BASIC ELECTRICAL ENGINEERING-BEE101/201

MIET GROUP OF INSTITUTIONS

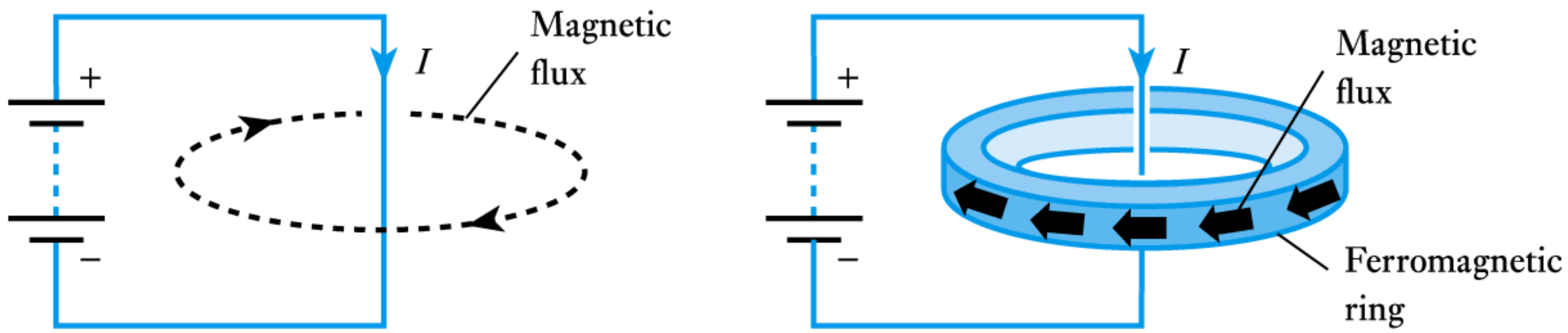
UNIT-III

TRANSFORMERS

Lecture 21

ELECTROMAGNETISM

- Adding a ferromagnetic ring around a wire will increase the flux by several orders of magnitude
  - Since, μ_r for ferromagnetic materials is **1000** or more



(a) The magnetic flux about a current-carrying wire in air

(b) The effect of adding a ferromagnetic ring

ELECTROMAGNETISM

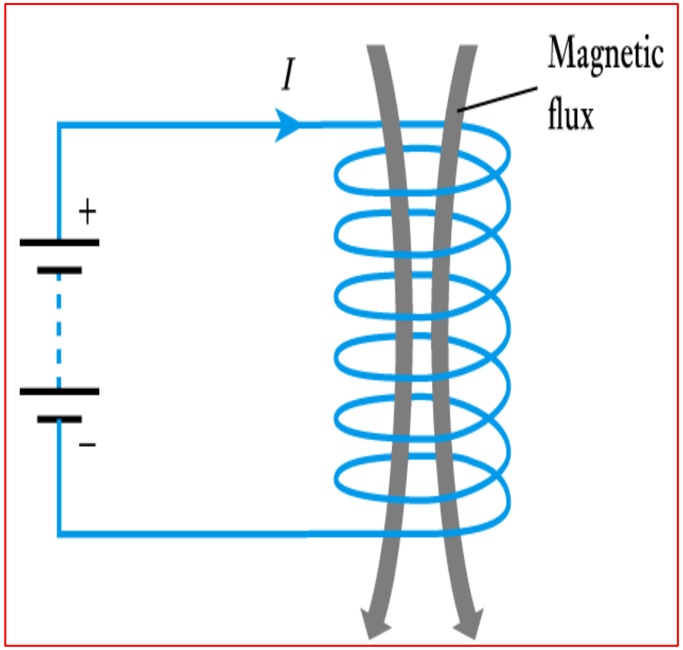
- When a **current-carrying** wire is formed into a **coil** the magnetic field is concentrated

- For a coil of N turns the M.M.F. (F) is given by

$$F = I N$$

- The magnetic field strength is given by,

$$H = \frac{IN}{l}$$



RELUCTANCE

- In a ***resistive*** circuit, the ***resistance, R*** is a measure of how the circuit opposes the flow of electricity
- In a ***magnetic*** circuit, the **reluctance, *S*** is a measure of how the circuit opposes the creation of magnetic flux.
- In a resistive circuit $R = V/I$
- In a magnetic circuit the units of reluctance are amperes turn per weber (AT/ Wb)

$$S = \frac{NI}{\phi} \text{ AT/ Wb}$$

INDUCTANCE

- When a circuit forms a single loop, the ***E.M.F.*** induced is given by the rate of change of the flux
- When a circuit contains many loops the resulting ***E.M.F.*** is the sum of those produced by each loop
- Therefore, if a coil contains ***'N'*** loops, the induced voltage ***'V'*** is given by:

$$\mathbf{V = N\frac{d\Phi}{dt}}$$

$$\text{where, } \frac{d\phi}{dt} \text{ is the rate of change of flux in Wb/s}$$

- This property, whereby an ***E.M.F.*** is induced as a result of changes in magnetic flux, is known as **inductance**.

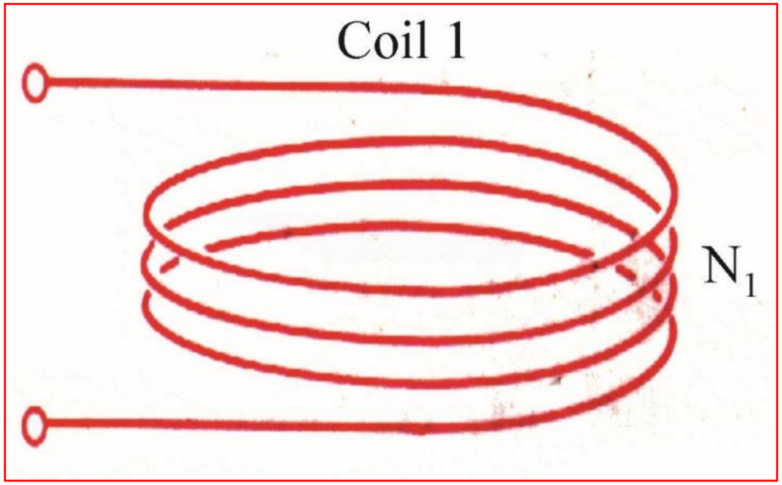
SELF-INDUCTANCE

- A changing current in a wire causes a changing magnetic field about it.
- A changing magnetic field induces an ***E.M.F.*** in conductors within that field.
- Therefore when the current in a coil changes, it induces an ***E.M.F.*** in the coil.
- This process is known as ***self-inductance***

$$V = L\frac{dI}{dt}$$

where *L*, is the inductance of the coil (Henry)

