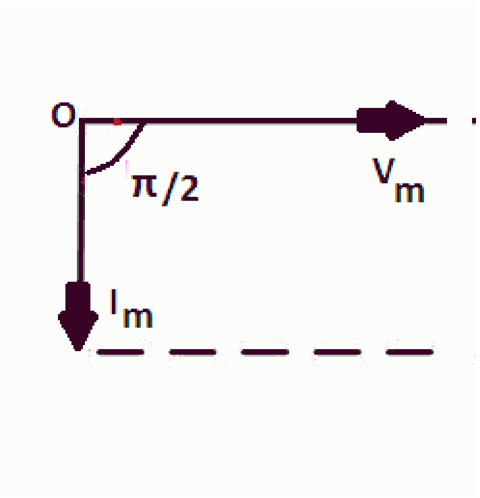
Pure L Circuit

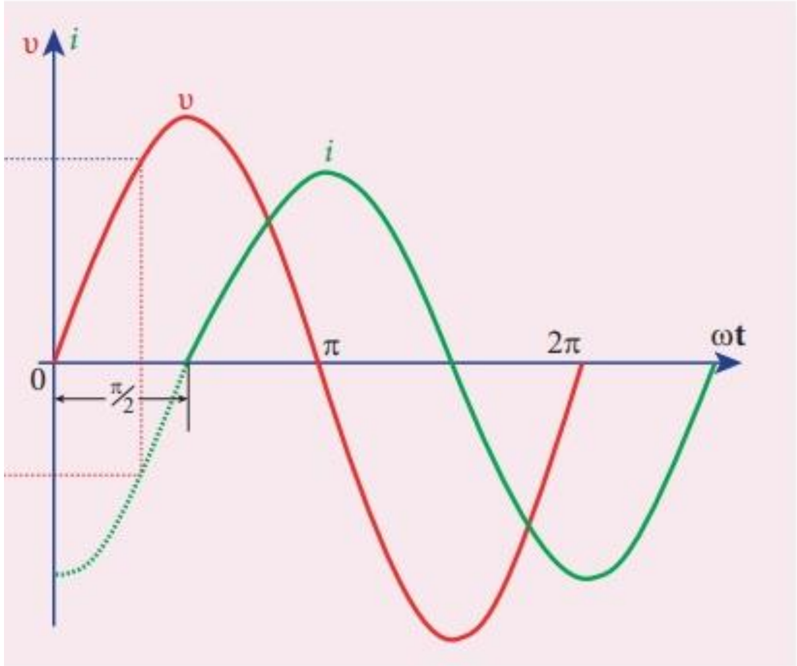
- $N\emptyset = Li$
- $N\frac{d}{dt}\emptyset = L\frac{d}{dt}i$
- $e_L = L\frac{d}{dt}i$
- $i = \frac{1}{L}\int e_L dt = \frac{1}{L}\int V_m \sin\omega t\, dt$
- $i = \frac{V_m}{\omega L}[-\cos\omega t] = I_m \sin(\omega t - {}^{\pi}/_{2})$
- $z = 0 + jX_L = j\omega L = \omega L\angle 90°$



Pure L Circuit: $v - i$ Relationship

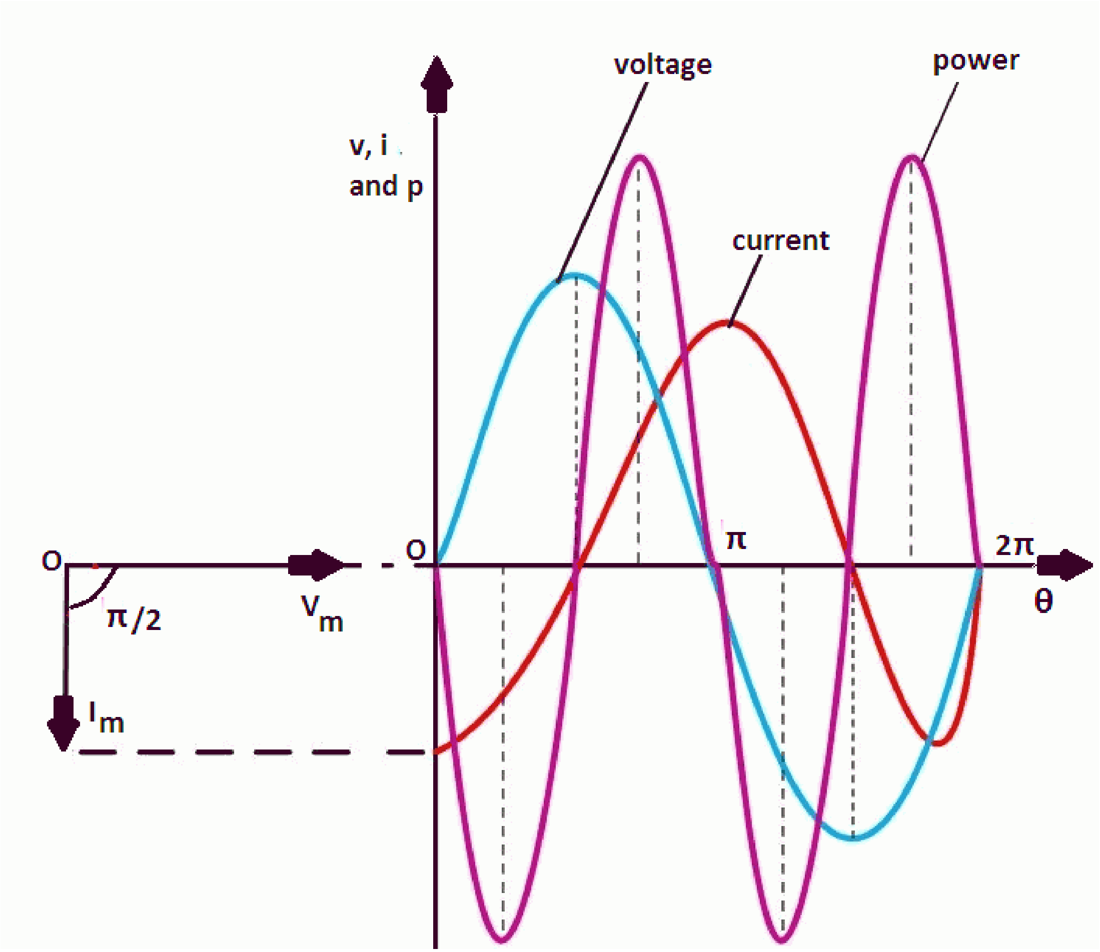
- Inductor current lags inductor voltage by 90 degrees





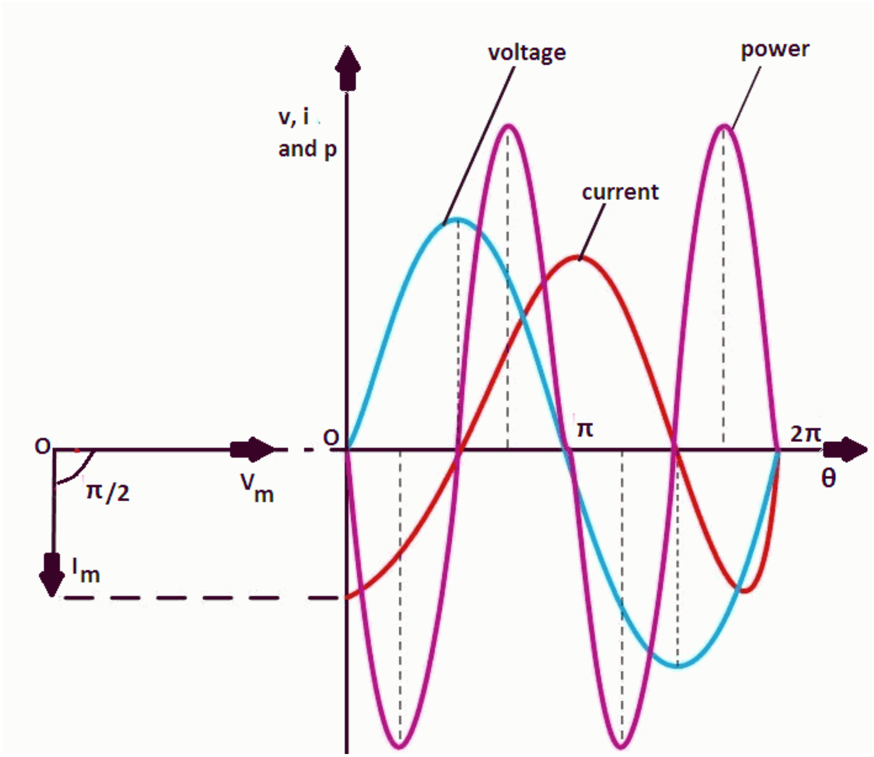
Pure L Circuit: Power

- From $0 - \pi/2$
  - $v = +ve$
  - $i = -ve$
  - $p = v * i = -ve$
- From $\pi/2 - \pi$
  - $v = +ve$
  - $i = +ve$
  - $p = v * i = +ve$
- From $\pi - 3\pi/2$
  - $v = -ve$
  - $i = +ve$
  - $p = v * i = -ve$
- From $3\pi/2 - 2\pi$
  - $v = -ve$
  - $i = -ve$
  - $p = v * i = +ve$



Pure L Circuit: Power

- Power at any instant is given as
- $p(\omega t) = v(\omega t) * i(\omega t)$
- $p(\omega t) = V_0 \sin \omega t * I_0 \sin(\omega t - \frac{\pi}{2}) = V_0 I_0 \sin \omega t \sin(\omega t - \frac{\pi}{2})$
- $p(\omega t) = V_0 I_0 \sin \omega t \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t$
- Average Power is given as
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} p(\omega t) d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2\pi} \int_0^{\pi} \sin 2\omega t \, d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2\pi} [\frac{-\cos 2\omega t}{2}]_0^{\pi} = 0$

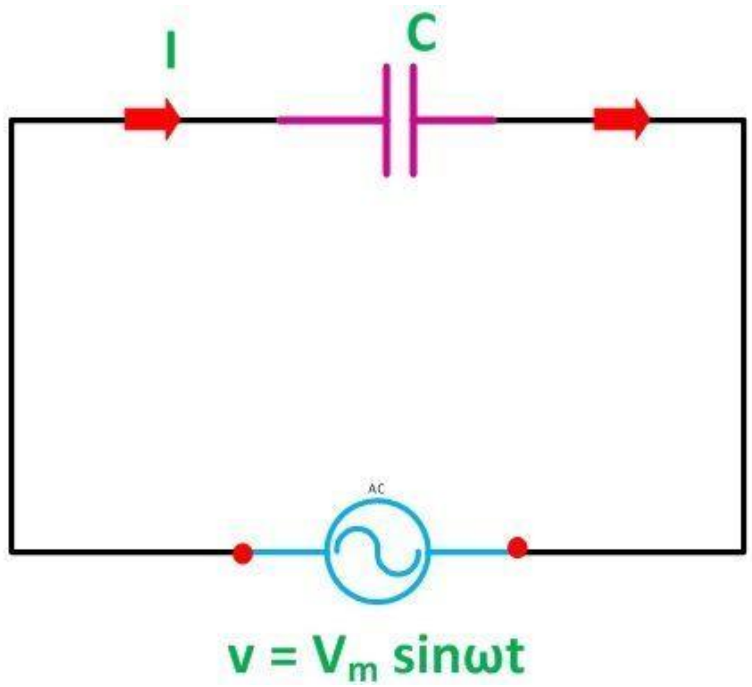


Pure L Circuit: Conclusions

- $Z = jX_L$
- $i\ lags\ v$ by 90°
- $\emptyset = 90°\ (\pi/2)$
- $\cos\emptyset = 0$
- Power transfer takes place from source to load ($p = +ve$) for half cycle, and from load to source ($p = -ve$) for next half cycle of power
- $P_{avg} = 0$
- $frequency(p) = 2frequency(v, i)$

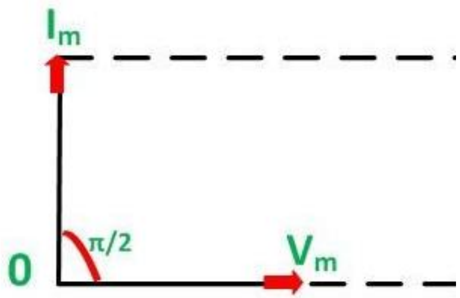
Pure C Circuit

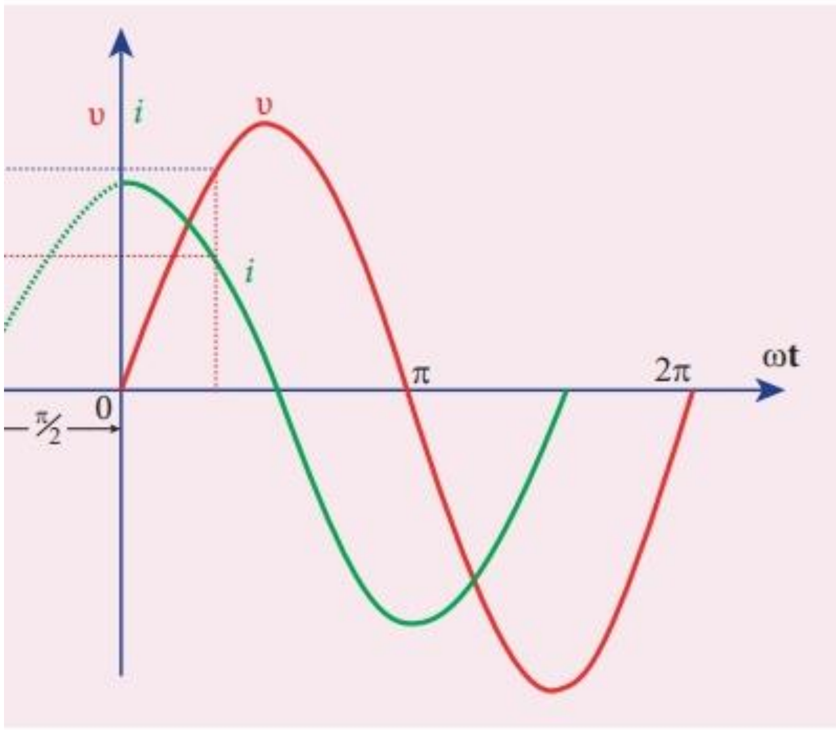
- $q = Cv$
- $\frac{d}{dt}q = C\frac{d}{dt}v$
- $i = C\frac{d}{dt}v = C\frac{d}{dt}(V_0 \sin\omega t)$
- $i = \omega C V_0 \cos\omega t$
- $i = \omega C V_0 \sin(\omega t + \frac{\pi}{2})$
- $I_0 = \frac{V_0}{X_C}$
- $X_C = 1/\omega C$