SELF-INDUCTANCE



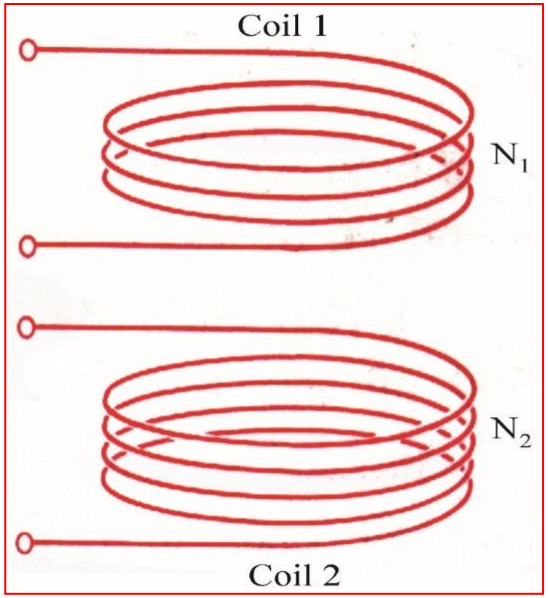
What is the effect of putting current into coil 1?
There will be “self flux”:

$$\Phi_B = LI$$

Faraday’s Law $\rightarrow$

$$\varepsilon = -L\frac{dI}{dt}$$

MUTUAL INDUCTANCE



Current I_2 in coil 2, induces magnetic flux F_{12} in coil 1.
"Mutual inductance" M_{12}:

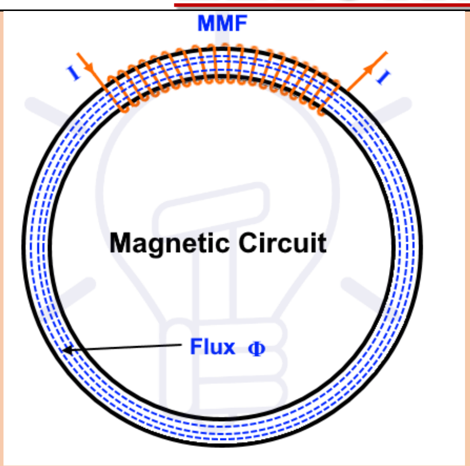
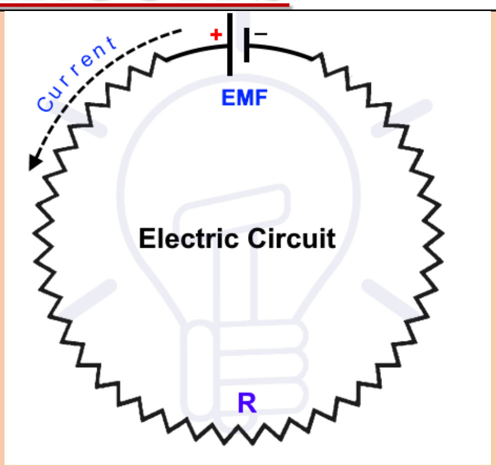
$$\Phi_{12} = M_{12} I_2$$

$$M_{12} = M_{21} = M$$

Change current in coil 2?
Induce EMF in coil 1:

$$\varepsilon_{12} = -M_{12} \frac{dI_2}{dt}$$

COMPARISON OF MAGNETIC AND ELECTRIC CIRCUITS

	
$Flux = \frac{M.M.F}{Reluctance}$	$Current = \frac{E.M.F}{Resistance}$
M.M.F(ampere-turns)	E.M.F.(volts)
Flux Φ (webers)	Current ***I*** (amperes)
Flux Density ***B*** (Wb/m^2)	Current Density (A/m^2)
Reluctance $S = \frac{1}{\mu A}\left[\frac{1}{\mu_0 \mu_r A}\right]$	Resistance $R = \rho \frac{l}{A}$
Permeance = $S = \frac{1}{Reluctance}$	Conductance= $\frac{1}{Resistance}$
Reluctivity	Resistivity
Permeability= $\frac{1}{Reluctivity}$	Conductivity= $\frac{1}{Resistivity}$
Total M.M.F=$\Phi S_1 + \Phi S_2 + \Phi S_3 + \ldots\ldots$	Total E.M.F = $IR_1 + IR_2 + IR_3 + \ldots\ldots$

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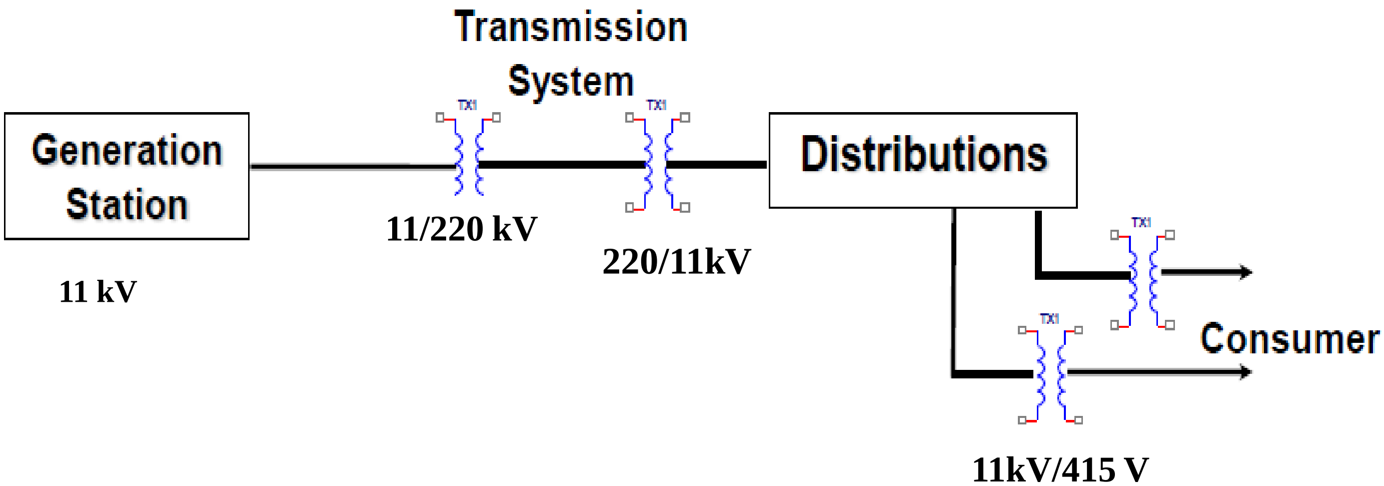
Lecture 22

Why we need transformer??



- The power transmission system using transformers has been shown above in figure.
- Transformers help improve safety and efficiency of power systems by raising and lowering voltage levels as and when needed.
- They are used in a wide range of residential and industrial applications, primarily and perhaps most importantly in the distribution and regulation of power across long distances.

TRANSFORMER

- The transformer is a **static device** which is used to **transfer electrical energy** from one ac circuit to another ac circuit.

- Input to a transformer and output from a transformer both are alternating quantities (AC).

- Electrical energy is generated and transmitted at an extremely high voltages.

- The voltage is to be then reduced to a lower value for its domestic and industrial use.

- This is done by using a transformer.

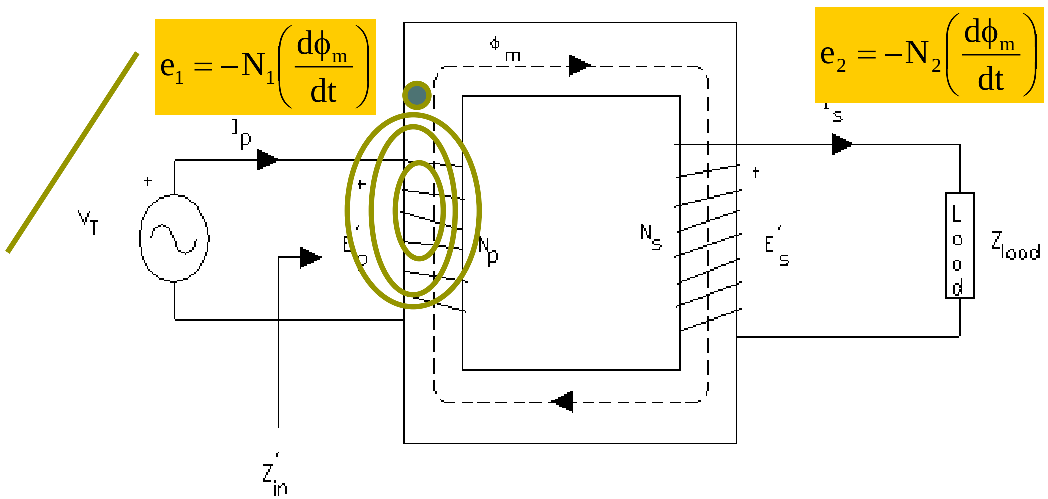
TRANSFORMER

Working principle of transformer:

- A Transformer is a static electrical device that transfers electrical energy between two or more circuits through **mutual induction** (electromagnetic induction).
- A varying current in one coil of the transformer produces a varying magnetic field, which in turn induces a varying **Electromotive Force (E.M.F)** or "voltage" in a second coil.
- Power can be transferred between the two coils through the magnetic field, without a metallic connection between the two circuits.
- Faraday's law of induction discovered in 1831 described this effect.
- Since the invention of the first constant-potential transformer in 1885, transformers have become essential for the transmission, distribution, and utilization of alternating current electrical energy.
- A wide range of transformer design is encountered in electronic and electric power applications.

TRANSFORMER ACTION

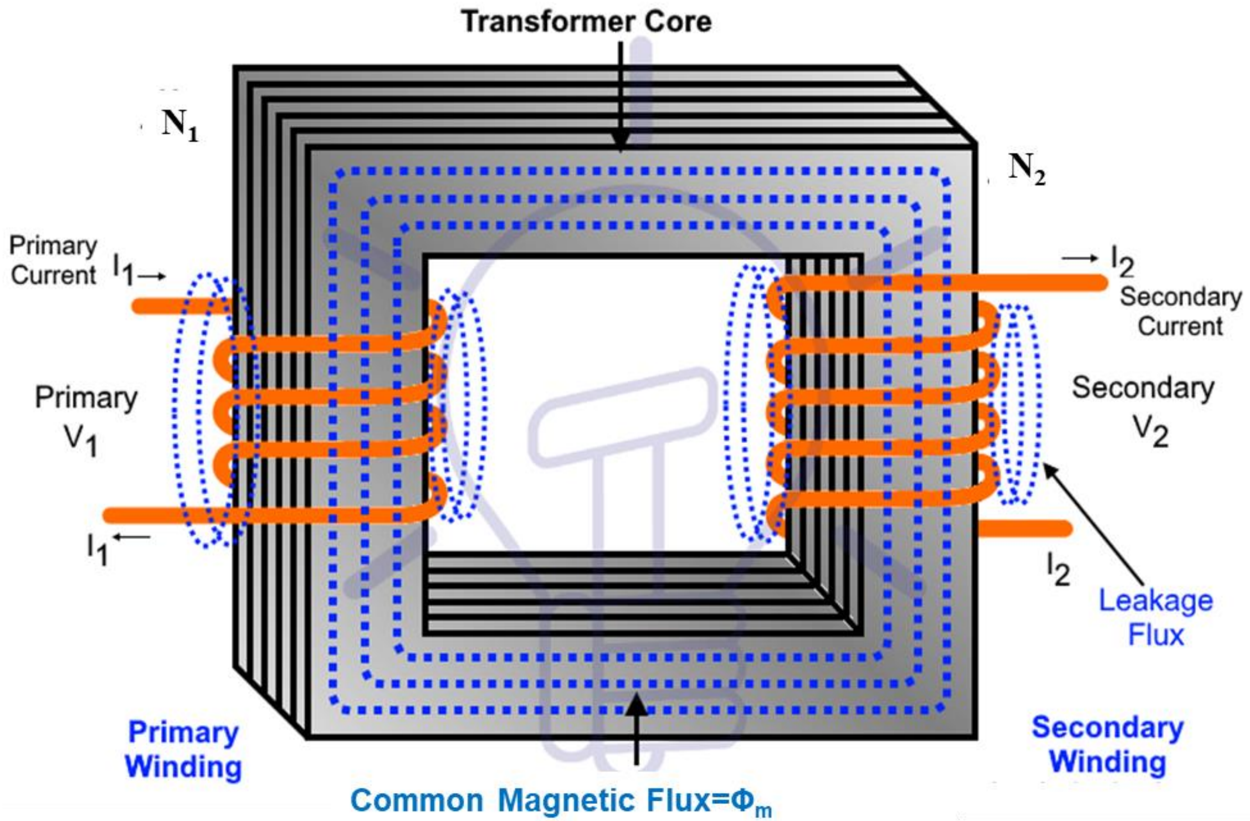
Induced voltage has opposite polarity from source



Principle: Mutual Induction

- ❖ Stationary coils, time varying flux due to ac current flow.
- ❖ Flux produced by one coil must link to other coil to induce voltage.