Pure C Circuit

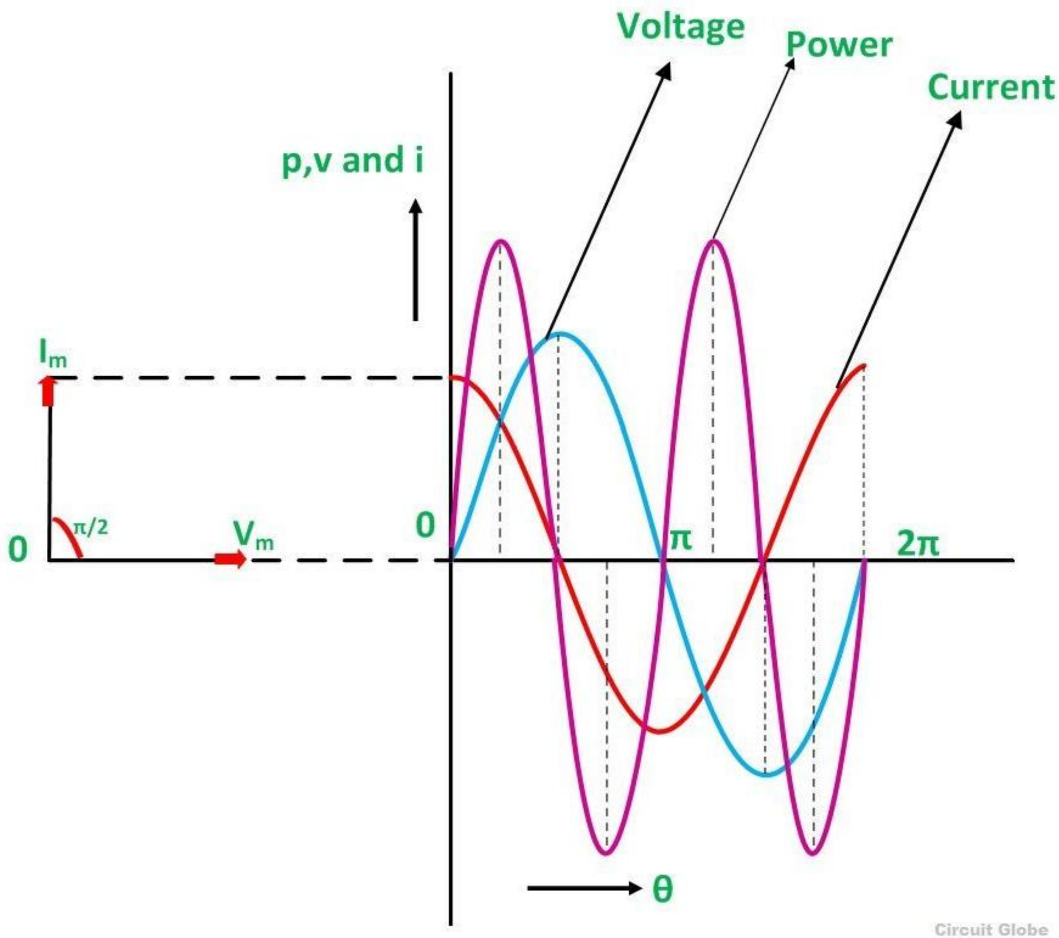
- $i \text{ leads } v \text{ by } 90°$





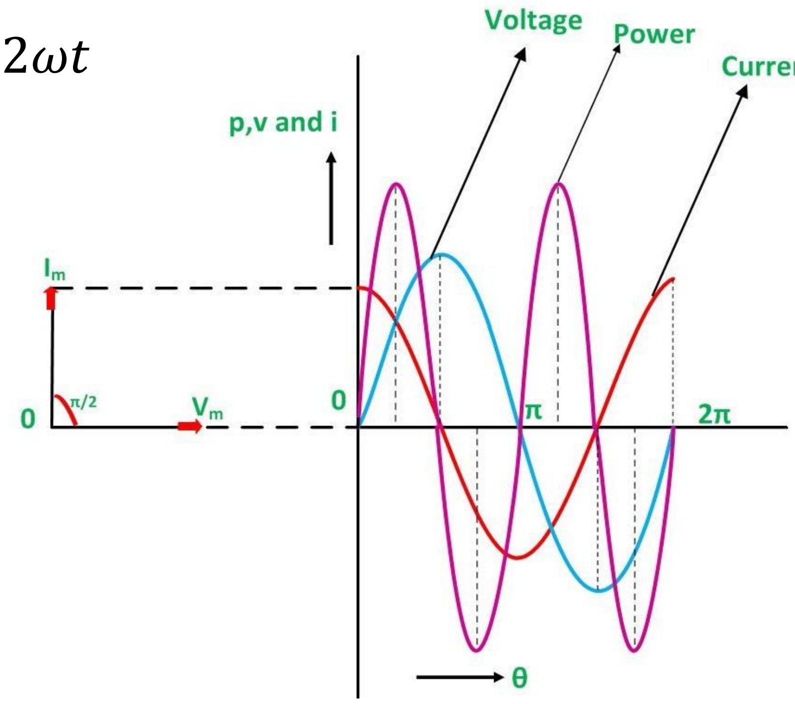
Pure C Circuit: Power

- From $0 - \pi/2$
  - $v = +ve$
  - $i = +ve$
  - $p = v * i = +ve$
- From $\pi/2 - \pi$
  - $v = +ve$
  - $i = -ve$
  - $p = v * i = -ve$
- From $\pi - 3\pi/2$
  - $v = -ve$
  - $i = -ve$
  - $p = v * i = +ve$
- From $3\pi/2 - 2\pi$
  - $v = -ve$
  - $i = +ve$
  - $p = v * i = -ve$



Pure C Circuit: Power

- Power at any instant is given as
- $p = v * i$
- $p = V_0 \sin \omega t * I_0 \sin(\omega t + \frac{\pi}{2}) = V_0 I_0 \sin \omega t \sin(\omega t + \frac{\pi}{2})$
- $p = V_0 I_0 \sin \omega t \cos \omega t = \frac{V_0 I_0}{2} \sin 2\omega t$
- Average Power is given as
- $P_{avg} = \frac{1}{T} \int_0^T p(\omega t) d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2\pi} \int_0^{\pi} \sin 2\omega t \, d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2\pi} [\frac{-\cos 2\omega t}{2}]_0^{\pi} = 0$



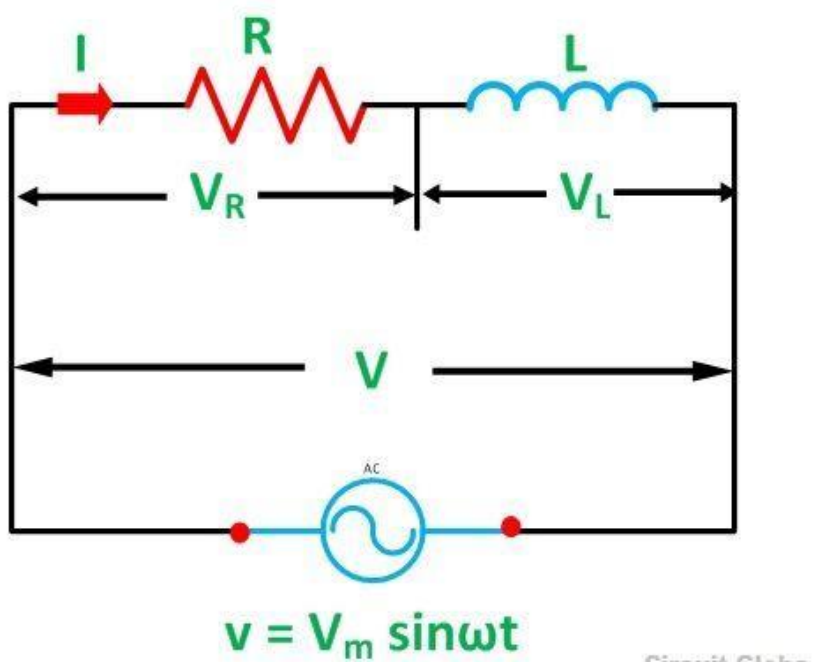
Pure C Circuit: Conclusions

- $Z = -jX_c$
- $i\ leads\ v$ by 90°
- $\emptyset = 90°\ (\pi/2)$
- $\cos\emptyset = 0$
- Power transfer takes place from source to load ($p = +ve$) for half cycle, and from load to source ($p = -ve$) for next half cycle of power
- $P_{avg} = 0$
- $f(p) = 2f(v, i)$

Lecture 12 & 13

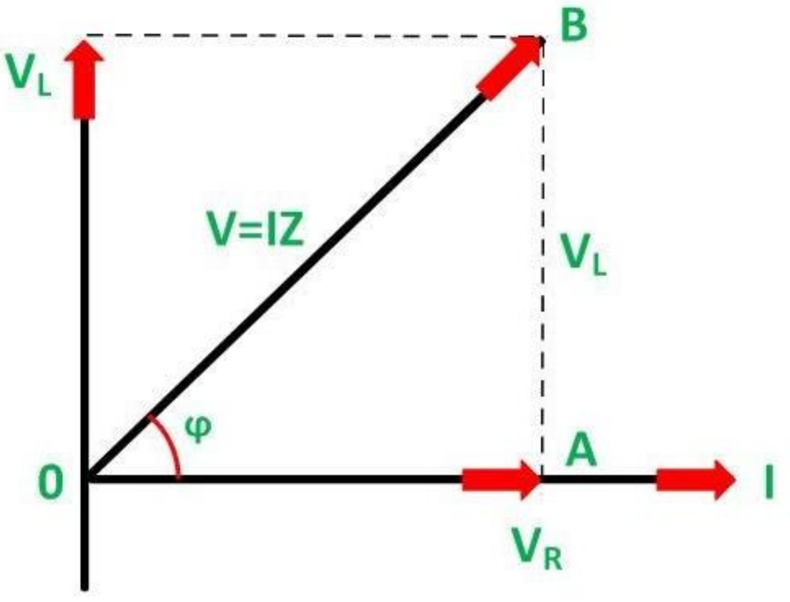
RL Circuit

- $v = V_0 \sin \omega t$



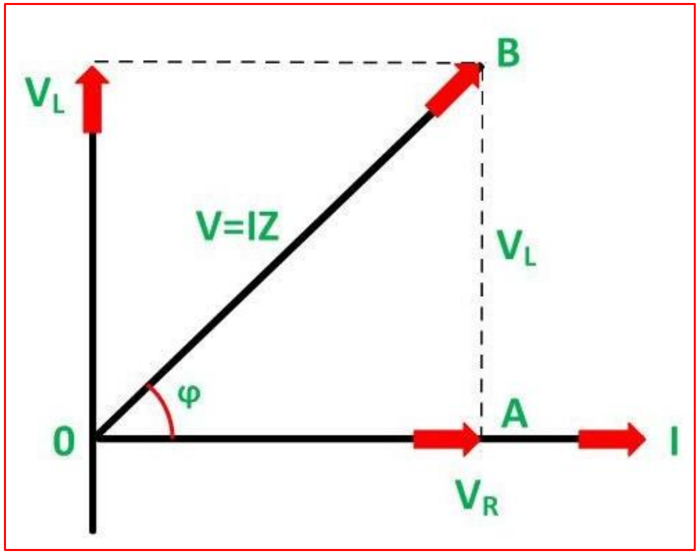
RL Circuit: $v - i\ phasor$

- $i_R = i_L = i$
- $i\ is\ in\ phase\ with\ v_R$
- $i\ lags\ v_L\ by\ 90°$



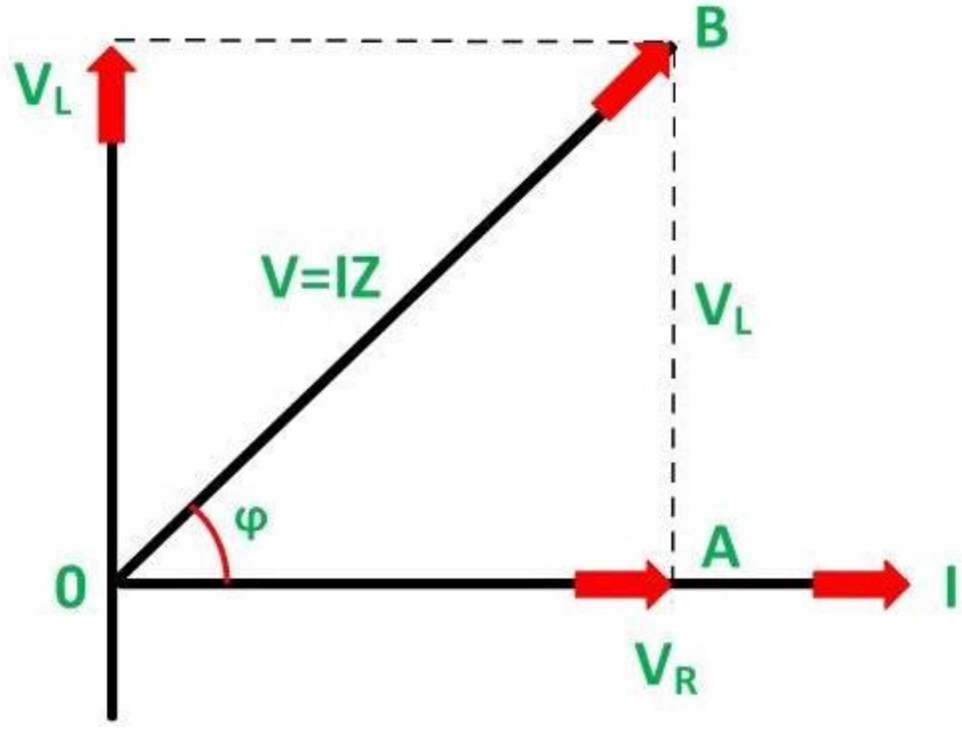
RL Circuit: Voltage Triangle

- $\underset{v}{\rightarrow} = \underset{v_R}{\rightarrow} + \underset{v_L}{\rightarrow}$
- $v^2 = {v_R}^2 + {v_L}^2$
- $(iZ)^2 = (iR)^2 + (iX_L)^2$
- $Z^2 = R^2 + {X_L}^2$