BASIC PRINCIPLE



- The primary winding is connected to the single – phase ac supply, an ac current starts flowing through it.
- The ac primary current produces an alternating flux (Φ) in the core.

BASIC PRINCIPLE

- **Most of this changing flux gets linked with the secondary winding through the core.**
- **The varying flux will induce voltage into the secondary winding according to the mutual induction.**
- **Voltage level change but frequency i.e. time period remains same.**
- **There is no electrical contact between the two winding, an electrical energy gets transferred from primary to the secondary.**
- **A simple transformer consists of two electrical conductors called the primary winding and the secondary winding.**
- **Energy is coupled between the windings by the time varying magnetic flux that passes through (links) both primary and secondary windings.**

BASIC PRINCIPLE

- As the primary and secondary windings link this flux, EMF is induced in these windings (Faraday's Law) and this opposes the supply voltage (Lenz's Law) and hence called '***Back EMF***'.
- **This Back EMF reduces the input current as per the relation:**

$$I_1 = \frac{V_1 - E_1}{z_1}$$

Where,

I_1 - Primary Current

V_1 - Supply Voltage

E_1 - Back EMF

Z_1 - Primary Impedance.

- Current in the secondary will flow if the secondary winding is closed.

Can the transformer operate on DC?

- Answer: ***NO***
- The transformer action does not take place with a direct current of constant magnitude.
- Because with a DC primary current, the flux produced in the core is **not alternating** but it is of constant value.
- As there is no change in the flux linkage with the secondary winding, **the induced EMF in the secondary is zero.**
- If DC is applied to the primary then there is a possibility of **transformer core saturation**.
- If core saturates the primary will draw **excessively large current**. Therefore application of DC should be avoided.
- Primary winding may **burn out due to the excessive current**.

REASON

$Z=R + jX_L$ (Ω) *f=0 in case of DC*

So, $X_L=0$ for DC

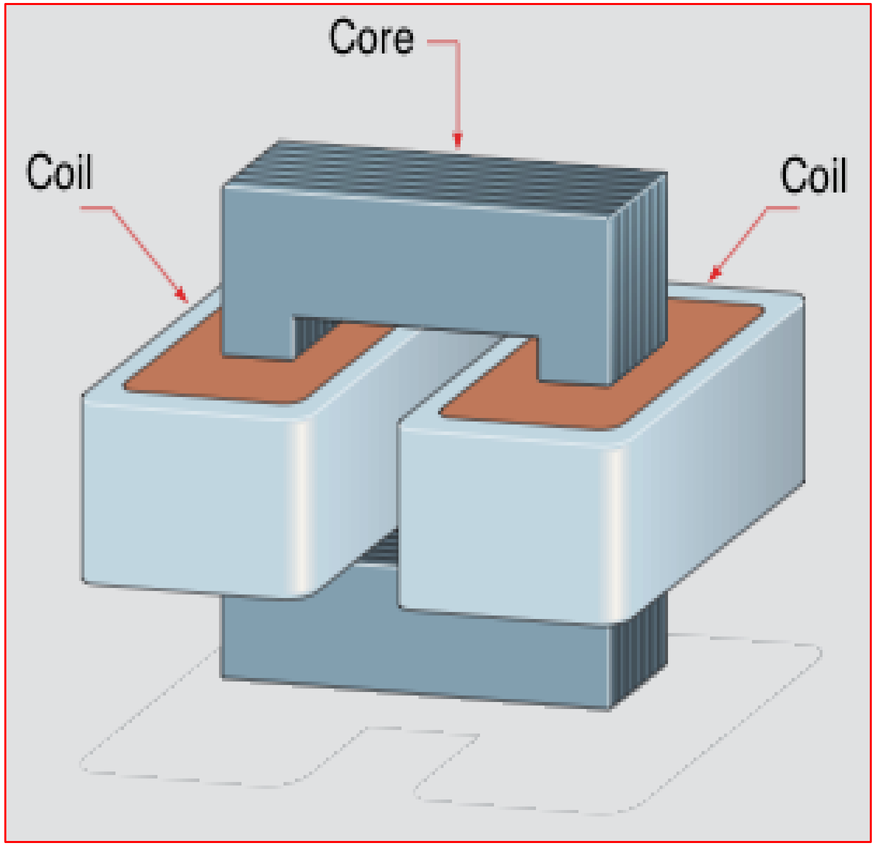
Z=R Minimum

$I=\frac{V}{R}$ {if R minimum, then current will be maximum}

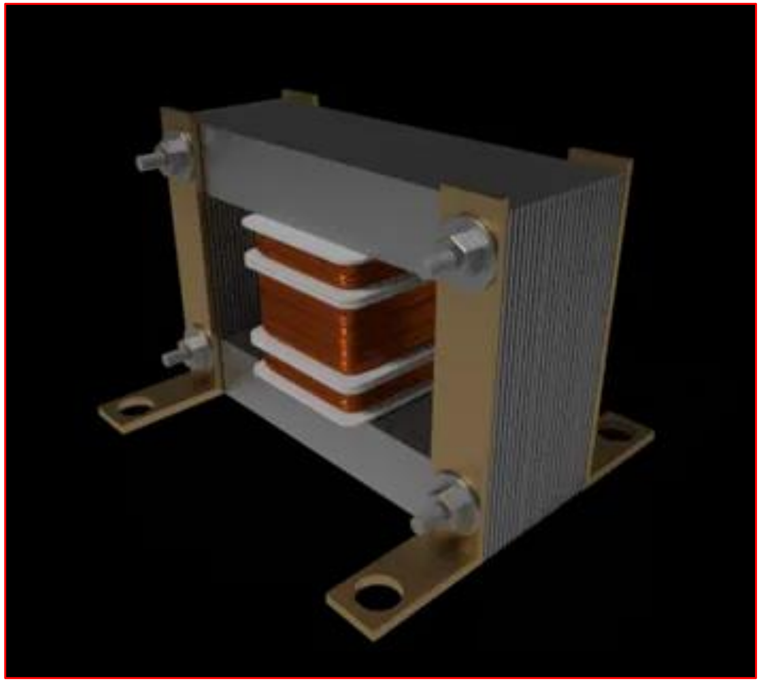
TYPES OF TRANSFORMER
(On the Basis of Design)