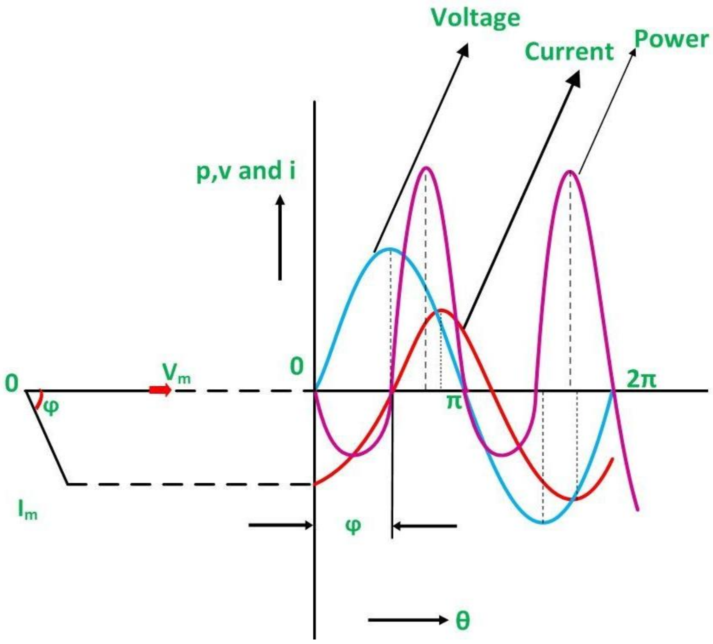
RL Circuit: Impedance Triangle

- Impedance of the Circuit
- $Z = R + jX_L$
- $Z = \sqrt{R^2 + {X_L}^2}$
- $\emptyset = \tan^{-1}\frac{X_L}{R}$
- Current in the Circuit
- $i = v/(R + jX_L) = \frac{v}{Z\angle\phi}$
- $i = \frac{v}{Z}\angle - \phi$
- $i(\omega t) = I_0 \sin(\omega t - \phi)$



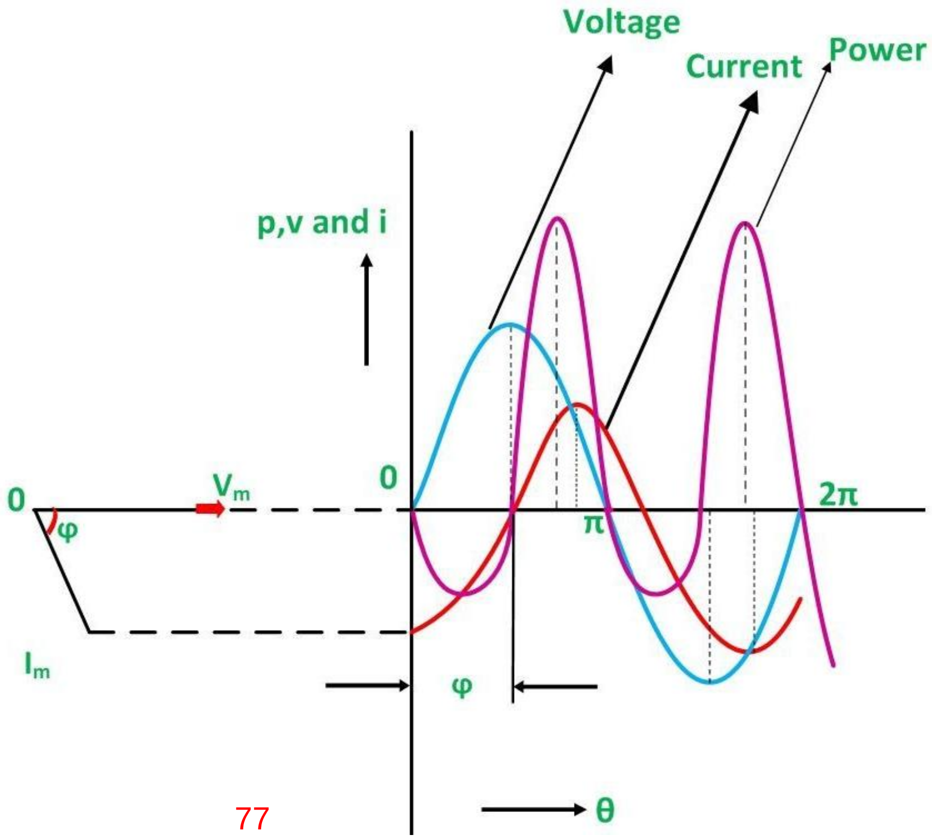
RL Circuit: Power

- $p = v * i$
- $p = V_0 \sin \omega t * I_0 \sin(\omega t - \emptyset)$
- $p = I_0 V_0 [\sin \omega t \sin(\omega t - \emptyset)]$
- $p = \frac{V_0 I_0}{2} [\cos \emptyset - \cos(2\omega t - \emptyset)]$
- $p = \frac{V_0 I_0}{2} \cos \emptyset - \frac{V_0 I_0}{2} \cos(2\omega t - \emptyset)$
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} p(\omega t) d(\omega t)$
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} [\frac{V_0 I_0}{2} \cos \emptyset - \frac{V_0 I_0}{2} \cos(2\omega t - \emptyset)] d(\omega t)$



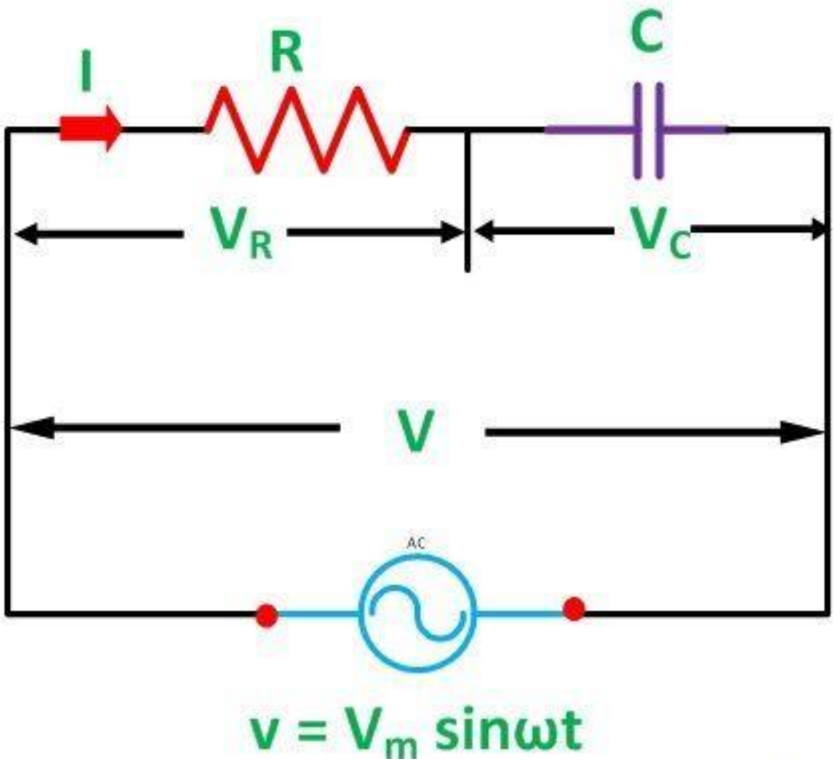
RL Circuit: Power cont…

- $\int_0^{\pi} \cos(2\omega t - \emptyset)\, d(\omega t) = \int_0^{\pi} [\cos 2\omega t \cos \emptyset + \sin 2\omega t \sin \emptyset] d(\omega t) = 0$
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} p(\omega t) d(\omega t) = \frac{1}{\pi} \int_0^{\pi} \frac{V_0 I_0}{2} \cos \emptyset\, d(\omega t) = \frac{V_0 I_0}{2\pi} \cos \emptyset \int_0^{\pi} d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2} \cos \emptyset$
- $P_{avg} = V_{rms} I_{rms} \cos \emptyset$



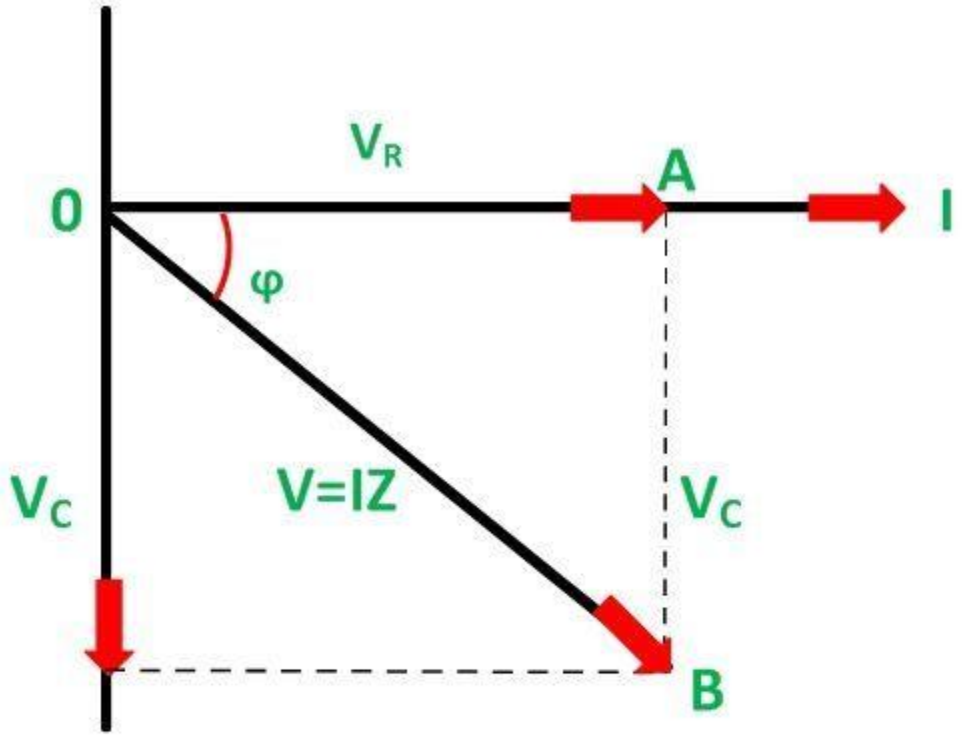
RC Circuit

- $v = V_0 \sin \omega t$



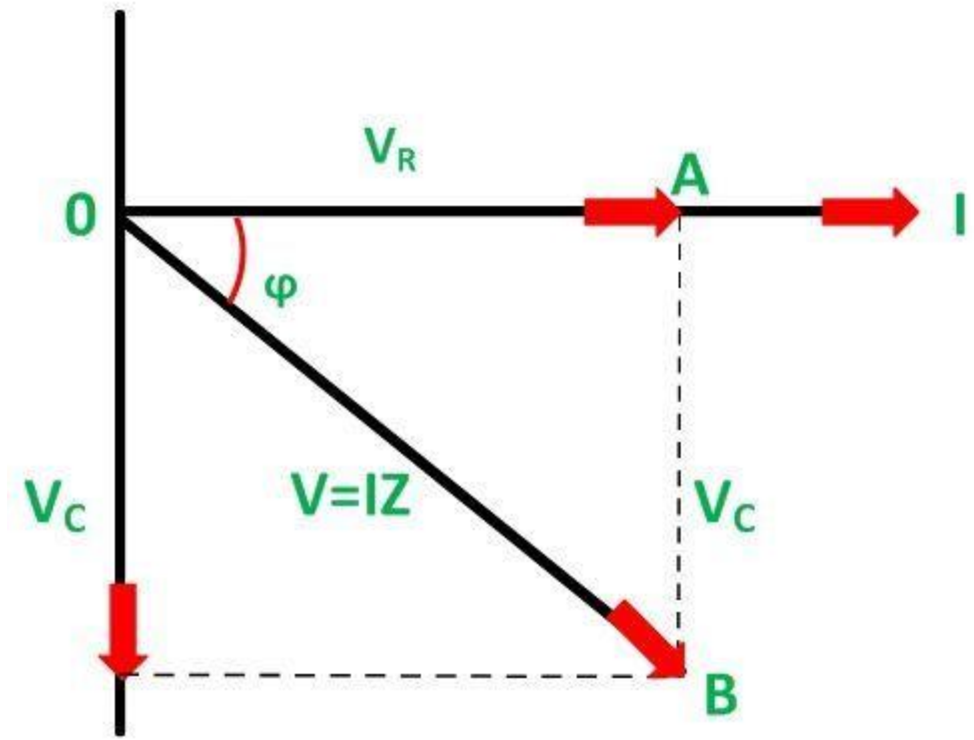
RC Circuit: $v - i\ phasor$

- $i_R = i_c = i$
- $i\ is\ in\ phase\ with\ v_R$
- $i\ leads\ v_c\ by\ 90°$



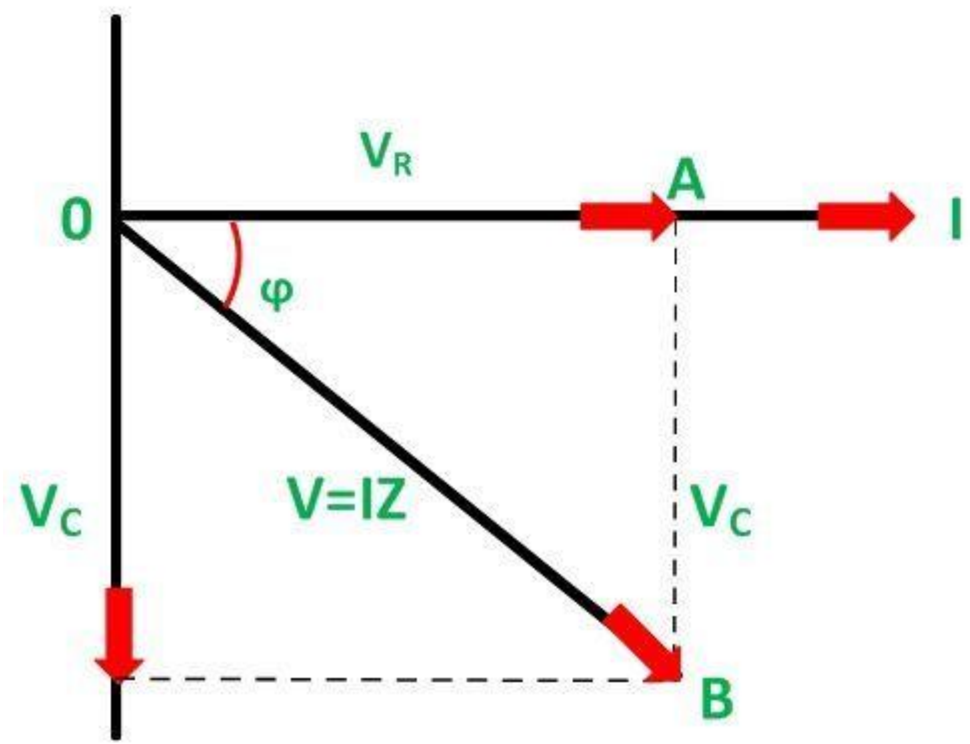
RC Circuit: Voltage Triangle

- $\vec{v} = \vec{v_R} + \vec{v_C}$
- $v^2 = {v_R}^2 + {v_C}^2$
- $(iZ)^2 = (iR)^2 + (iX_c)^2$
- $Z^2 = R^2 + {X_c}^2$