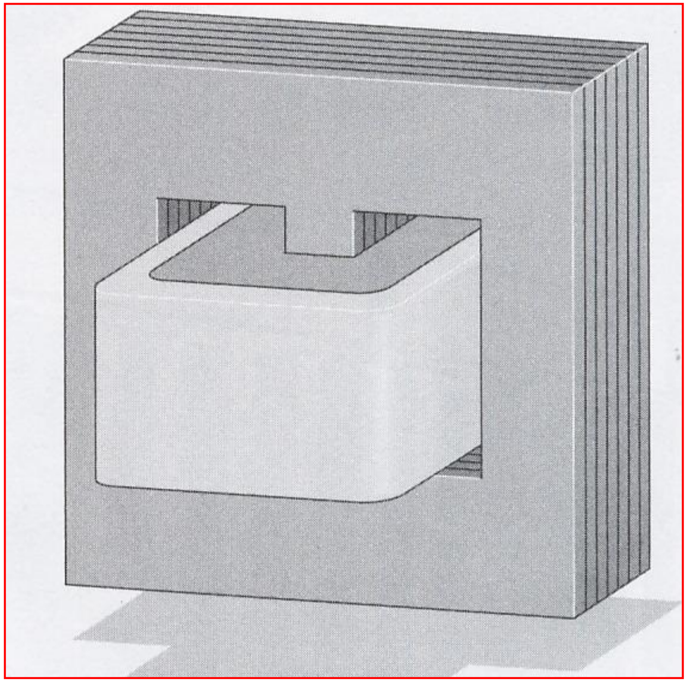
- The transformer are of different types depending on the arrangement of the core and the winding as follows:
  - **Core Type**
  - **Shell Type**
  - **Berry Type**
- The magnetic core is a stack of thin silicon-steel laminations about 0.35 mm thick for 50 Hz transformer.
- In order to reduce the eddy current losses, these laminations are insulated from one another by thin layers of varnish.

CORE TYPE TRANSFORMER



SHELL TYPE TRANSFORMER





Difference between core type and shell type transformer

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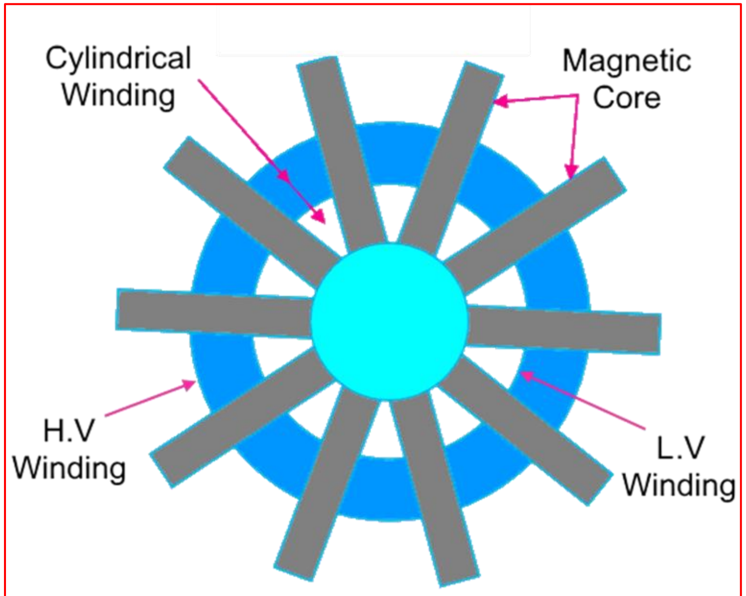
Sr. No	Core Type Transformer	Shell Type Transformer
1.	The core has only one window.	The core has two windows.
2.	Winding encircles the core.	Core encircles the windings.
3.	Cylindrical windings are used.	Sandwich type windings are used.
4.	Easy to repair.	It is not so easy to repair.
5.	Better cooling since more surface is exposed to the atmosphere.	Cooling is not very effective.

BERRY TYPE TRANSFORMER

- It has **distributed magnetic circuit**.
- The core type is like the spikes of a wheel.
- The transformers are generally placed in tightly fitted sheet metal tanks.
- Tanks are made up of high quality steel plate, formed and welded into a rigid structure.
- The tanks are filled with special insulating oil.
- All the joints are painted with light blue chalk solution disclosing even a minute leak.