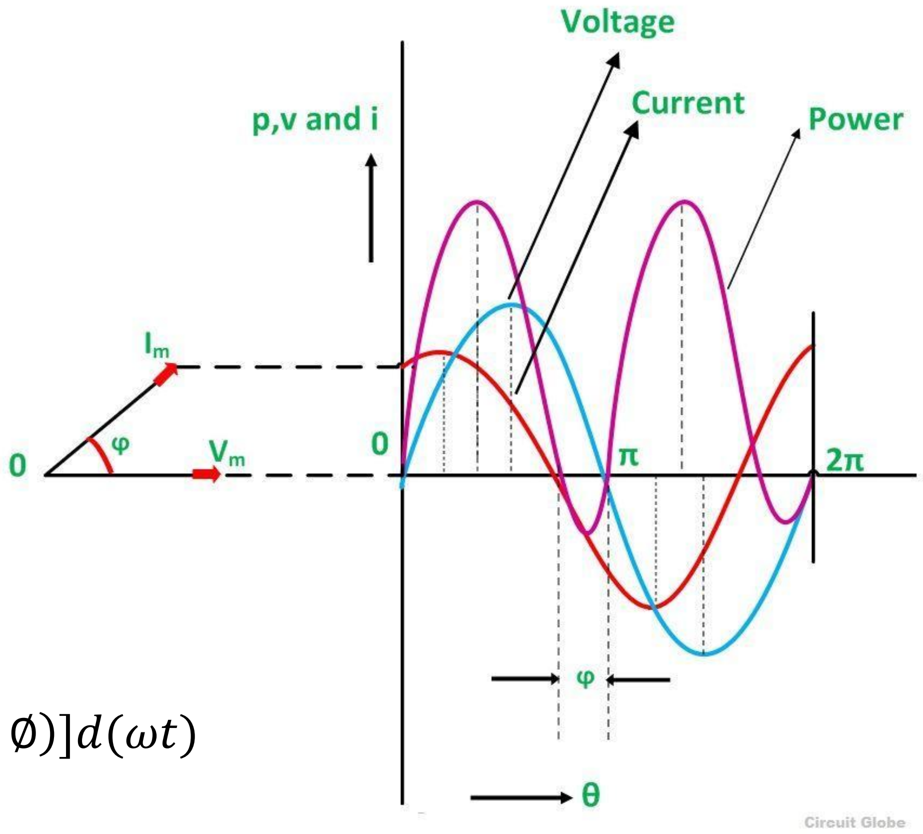
RC Circuit: Impedance Triangle

- Impedance of the Circuit
- $Z = R - jX_c$
- $Z = \sqrt{R^2 + {X_c}^2}$
- $\emptyset = \tan^{-1}(\frac{-X_C}{R})$
- Current in the Circuit
- $i = v/(R - jX_C) = \frac{v}{Z\angle -\phi}$
- $i = \frac{v}{Z}\angle\phi$
- $i(\omega t) = I_0 \sin(\omega t + \phi)$



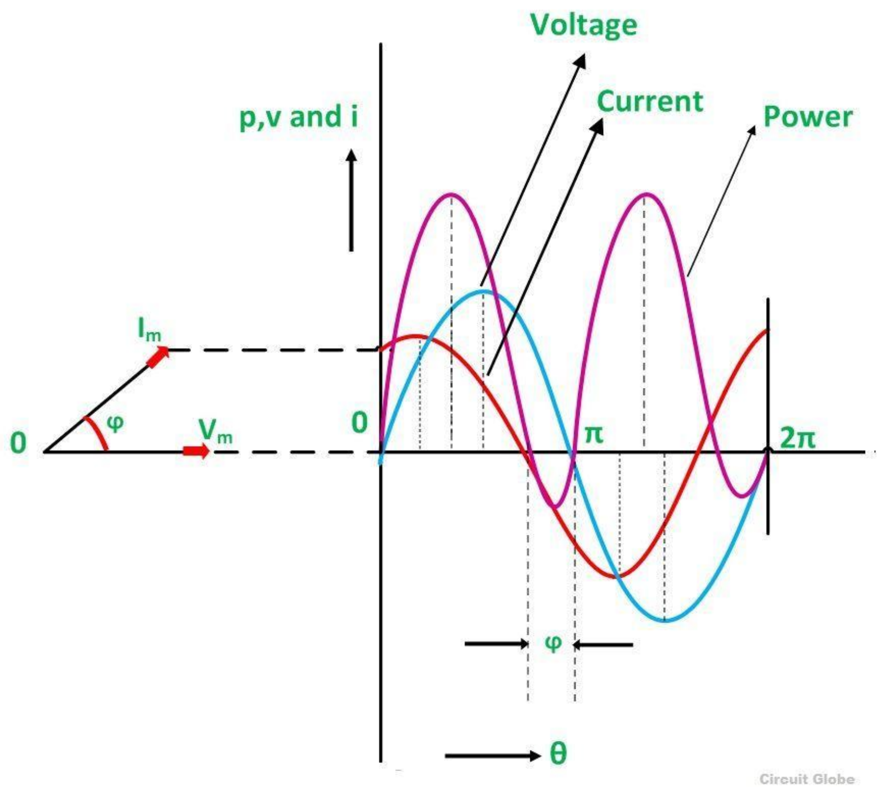
RC Circuit: Power

- $p = v * i$
- $p = V_0 \sin \omega t * I_0 \sin(\omega t + \emptyset)$
- $p = V_0 I_0 [\sin \omega t \sin(\omega t + \emptyset)]$
- $p = \frac{V_0 I_0}{2} [\cos(-\emptyset) - \cos(2\omega t + \emptyset)]$
- $p = \frac{V_0 I_0}{2} \cos \emptyset - \frac{V_0 I_0}{2} \cos(2\omega t + \emptyset)$
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} p(\omega t) d(\omega t)$
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} [\frac{V_0 I_0}{2} \cos \emptyset - \frac{V_0 I_0}{2} \cos(2\omega t + \emptyset)] d(\omega t)$



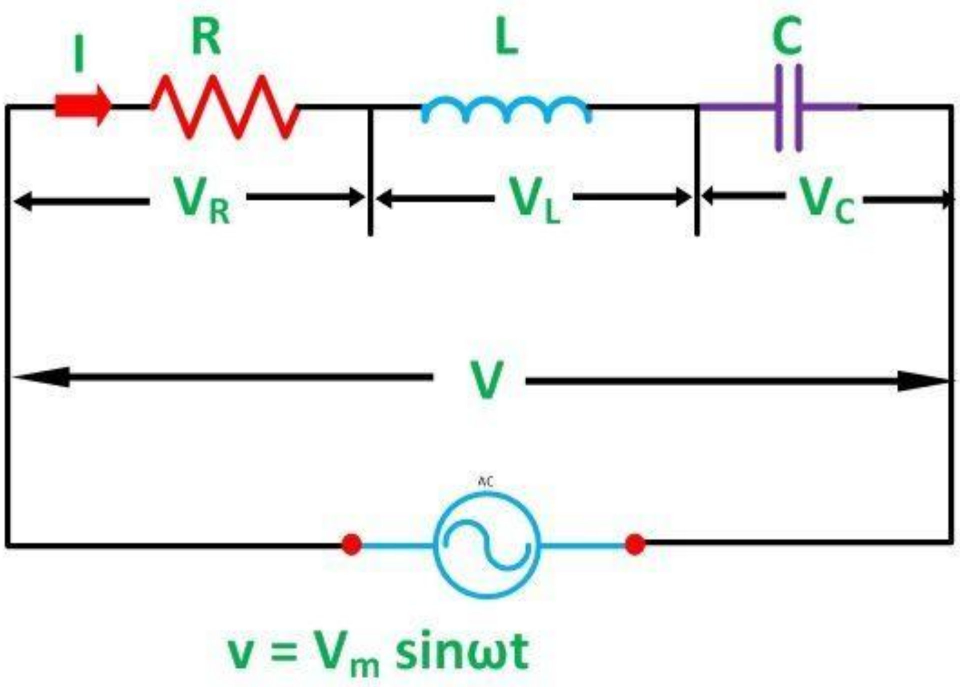
RC Circuit: Power cont…

- $\int_0^{\pi} \cos(2\omega t + \emptyset)\, d(\omega t) = \int_0^{\pi} [\cos 2\omega t \cos \emptyset - \sin 2\omega t \sin \emptyset] d(\omega t) = 0$
- $P_{avg} = \frac{1}{\pi} \int_0^{\pi} p(\omega t) d(\omega t) = \frac{1}{\pi} \int_0^{\pi} \frac{V_0 I_0}{2} \cos \emptyset\, d(\omega t) = \frac{V_0 I_0}{2\pi} \cos \emptyset \int_0^{\pi} d(\omega t)$
- $P_{avg} = \frac{V_0 I_0}{2} \cos \emptyset$
- $P_{avg} = V_{rms} I_{rms} \cos \emptyset$



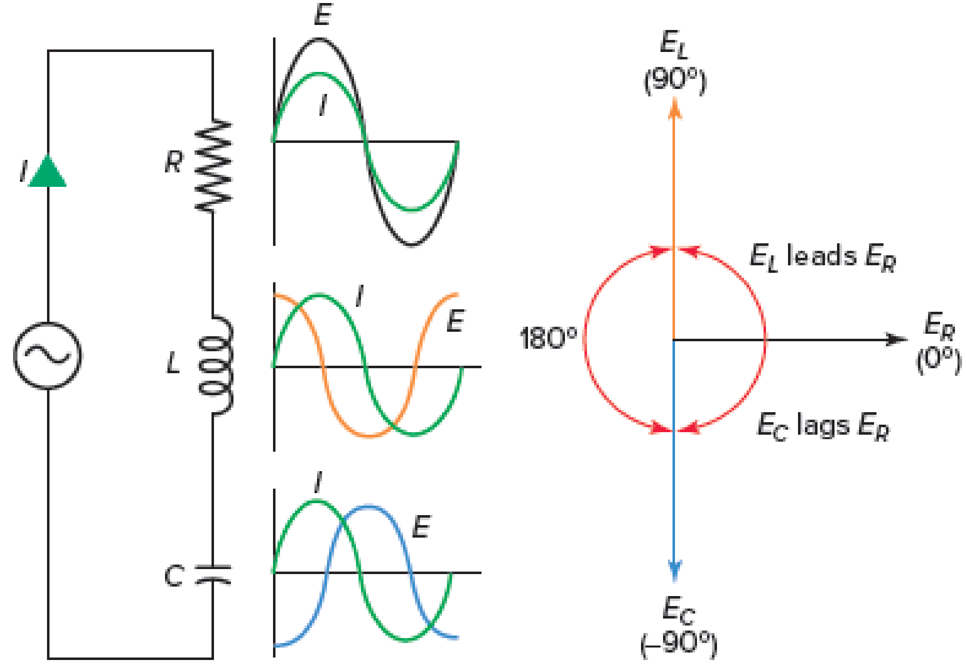
RLC Circuit

- $v = V_0 \sin wt$
- $i = I_0 \sin(wt \pm \emptyset)$
- $\vec{v} = \overrightarrow{v_R} + \overrightarrow{v_L} + \overrightarrow{v_C}$
- Impedance of the Circuit
- $Z = R + j(X_L - X_C)$
- Current in the Circuit
- $i = v/(R + j(X_L - X_C))$
- $p = v * i$
- $P_{avg} = V_{rms} I_{rms} \cos \emptyset$



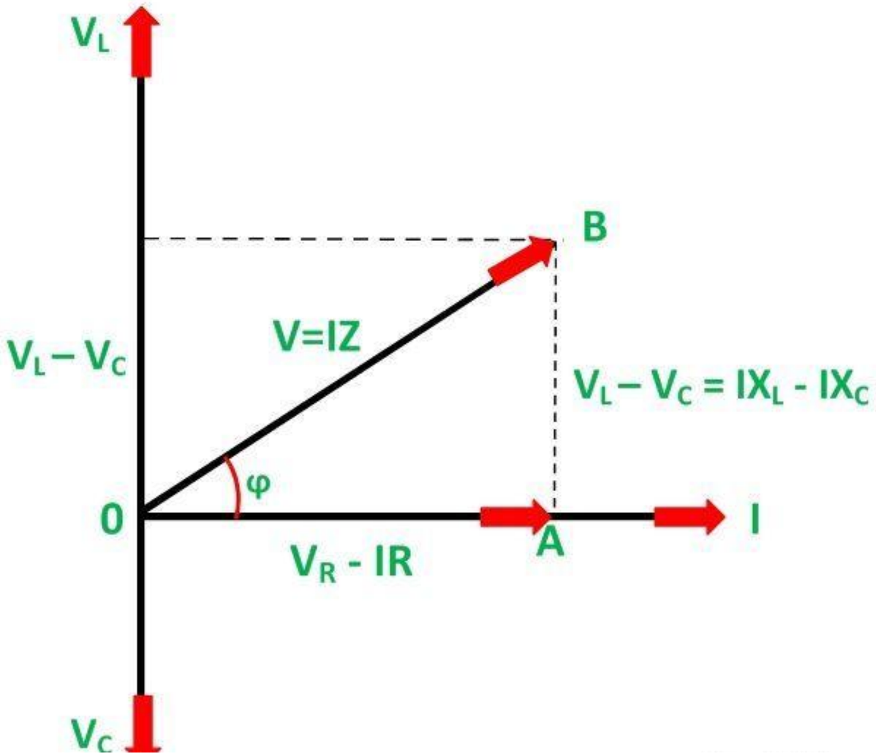
RLC Circuit: $v - i\ phasor$

- $i_R = i_L = i_C = i$
- $i\ is\ in\ phase\ with\ v_R$
- $i\ leads\ v_C\ by\ 90°$
- $i\ lags\ v_L\ by\ 90°$



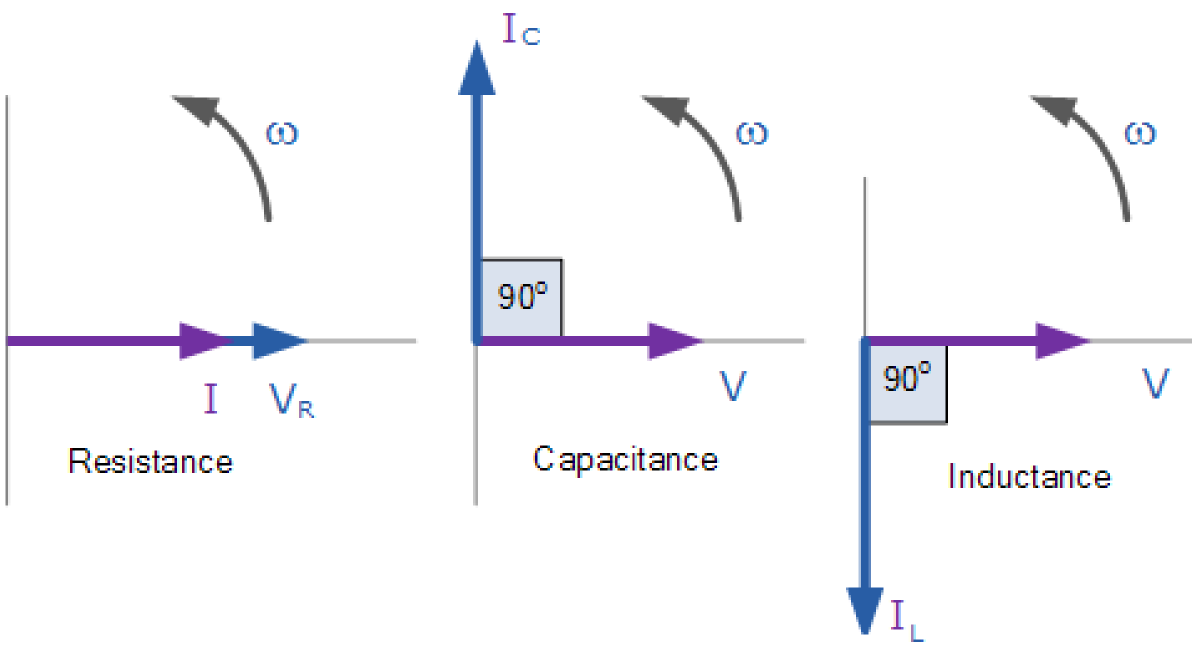
RLC Circuit: Voltage Triangle

- $\vec{v} = \overrightarrow{v_R} + \overrightarrow{v_L} + \overrightarrow{v_C}$
- $v^2 = {v_R}^2 + (v_L - v_C)^2$
- $(iZ)^2 = (iR)^2 + (i(X_L - X_c))^2$
- $Z^2 = R^2 + (X_L - X_c)^2$
- $\emptyset = \tan^{-1} \frac{(v_L - v_C)}{v_R}$



RLC Circuit

- Case 1- $X_L > X_C$ Circuit behaves as RL circuit
- Case 2- $X_L < X_C$ Circuit behaves as RC circuit
- Case 3- $X_L = X_C$ Circuit behaves as pure R circuit

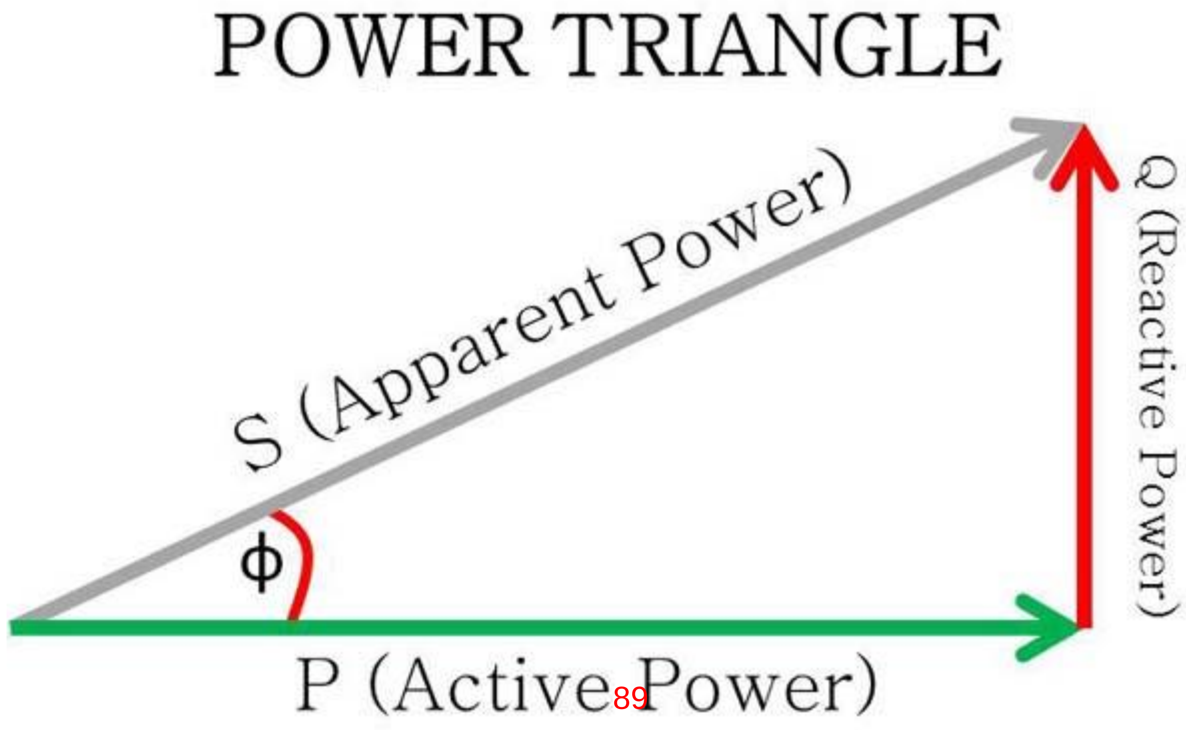


Types of Power

- ***Total/Apparent/Gross power***
  - The total power sent from the source end is called apparent power, and is denoted by S. It's unit of measurement is Volt-Amp (VA).
- ***Active/Real/Average/Net power***
  - The fraction of total power which is received/utilized/dissipated by load is called active power.
  - It is denoted by P, and it's unit of measurement is Watts (W).
  - This power is responsible for the actual work done.
- ***Reactive/Imaginary power***
  - That fraction of total power which gets bounced back from the load end towards source end.
  - Denoted by Q, and it's unit of measurement is Volt-Amp-reactive (VAr)

Types of Power

- $P = V_{rms} * I_{rms} \cos \emptyset$
- $Q = V_{rms} * I_{rms} \sin \emptyset$
- $S = P + jQ = V_{rms} * I_{rms}$



Problem

- In a series circuit voltage and current are given as
  - $v = 283 \sin 314t$
  - $i = 4 \sin(314t - 45)$
- Find
  - Impedance
  - Circuit parameters
  - Power factor and power
  - Phasor diagram

Solution

- Impedance
  - $z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{283\angle 0}{4\angle -45} = 70.75\angle 45\Omega$
  - $z = 50.02 + j50.02\Omega$
- Circuit parameters
  - Impedance in rectangular form is given as
    - $Z = R + jX_L$
  - Hence,
    - $R = 50.02\Omega$
    - $X_L = 50.02\Omega = \omega L$
    - $L = \frac{50.02}{314} = 0.1593$Henry
- Power factor
  - $\cos\phi = \cos 45 = 0.7071$ (lag)
- Power
  - $S = V_{rms}I_{rms} = \frac{283}{\sqrt{2}}\frac{4}{\sqrt{2}}$ =566VA
  - $P = V_{rms}I_{rms}\cos\phi = 400.22$W
  - $Q = V_{rms}I_{rms}\sin\phi = 400.22$VAR

Problem

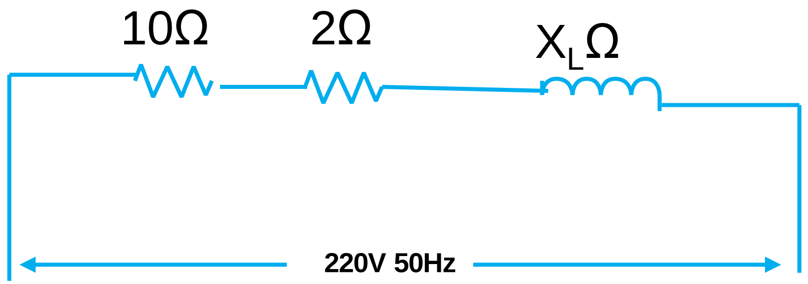
- Given
  - $v = 200 \sin 377t$
  - $i = 8 \sin(377t - 30)$
- Find
  - Impedance
  - Circuit parameters
  - Power factor and power
  - Phasor diagram

Solution

- Impedance
  - $z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{200\angle 0}{8\angle -30} = 25\angle 30\Omega$
  - $z = 21.65 + j12.5\Omega$
- Circuit parameters
  - Impedance in rectangular form is given as
  - $z = R + jX_L$
  - Hence,
  - $R = 21.65\Omega$
  - $X_L = 12.5\Omega = \omega L$
  - $L = \frac{12.5}{377} = 0.0331\text{Henry}$
- Power factor
  - $\cos\phi = \cos 30 = 0.866$ (lag)
- Power
  - $S = V_{rms}I_{rms} = \frac{200}{\sqrt{2}}\frac{8}{\sqrt{2}} = 800\text{VA}$
  - $P = V_{rms}I_{rms}\cos\phi = 692.82\text{W}$
  - $Q = V_{rms}I_{rms}\sin\phi = 400\text{VAR}$

Problem

- A non-inductive resistance of 10Ω is connected in series with an inductive coil across 200V, 50Hz supply. The current drawn by the series combination is 10A. The resistance of the coil is 2 Ω. Determine:
  - Inductance of the coil
  - Power factor
  - Voltage across the coil



Solution

- Total impedance of the circuit
  - $Z = (R + r) + jX_L = 12 + jX_l$
- Impedance is also given as
  - $Z = \frac{V}{I} = \frac{200}{10} = 20\Omega = \sqrt{R^2 + X_L^2}$
- Inductance of the coil is given as
  - $X_L = \sqrt{256} = 16\Omega$
  - $L = \frac{16}{314} = 0.0509\text{H}$
- Power factor
  - $\cos\phi = \frac{R+r}{Z} = \frac{12}{20} = 0.6(\text{lag})$
- Voltage across coil is
  - $V_{coil} = IZ_{coil}$
  - $V_{coil} = 10 * 16.12 = 161.245\text{V}$

Problem

- Given
  - $R = 10\Omega$
  - $L = 1/3\text{H}$
  - $C = 1/6\text{F}$
  - $v = 200 \sin 3t$
- Find,
  - Z
  - I
  - $\cos\phi$
  - $v_R v_L v_C$
  - P,Q,S

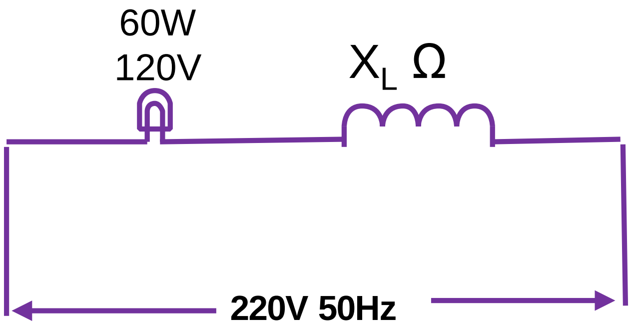
Solution

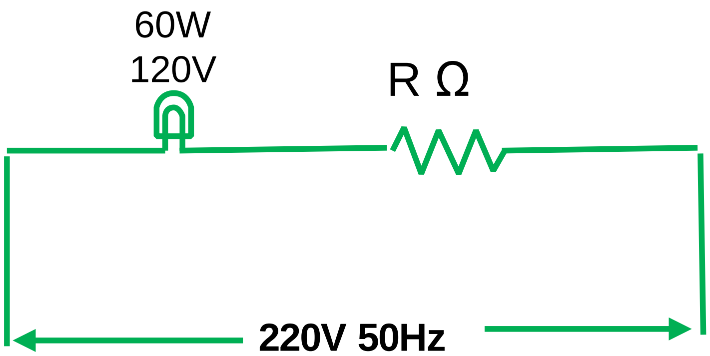
- $v = \frac{200}{\sqrt{2}}\angle 0,\ \omega = 3\text{rad/sec}$
- $X_L = \omega L = 3 * \frac{1}{3} = 1\Omega$
- $X_C = \frac{1}{\omega C} = \frac{6}{3} = 2\Omega$

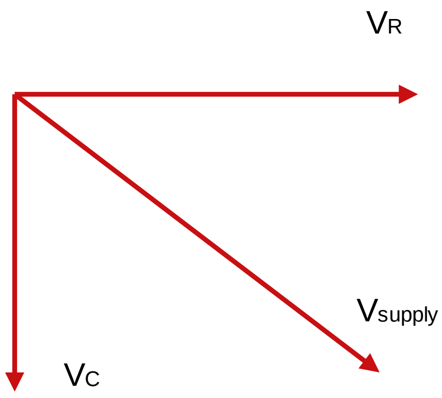
- Impedance
  - $Z = R + j(X_L - X_C) = 10 - j$
  - $Z = 10.04\angle - 5.71$
- Current
  - $I = \frac{V}{Z} = \frac{\frac{200}{\sqrt{2}}\angle 0}{10.04\angle{-5.71}}$
  - $I = 14.08\angle 5.71$
- Power factor
  - $\cos\phi = \cos 5.71 = 0.99(\text{lead})$
- Voltages across elements
  - $V_R = 14.08 * 10 = 140.8\text{V}$
  - $V_L = 14.08 * 1 = 14.08\text{V}$
  - $V_C = 14.08 * 2 = 28.16\text{V}$
- Power
  - $S = V_{rms}I_{rms} = \frac{200}{\sqrt{2}} * 14.08 = 1991.21\text{VA}$
  - $P = V_{rms}I_{rms}\cos\phi = 1981.33\text{W}$
  - $Q = V_{rms}I_{rms}\sin\phi = 1981.1\text{VAR}$

Problem

- A 120V 60W lamp is to be operated on 220V 50 Hz supply mains. In order that lamp should operate on correct voltage rating, calculate the value of
  - Non-inductive resistance
  - Pure inductance





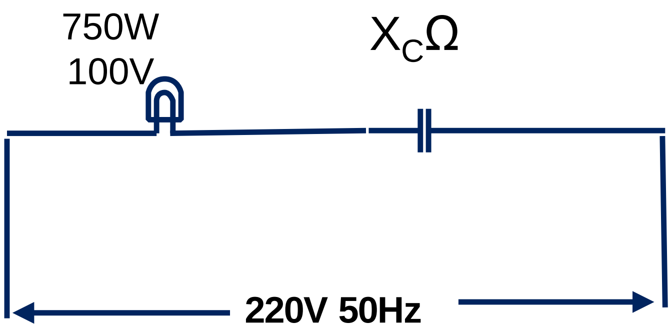


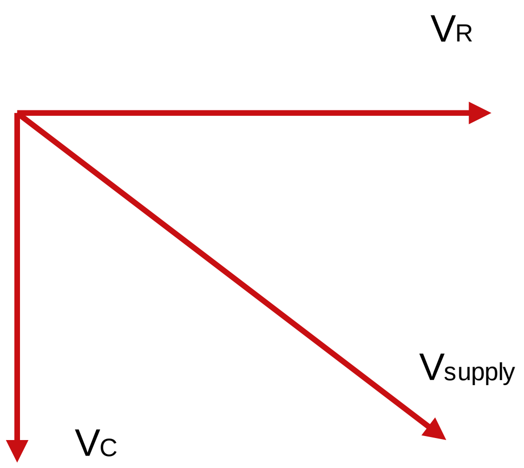
Solution

- Lamp is pure resistive
- Current drawn by lamp at rated values is
  - $I = \frac{P}{V} = \frac{60}{120} = 0.5\text{A}$
- Let a pure resistance of value R is added in series for proper voltage distribution, then
  - $V_{supply} = V_{lamp} + V_R$
  - $V_R = 220 - 120 = 100\text{V}$
  - $R = \frac{V}{I} = \frac{100}{0.5} = 200\Omega$
- If a pure inductor is added for proper voltage distribution then voltage across the coil is
  - $\overrightarrow{V_{supply}} = \overrightarrow{V_{lamp}} + \overrightarrow{V_{coil}}$
  - $\overrightarrow{V_{coil}} = \sqrt{220^2 - 120^2} = 184.39\text{V}$
  - $X_{coil} = \frac{V_{coil}}{I_{coil}} = \frac{184.39}{0.5} = 368.78\text{V}$
  - $L = \frac{368.78}{2\pi * 50} = 1.1738\text{H}$

Problem

- A metal filament lamp rated at 750W 100V, is to be connected in series with a capacitor, across a 230V 50Hz supply. Calculate the value of capacitance required.