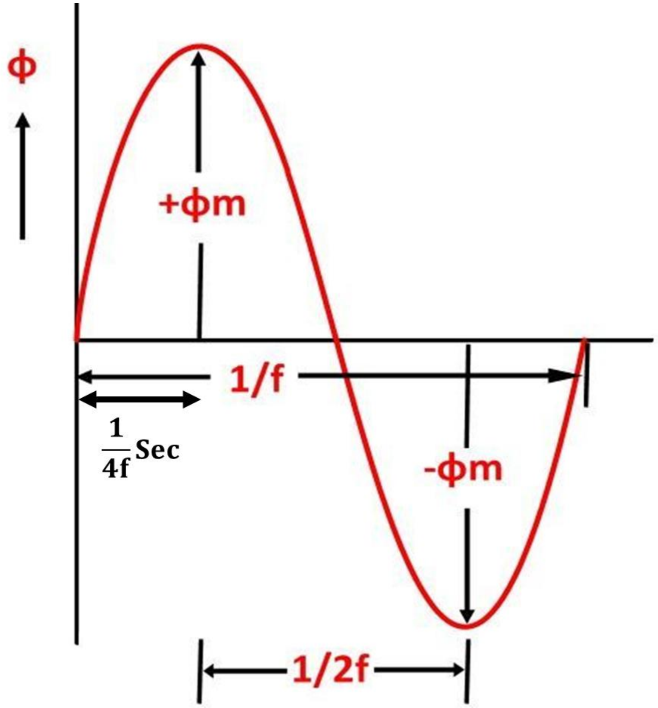
Berry Type Transformer

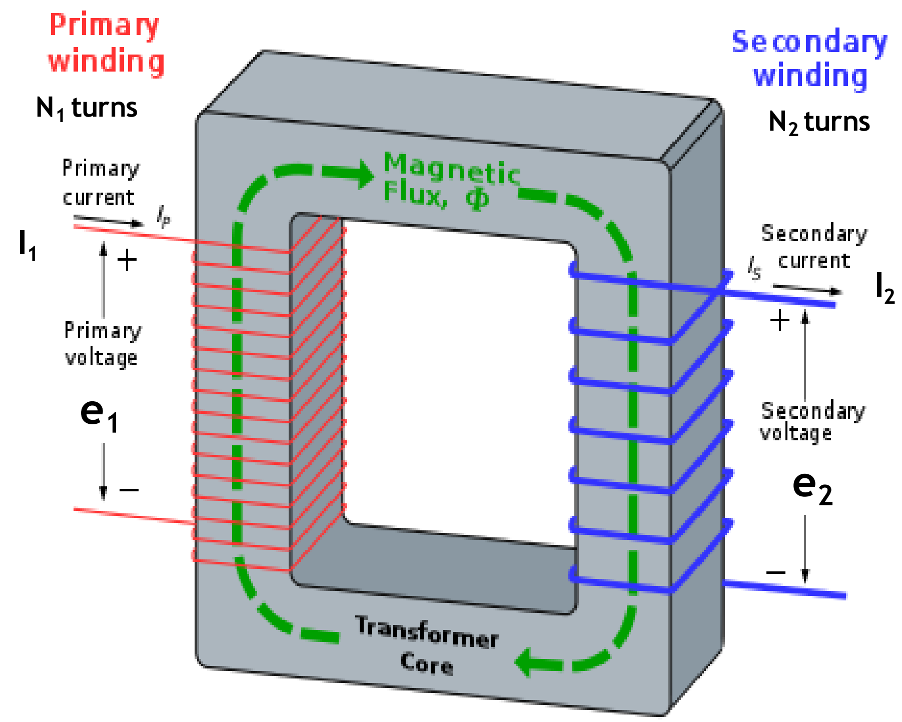
https://www.electricaltechnology.org/2019/07/types-of-transformers.html

APPLICATIONS OF TRANSFORMER

- **Step – up and Step – down Voltage**
- **Measurement of current in single and three phase system**
- **Measurement of voltage in single and three phase system**
- **Measurement of Power**
- **Measurement of Energy**

E.M.F EQUATION OF TRANSFORMER





The primary winding draws a current when it is connected to an alternating voltage source this sinusoidal current produces a sinusoidal flux Φ that can be expressed as:

E.M.F EQUATION OF TRANSFORMER

Let, Φ_m: The maximum amount of flux is presented in Weber unit
f: The source frequency based on Hz
N_1: Turns number in the first winding
N_2: Turns number in the second winding
Φ: Flux per turn based on Weber unit

When an alternating (sinusoidal) voltage is applied to the primary winding of a transformer, an alternating (sinusoidal) flux, as shown in previous slide is set up in the iron core which links both the windings (Primary and Secondary).

As illustrated in figure, the magnetic flux increases from zero to its maximum value ϕ_{max} in one-fourth of a cycle i.e. in $\frac{1}{4f}$ second.

So, Average rate of change of flux,

$$\frac{d\phi}{dt} = \frac{\phi_{max}}{1/4f} = 4\ f\ \phi_{max}$$

Since, its average EMF induced per turns in volts is equal to the average rate of change of flux,

Average EMF induced per turn = $4\ f\ \phi_{max}$ Volts

E.M.F EQUATION OF TRANSFORMER

Since, flux ϕ varies sinusoidally, EMF induced will be sinusoidal and form factor for sinusoidal wave is 1.11 i.e. the RMS or effective value is 1.11 times the average value.

∴ RMS value of EMF induced per turn $= 1.11 * 4 \text{ f } \phi_{max}$ ……….. (1)

If the number of turns on primary and secondary windings are N_1 and N_2 respectively, then

RMS value of EMF induced in primary,

E_1 = EMF induced per turn $*$ number of primary turns

$$= 4.44 \text{ f } \phi_{max} * N_1$$

$$\mathbf{E_1 = 4.44 \text{ f } N_1 \phi_{max} \text{ Volts}} \text{ ………….. (2)}$$

Similarly, RMS value of EMF induced in the secondary,

$$\mathbf{E_2 = 4.44 \text{ f } N_2 \phi_{max} \text{ Volts}} \text{ ……………. (3)}$$

Alternative method for E.M.F equation

Let,

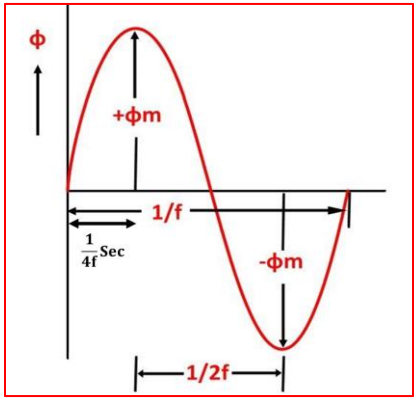
Φ_m: The maximum amount of flux is presented in Weber unit

f: The source frequency based on Hz

N_1: Turns number in the first winding

N_2: Turns number in the second winding

Φ: Flux per turn based on Weber unit



As presented in the last figure, the flux varies from + Φ_m to – Φ_m in ½ f seconds or half a period. According to Faraday's Law, consider E_1 as the EMF produced in the basic winding