Solution

- Lamp is purely resistive
- Current at rated supply is
  - $I = \frac{P}{V} = \frac{750}{100} = 7.5\text{A}$
- Now, the voltage across required capacitor will be
  - $\overrightarrow{V_{supply}} = \overrightarrow{V_{lamp}} + \overrightarrow{V_{capacitor}}$
  - $\overrightarrow{V_{capacitor}} = \sqrt{230^2 - 100^2} = 207.1231\text{V}$
  - $X_{capacitor} = \frac{V_{capacitor}}{I_{capacitor}} = \frac{207.1231}{7.5} = 27.6164\text{V}$
  - $C = \frac{1}{2\pi * 50 * 27.6164} = 115.261\mu F$

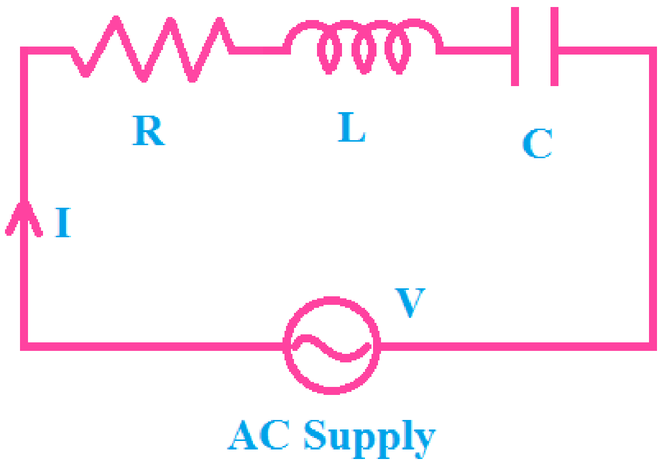
Lecture 14

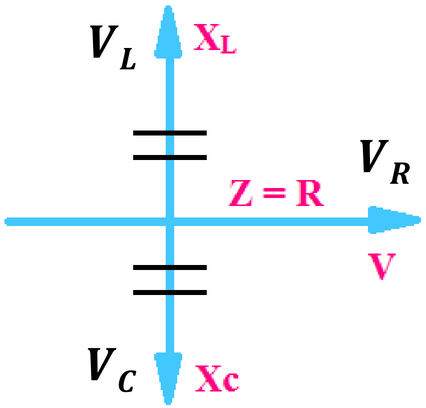
Resonance

- *Definition*
  - When the natural frequency of any system gets matched with the frequency of driving force, the system responds with maximum amplitude, this condition is known as **resonance**.
  - When the frequency of any electrical network or circuit gets matched with the supply frequency, the electrical circuit starts resonating with maximum amplitude at that particular frequency. This condition is known as **resonance.**

Resonance: Series RLC Circuit

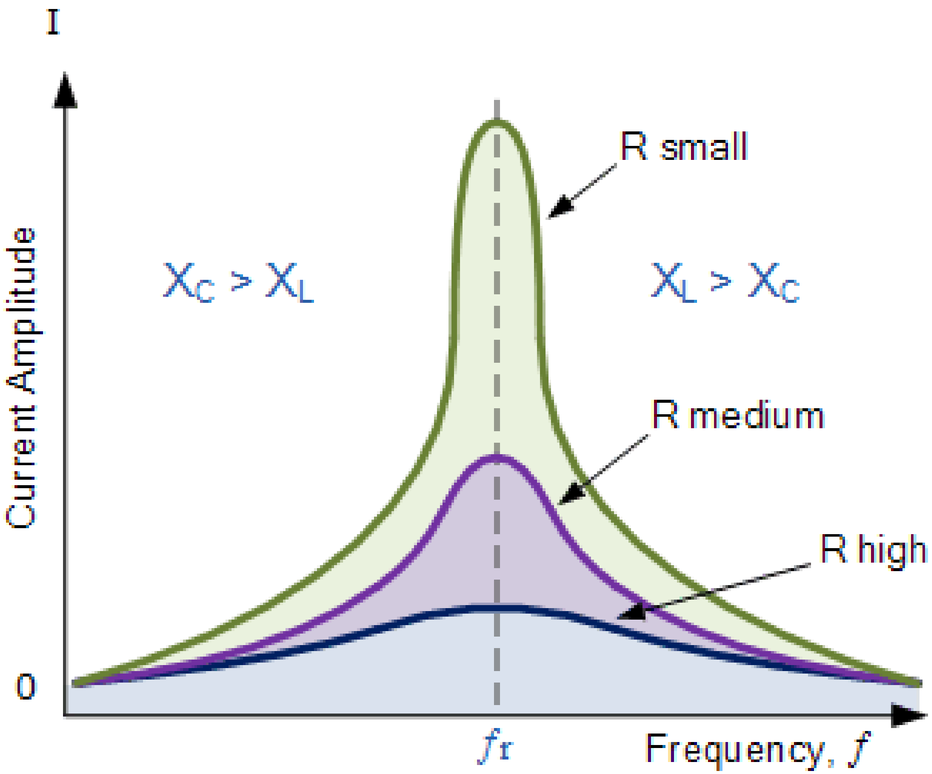
- At $X_L = X_C$
- Circuit behaves as pure R circuit
- Current maximum
- Impedance minimum
- $\emptyset = 0$
- $\cos\emptyset = 1$





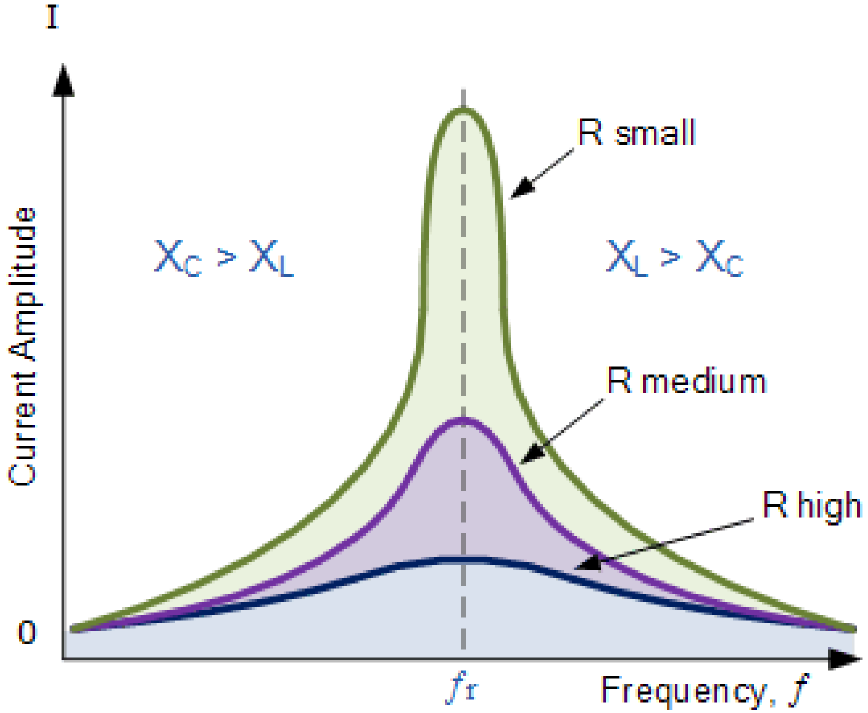
Resonance: Resonant frequency

- At
  - $f = f_r$
  - $X_L = X_C$
  - $2\pi f_r L = \frac{1}{2\pi f_r C}$
  - $f_r = \frac{1}{2\pi\sqrt{(LC)}}$



Resonance: Resonance Curve

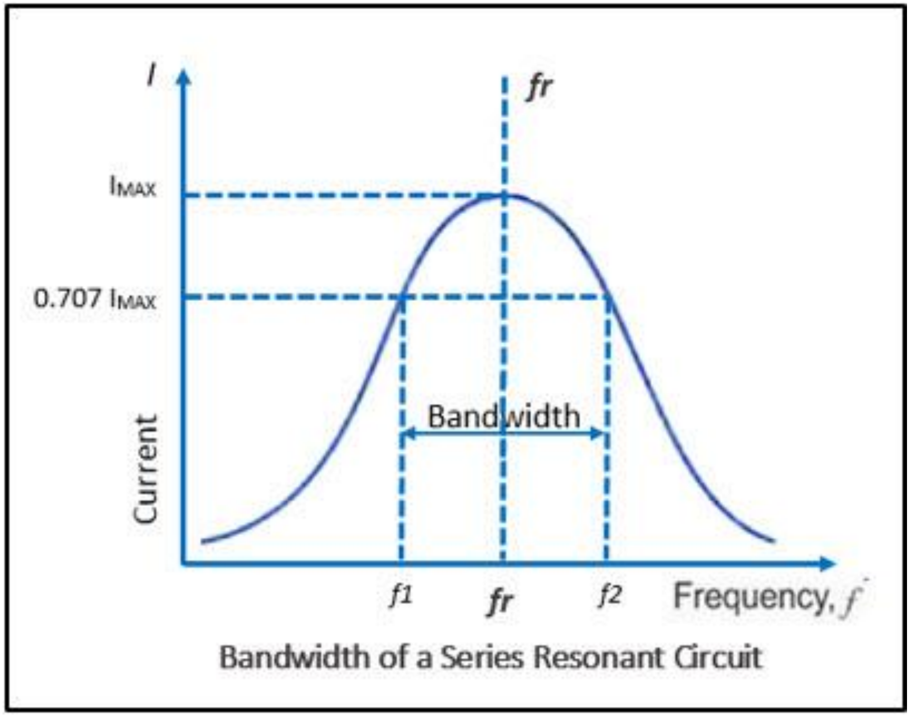
- Trace of current as a function of frequency
- At
  - $f = f_r, pure\ R\ circuit$
  - $f > f_r, RL\ circuit$
  - $f < f_r, RC\ circuit$



Lecture 15

Resonance: Bandwidth

- Frequency range between half power frequencies ($f_1 f_2$) is defined as bandwidth of resonance curve and is given as
  - $Bandwidth = (f_2 - f_1)\,Hz$
- At f_{r},
  - Current maximum-I
  - Power transfer maximum-*P*
- At $f_1 f_2$,
  - Power half-P/2
  - Current-I/√2



Prove $f_0 = \sqrt{f_1 f_2}$

- At half power frequencies
- $I = \frac{I_m}{\sqrt{2}} = \frac{V}{\sqrt{2}R}$
- Generalized equation for current is given as,
- $I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$
- Equating both
- $\frac{V}{\sqrt{2}R} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$

$f_0 = \sqrt{f_1 f_2}$: cont…

- $2R^2 = R^2 + (\omega L - \frac{1}{\omega C})^2$
- $(\omega L - \frac{1}{\omega C})^2 = R^2$
- $\omega L - \frac{1}{\omega C} = \pm R$
- $\omega_1 L - \frac{1}{\omega_1 C} = -R$
- $\omega_2 L - \frac{1}{\omega_2 C} = +R$
- $(\omega_1 + \omega_2)L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right)\frac{1}{C} = 0$
- $(\omega_1 + \omega_2)L = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right)\frac{1}{C}$
- $\omega_1 \omega_2 = \frac{1}{LC} = \omega_r^2$
- $\omega_r = \sqrt{\omega_1 \omega_2}$
- $f_r = \sqrt{f_1 f_2}$

Bandwidth Derivation

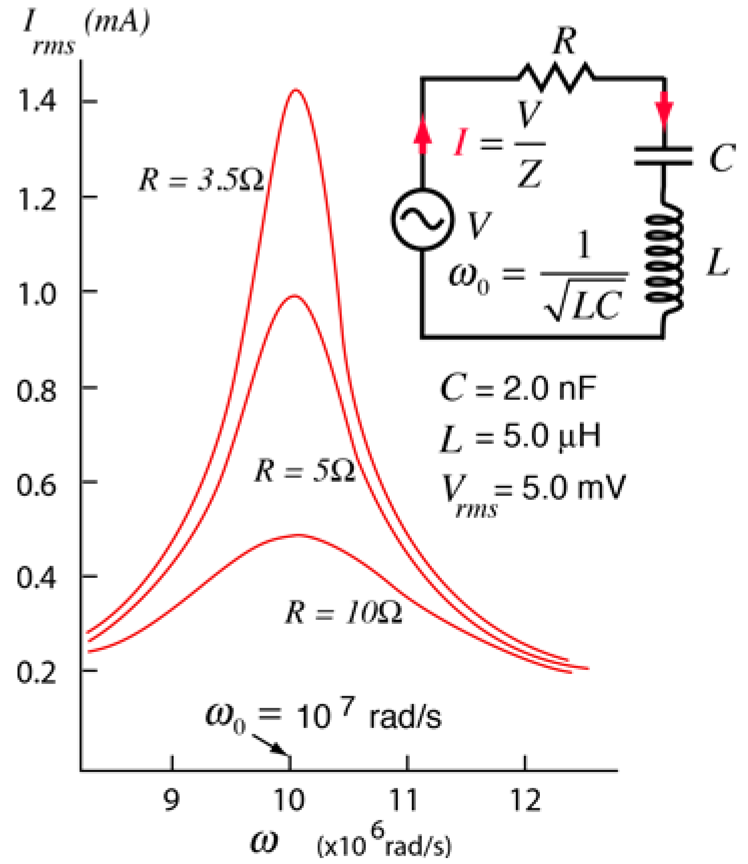
- Given
  - $\omega L - \frac{1}{\omega C} = \pm R$, from previous slide
- Case 1
  - $\omega L - \frac{1}{\omega C} = R$
  - $\frac{\omega^2 LC - 1}{\omega C} = R$
  - $\omega^2 LC - \omega CR - 1 = 0$
- By Shreedharacharya formula
  - $\omega = \frac{+CR \pm \sqrt{C^2R^2 + 4LC}}{2LC} = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = \frac{R}{2L} \pm \omega_r$
- Case 2
  - $\omega = -\frac{R}{2L} \pm \omega_r$

Bandwidth Derivation cont…

- ω can’t be negative, hence roots are,
  - $\omega_1 = \omega_r - \frac{R}{2L}$
  - $\omega_2 = \omega_r + \frac{R}{2L}$
- And,
  - $f_1 = f_r - \frac{R}{4\pi L}$
  - $f_2 = f_r + \frac{R}{4\pi L}$
- Also,
  - $\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$
  - $\Delta f = f_2 - f_1 = \frac{R}{2\pi L}$

Resonance: Quality Factor

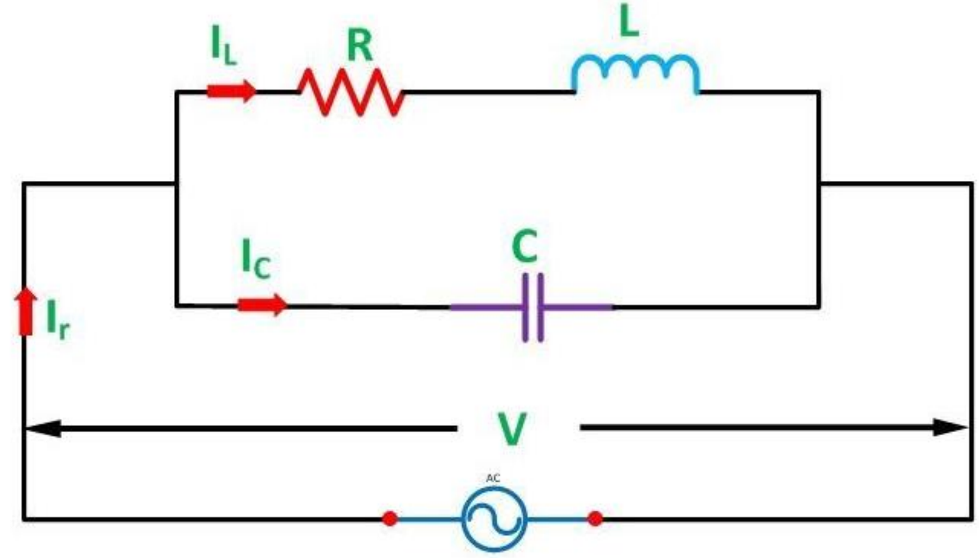
- Shows the quality of resonance curve
- Defined as the magnification ratio of voltage across L or C to the supply voltage
- Quality factor,
- $Q = \dfrac{V_L \ or \ V_C}{V_{supply}} = \dfrac{i(X_L \ or \ X_C)}{iR} = \dfrac{X_L \ or \ X_C}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$



Lecture 16 & 17

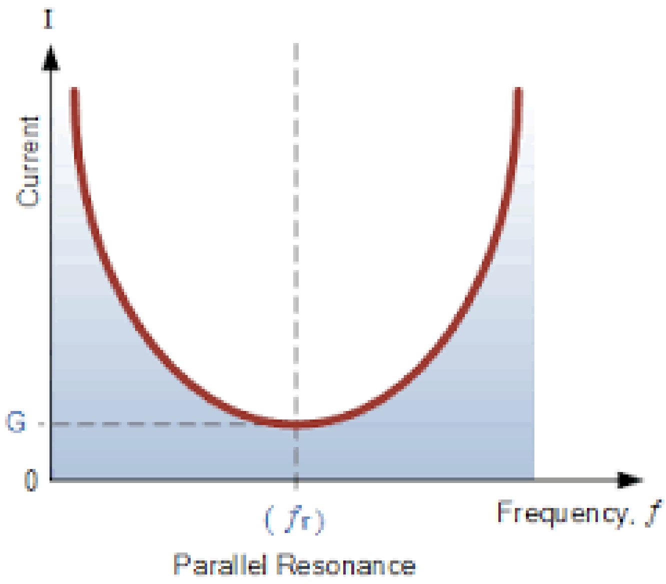
Resonance: Parallel RLC Circuit

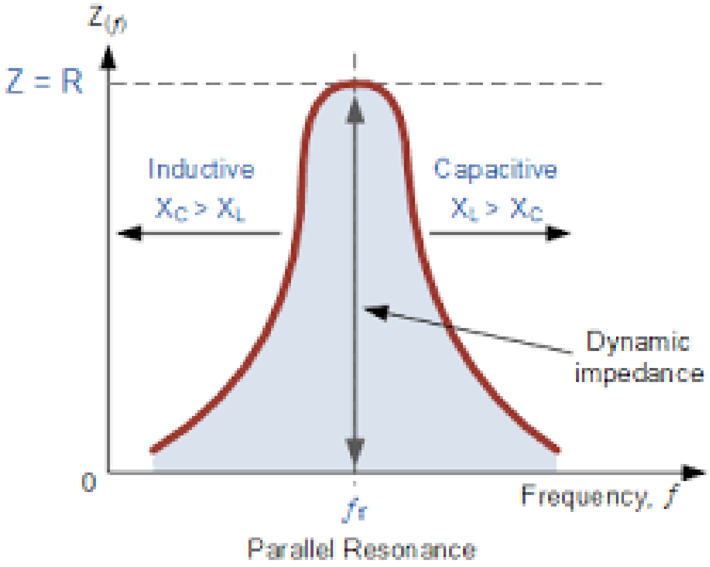
- $X_L = X_C \; or Y_L = Y_C$
- RLC Circuit behaves as R circuit
- Current minimum
- Impedance maximum



Parallel Resonance: cont…

- Impedance maximum
- Current minimum

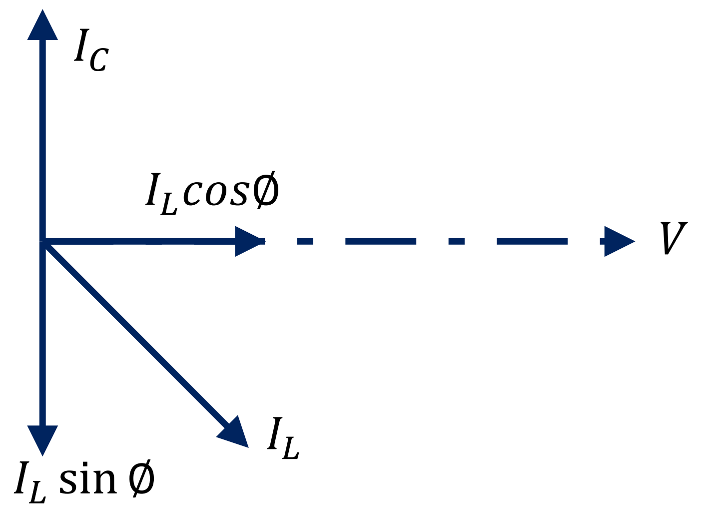




Parallel Resonance: Resonant Frequency

- $I_C = I_L \sin \emptyset$
- $\frac{V}{X_C} = \frac{V}{Z_L} * \frac{X_L}{Z_L}$

- Where, Z_L is the impedance of the coil and is given by,
  - $Z_L = \sqrt{R^2 + X_L^2}$
  - $X_L X_C = {Z_L}^2$
  - $\frac{\omega L}{\omega C} = R^2 + \omega^2 L^2$
  - $\omega^2 L^2 = \frac{L}{C} - R^2$
  - $\omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$
  - $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$



Parallel Resonance: Resonant Frequency cont…

- $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
- If resistance is neglected, then
- $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$
- Thus, if resistance is neglected, then the resonant frequency of parallel circuit is equal to the resonant frequency of series circuit. Also, at resonance the net susceptance is zero.
- Hence net reactive component of current is zero and the supply current is given by only active component as
- $I = I_L \cos \emptyset = \frac{V}{Z_L}\frac{R}{Z_L} = \frac{VR}{Z_L^2} = \frac{VR}{L/C} = \frac{V}{L/CR}$

Parallel Resonance: Resonant Frequency cont…

- Hence, at parallel resonance the net impedance is given by
  - $Z_D = L/CR$
- And is known as dynamic impedance of the parallel circuit at resonance.
- The nature of this impedance is resistive only.

Parallel Resonance: Quality Factor

- Shows the quality of resonance curve
- Defined as the magnification ratio of current flowing through L or C to the supply current
- Quality factor,
- $Q = \frac{I_L}{I_{supply}} = \frac{V/Z_L}{V/Z_D} = \frac{Z_D}{Z_L} = \frac{L/RC}{\sqrt{L/C}} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Problem

- A series RLC circuit has R=10Ω, L=0.1H, C=8μF. Determine,
  - Resonant frequency
  - Q-factor of the circuit
  - The half power frequency

Solution

- Resonant frequency
  - $f_r = \frac{1}{2\pi\sqrt{(LC)}} = \frac{1}{2\pi\sqrt{0.1*8*10^{-6}}} = 177.94\text{Hz}$
- Q-factor
  - $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{0.1}{8*10^{-6}}} = 11.18$
- Half power frequency
  - $f_1 = f_r - \frac{R}{4\pi L} = 177.94 - 7.95 = 169.99\text{Hz}$
  - $f_2 = f_r + \frac{R}{4\pi L} = 177.94 + 7.95 = 185.89\text{Hz}$

Lecture 18

Power Factor

- *Definition*
  - The factor of real power to the total or apparent power is called power factor.
    - $pf = cos\phi = \frac{RealPower}{ApparentPower} = \frac{R}{Z} = \frac{V_R}{V_{supply}} = \frac{RealFactor}{TotalFactor}$
  - The fraction of utilized power by load to the total power sent by source is called power factor.

Disadvantages of Low Power Factor

- Large generators and transformers are required to deliver the same load at low power factor.
- More conductor material is required in transmission lines due to large current.
- Copper losses are more at low pf.
- Low lagging pf leads to large voltage drop in transformers, generators and transmission lines, which results poor regulation.

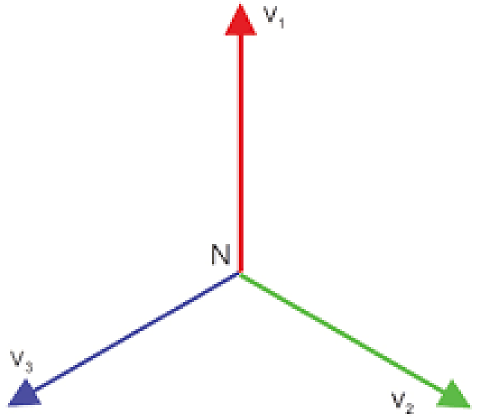
Causes of Low Power Factor

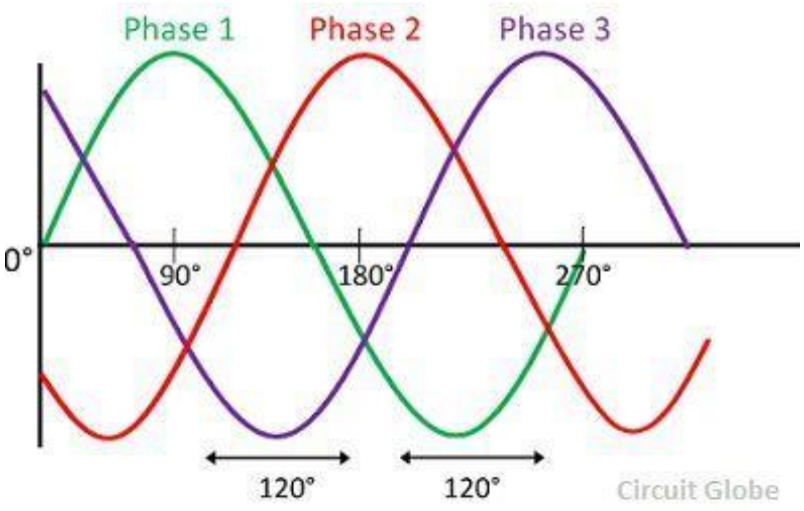
- All ac motors and transformers operate at low pf. Pf decreases with decrease in load.
- Industrial heating furnaces like arc and induction furnaces operate at low lagging pf.