$$E_1 = -\frac{d\psi}{dt}$$

Where,

$$\psi = N_1 \Phi$$

AS ϕ is generated base on the AC supply

Therefore,

$$\Phi = \Phi_m . \sin\omega t$$

E.M.F EQUATION OF TRANSFORMER

$$\Rightarrow E_1 = -N_1 \frac{d(\Phi_m \sin\omega t)}{dt}$$

$$\Rightarrow E_1 = -N_1 \omega \Phi_m \cos\omega t$$

$$\Rightarrow E_1 = N_1 \omega \Phi_m (\sin\omega t - \frac{\pi}{2})$$

So, the produced EMF lags the flux by 90º. Therefore, the maximum rate of EMF can be evaluated as:

$$E_{1max} = N_1 \omega \Phi_m$$

But, ω=2πf, So

$$E_{1max} = 2\pi f N_1 \Phi_m$$

RMS value or Root Mean Square can be calculated as:

$$E_1 = \frac{E_{1max}}{\sqrt{2}}$$

E.M.F EQUATION OF TRANSFORMER

By using the value of E_{1max} in the last equation, we can get

$$E_1 = \sqrt{2}\,\pi f\, N_1\, \Phi_m$$

If we put the value of $\pi = 3.14$ in the previous equation, we can obtain the value of E_1 as

$$E_1 = 4.44 f\, N_1 \Phi_m$$

Similarly;

$$E_2 = \sqrt{2}\,\pi f\, N_2\, \Phi_m$$

Or

$$E_2 = 4.44 f\, N_2 \Phi_m$$

Now, based on the equation of E_1 and E_2, we can obtain:

E.M.F EQUATION OF TRANSFORMER

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The above formula is introduced the turn coefficient where **K** is called the transformation ratio.

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

$$K \Rightarrow \text{Voltage Transformation Ratio}$$

$$\text{Where, } K = \frac{N_2}{N_1}$$

Thus,

1. If $N_2 > N_1$ i.e. $K > 1$, $E_2 > E_1 \Rightarrow$ Step up Transformer

2. If $N_2 < N_1$ i.e. $K > 1$, $E_2 < E_1 \Rightarrow$ Step down Transformer

3. If $N_2 = N_1$ i.e. $K = 1$, $E_2 = E_1 \Rightarrow$ Isolation or 1 : 1 Transformer

$\Rightarrow$ Current Ratio :

For an Ideal Transformer, there are no losses.

So, Input (VA) = Output (VA)

$$\therefore \; V_1 I_1 = V_2 I_2$$

$$\therefore \; \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

E.M.F EQUATION OF TRANSFORMER

Rating of Transformer:

The rating of transformer is in Volt-Ampere **or** kVA/MVA. While designing the transformer, there is no idea about load and its nature so its rating is expressed in VA/kVA/MVA.

Moreover, Cu loss is directly proportional to $\mathbf{I^2}$ and core loss us directly proportional to the V_{supply}.

These losses doesn't depend on load power factor. So rating of transformer is in VA/kVA/MVA not in kW/MW.

Problem: A 3300V/200V, 50 Hz, 100 kVA transformer has its low voltage winding with 80 turns.

Calculate:-

i) The Currents in both windings

ii) Number of turns of high voltage winding.

iii) Maximum value of flux, transformer is fully loaded.

Solution : $3300/200V, 100\,kVA,\ f = 50Hz, N_2 = 80$

$$\text{(i) Primary Current}(I_1)_{F.L.} = \frac{kVA * 1000}{3300} = 30.303\,A$$

$$\text{Secondary Current}(I_2)_{F.L.} = \frac{kVA * 1000}{V_2} = \frac{100 * 10^3}{200} = 500\,A$$

$$\text{(ii)} \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{E_2}{E_1} = K \quad \therefore N_1 = \frac{V_2}{V_1} * N_2 = \frac{3300}{200} * 80 \Rightarrow N_1 = 1320$$

$$\text{(iii) As, } E_1 = 4.44\, f\, \phi_m\, N_1 \qquad \therefore \phi_m = \frac{E_1}{4.44 f_1 N_1} = \frac{3300}{4.44 * 50 * 1320}$$

Problem:

For a single phase transformer having primary and secondary turns of 440 and 880 respectively, determine the transformer kVA rating if half load secondary current is 7.5 A and maximum value of core flux is 2.25 mWb

Solution :

Given : $N_1 = 440, N_2 = 880, (I_2)_{H.L.} = 7.5\,A$

$$\phi_m = 2.25 * 10^{-3}\ wb$$

$$\text{As,} \quad E_2 = 4.44\, f\, \phi_m\, N_2$$

$$\therefore E_2 = 4.44 * 50 * 2.25 * 10^{-3} * 880$$

$$E_2 = 439.56\, V$$

$$\text{Secondary Current} (I_2)_{F.L.} = \frac{(kVA)_{rating} * 1000}{E_2}$$

$$(I_2)_{F.L.} = 7.5 * 2 = 15\, A$$

$kVA = 6.5934 kVA$.........ANS.

Lecture 23

IDEAL TRANSFORMER AND IT'S CHARACTERISTICS

- An ideal transformer is an imaginary transformer which has
  - No copper losses (no winding resistance)
  - No iron loss in core
  - No leakage flux
- In other words, an ideal transformer gives output power exactly equal to the input power.
- The efficiency of an ideal transformer is 100%.
- Actually, it is impossible to have such a transformer in practice, but ideal transformer model makes problems easier.

CHARACTERISTICS OF IDEAL TRANSFORMER

- **Zero winding resistance**: It is assumed that, resistance of primary as well as secondary winding of an ideal transformer is zero. That is, both the coils are purely inductive in nature.
- **Infinite permeability of the core:** Higher the permeability, lesser the **MMF** required for flux establishment. That means, if permeability is high, less magnetizing current is required to magnetize the transformer core.
- **No leakage flux:** Leakage flux is a part of magnetic flux which does not get linked with secondary winding. In an ideal transformer, it is assumed that entire amount of flux get linked with secondary winding (**that is, no leakage flux**).
- **100% efficiency:** An ideal transformer does not have any losses like hysteresis loss, eddy current loss etc. So, the output power of an ideal transformer is exactly equal to the input power. Hence, 100% efficiency.

EFFECTS OF VOLTAGE AND FREQUENCY VARIATIONS ON TRANSFORMER

- The effects of voltage and frequency variation on the transformer has been discussed.

- Usually, power transformers are not subjected to wide frequency variations. Also, modest voltage variations occur on power transformers. But it is interesting to consider their effects thereof.