Lecture 19 & 20

Three Phase System

- Balanced
- Unbalanced



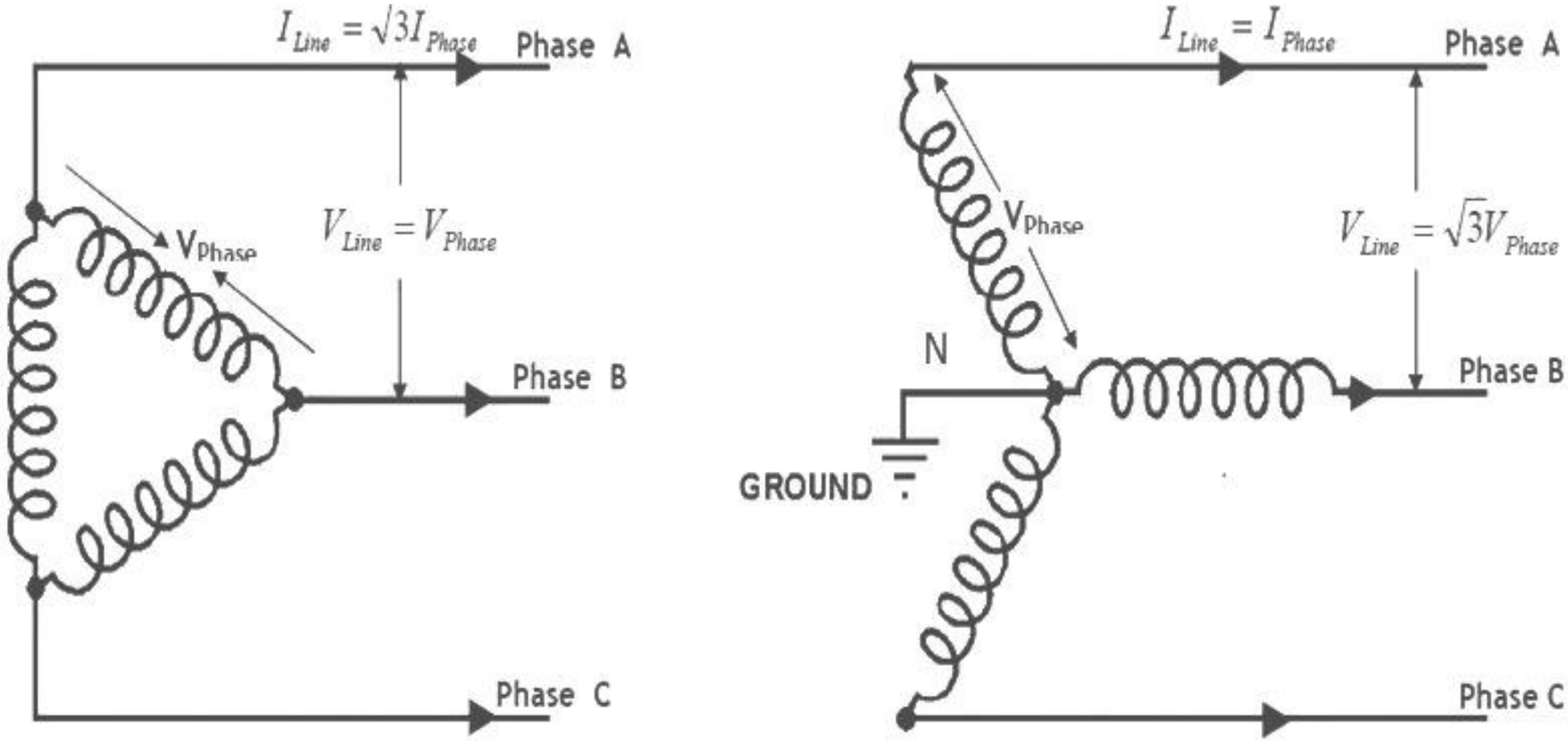


Three Phase vs Single Phase

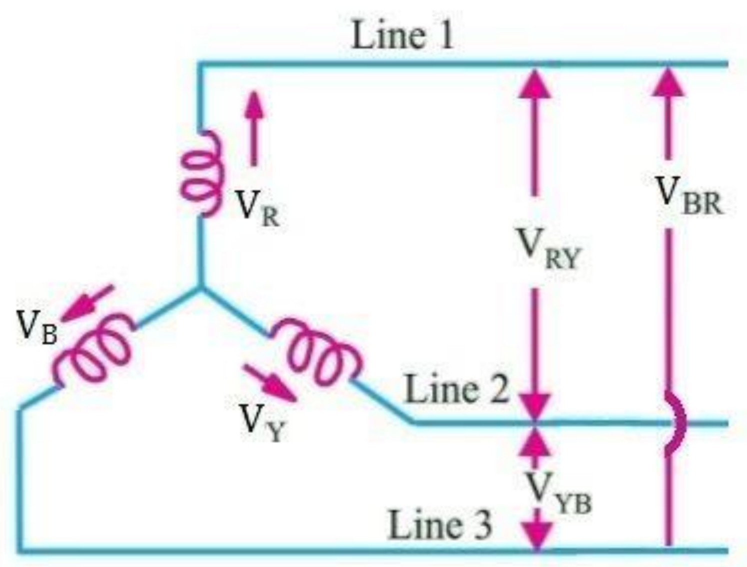
- The rating of a machine increases with increase in number of phases.
- Power factor of a single-phase motor is lower than that of a three-phase motor of same rating.
- Three phase motor has more output (>1.5 times) than single phase motor.
- Three phase system is more reliable and capable than single phase system
- Three phase system have higher efficiency compared to single phase system.

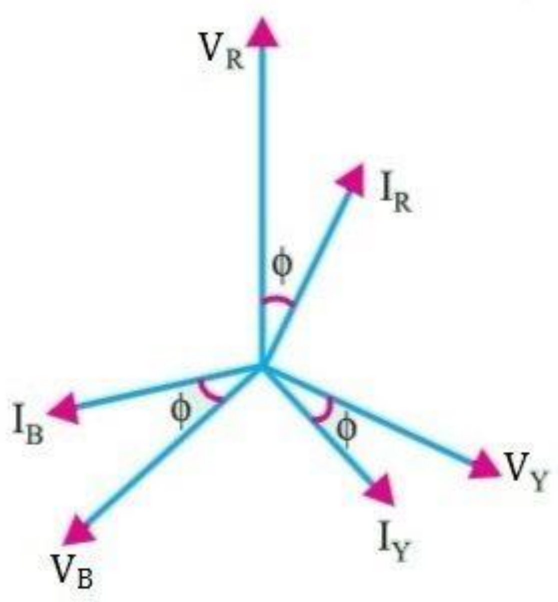
Important Terminologies

- *Line Voltage*
  - Voltage difference between any two phases.
- *Phase Voltage*
  - Voltage across any one phase of load.
- *Line Current*
  - Current flowing through line between source and load
- *Phase Current*
  - Current flowing through any one phase of load.


$I_{Line} = \sqrt{3} I_{Phase}$
Phase A
$V_{Line} = V_{Phase}$
V_{Phase}
Phase B
Phase C
$I_{Line} = I_{Phase}$
Phase A
V_{Phase}
$V_{Line} = \sqrt{3} V_{Phase}$
N
Phase B
GROUND
Phase C

Three Phase System: Star





Since $I_R = I_Y = I_B = I_{ph}$

Therefore $I_L = I_{ph}$

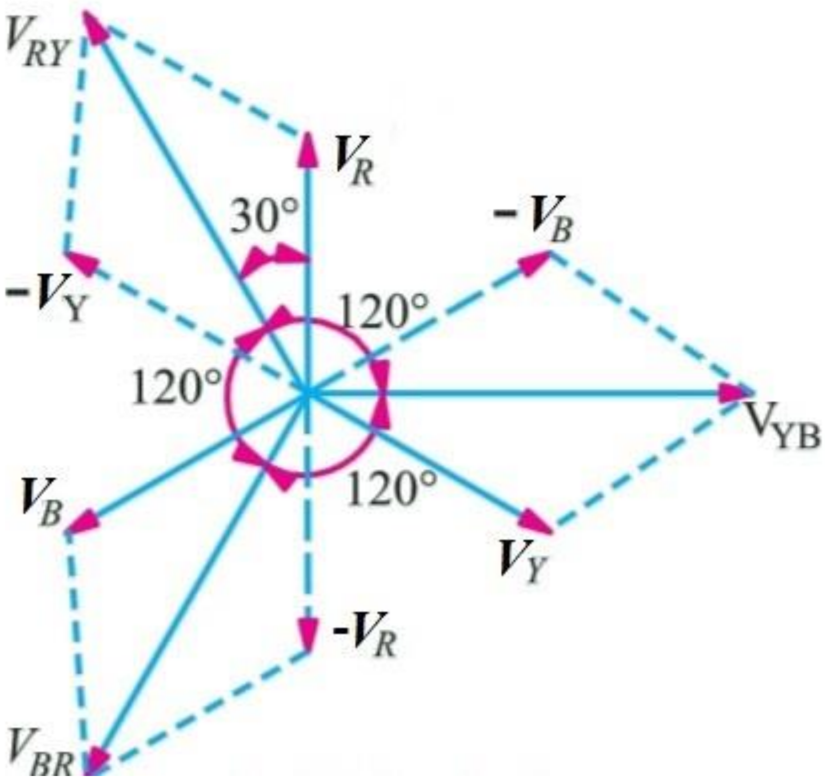
Line current = Phase current

$$V_{RY} = V_R - V_Y$$

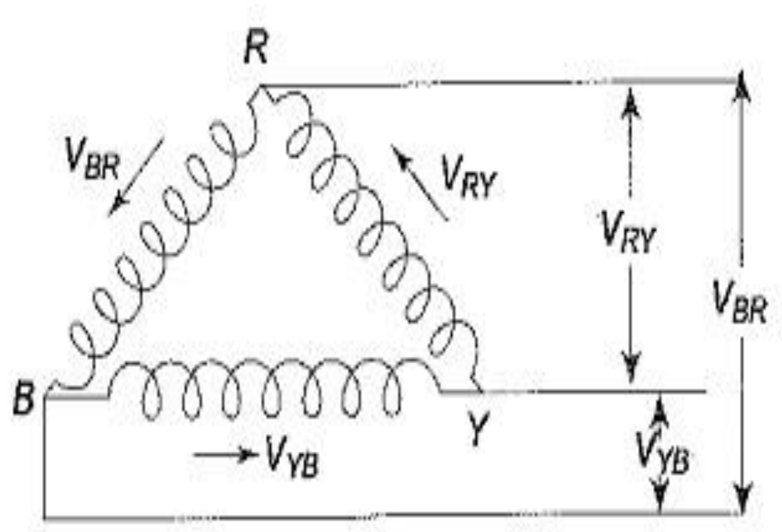
$$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

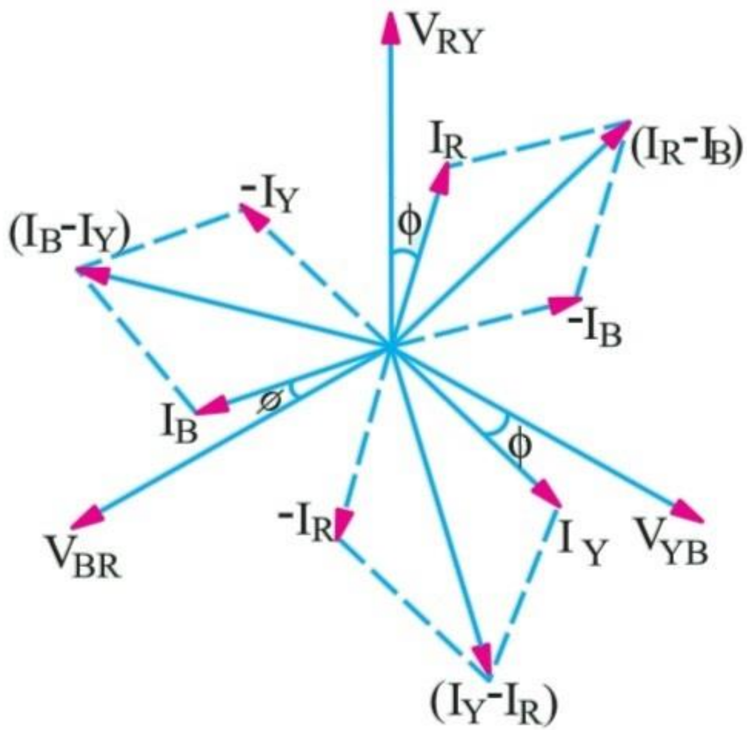
Three Phase System: Star cont…

- $|V_{RY}| = \sqrt{|V_R|^2 + |V_Y|^2 + 2|V_R||V_Y|\cos 60}$
- Angle between V_R and V_Y is 60 degrees
- $|V_{RY}| = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2 \cos 60}$
- $|V_{RY}| = \sqrt{3V_{ph}^2}$
- $V_L = \sqrt{3}V_{ph}$
- Line voltage = $\sqrt{3}$ phase voltage



Three Phase System: Delta

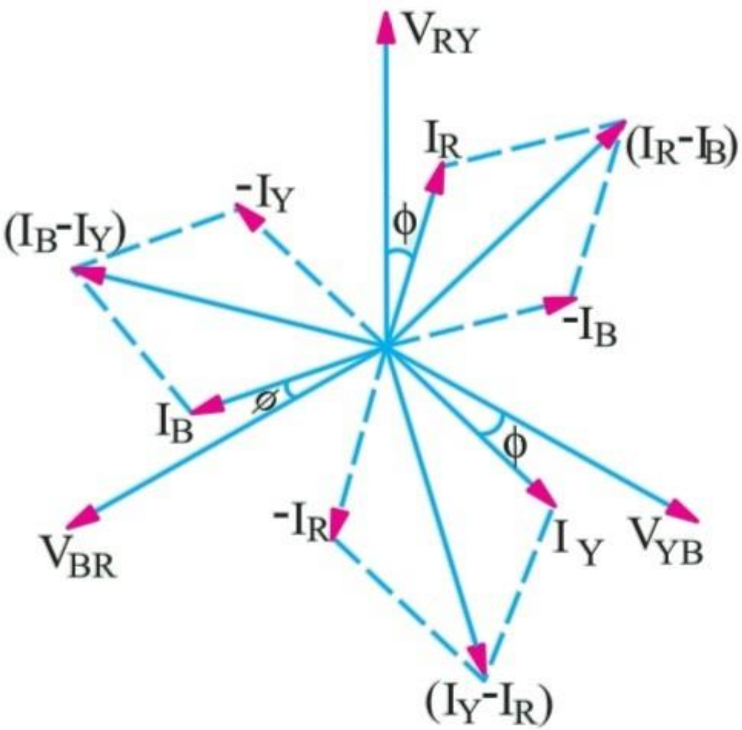




- $V_{RY} = V_{YB} = V_{BR} = V_L = V_{ph}$

Three Phase System: Delta cont…

- $|I_L| = \sqrt{|I_{RY}|^2 + |I_{YB}|^2 + 2|I_R||Y_Y|\cos 60}$
- Angle between I_R and I_Y is 60 degrees
- $|I_{RY}| = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2 \cos 60}$
- $|I_{RY}| = \sqrt{3I_{ph}^2}$
- $I_L = \sqrt{3}I_{ph}$
- Line current = $\sqrt{3}$ phase current



Three Phase System: Power (star and delta)

- $P_{total} = \sqrt{3}V_L I_L \cos\emptyset$ W
- $Q_{total} = \sqrt{3}V_L I_L \sin\emptyset$ VAR
- $S_{total} = \sqrt{3}V_L I_L$ VA

Problem

- A three-phase voltage source has a phase voltage of 120V and supplies a star connected load having impedance of 36+j48Ω per phase. Calculate
  - Line voltage
  - Line current
  - Power factor
  - Total three phase power supplied to load.

Solution

- Line voltage
  - $V_L = \sqrt{3}V_{ph} = \sqrt{3} * 120$
  - $V_L = 207.8\text{V}$
- Line current
  - $I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{120}{60} = 2\text{A}$
- Power factor
  - $\cos \emptyset = \frac{36}{60} = 0.6$
- Three phase power
  - $P = \sqrt{3}V_L I_L \cos 30 = 432\text{W}$

Problem

- A star connected load has a three phase resistance of 8Ω and an inductive reactance of 6Ω in each phase. It is fed from a 400V balanced three phase supply.
- Determine
  - Line current
  - Power factor
  - Active and reactive power

Solution

- $V_L = \sqrt{3}V_{ph}$
- $V_{ph} = \frac{400}{\sqrt{3}} = 230.94\text{V}$
- $Z_{ph} = \sqrt{6^2 + 8^2} = 10\Omega$
- Line current
  - $I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.94}{10} = 23.09\text{A}$
- Power factor
  - $\cos \emptyset = \frac{8}{10} = 0.8$
- Three phase power
  - $P = \sqrt{3} * 400 * 23.09 * \cos 36.86 = 12.8\text{KW}$
  - Q= $\sqrt{3} * 400 * 23.09 * \sin 36.86 = 9.6\text{KVAR}$