- Variation of voltage and frequency affects the iron losses (hysteresis and eddy current losses) in a transformer. If the flux variations are sinusoidal, then hysteresis loss (P_h) and eddy current losses (P_e) varies according to the following relations:

$$P_h \propto f(\varphi_{max})^x$$

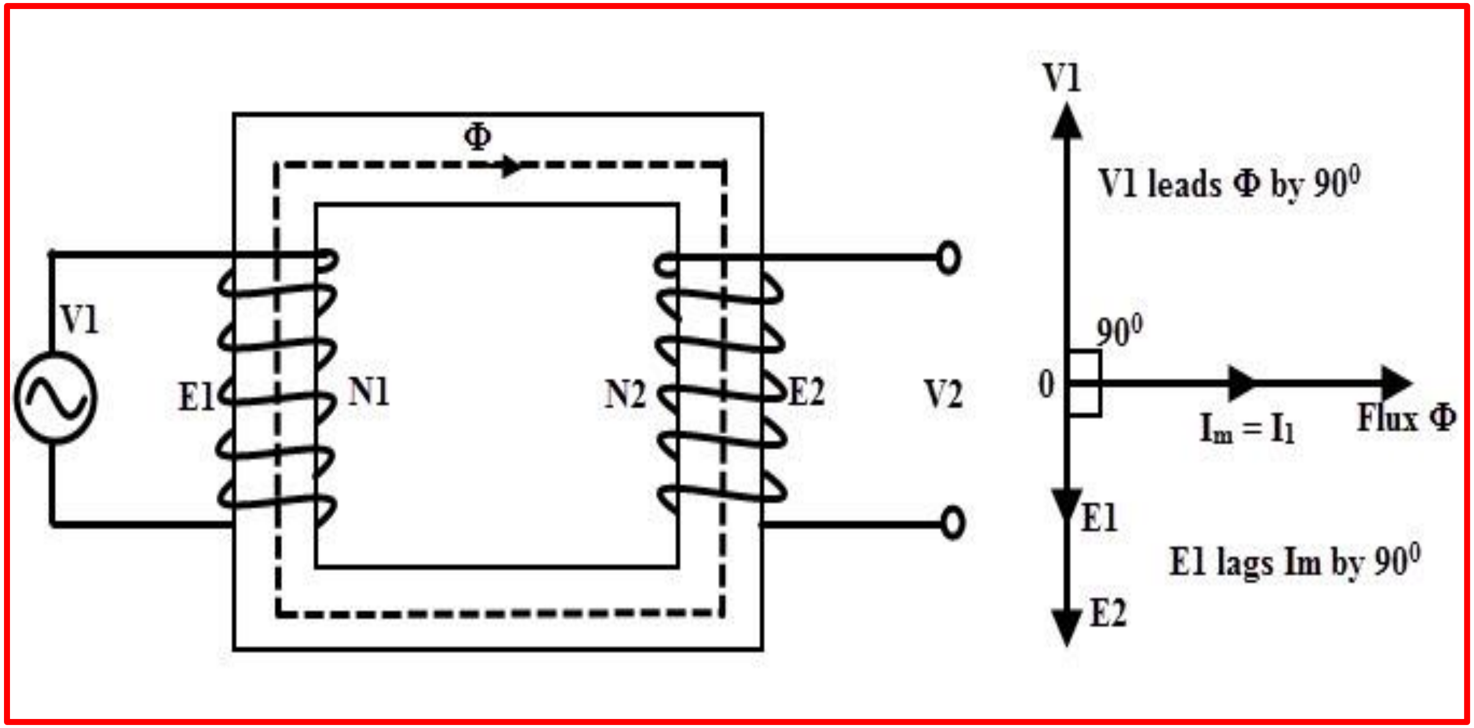
- x= 1.5 to 2.5 depending on the grade of iron used in the transformer core

$$P_e \propto f^2(\varphi_{max})^2$$

NOW CONSIDER SOME OPERATING CONDITION OF TRANSFORMER

- If the transformer voltage and frequency changes in the same proportion, The flux density (B_{max}) will remain unchanged ($V/f \propto \varphi_{max}$). So the no-load current will also remain unaffected.
- The transformer can be operated safely with a frequency less than rated one with correspondingly reduced voltage. In this case, iron losses will be reduced.
- If the transformer is operated with increased voltage and frequency in the same proportion, the core loss may increase to an intolerable level.
- If the frequency is increased with constant supply voltage ($V/f \propto \varphi_{max}$) – Hysteresis loss reduced and eddy current loss unaffected.
- Some increase in voltage could, therefore, be tolerated at higher frequencies, but exactly how much depends on the relative magnitude of the hysteresis and eddy current losses and the grade of iron used in the transformer core.

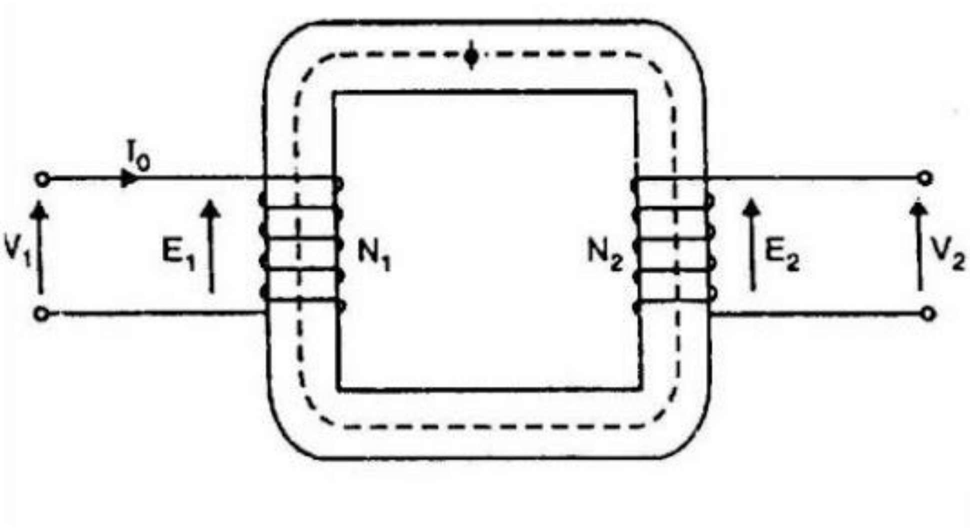
IDEAL TRANSFORMER ON NO LOAD

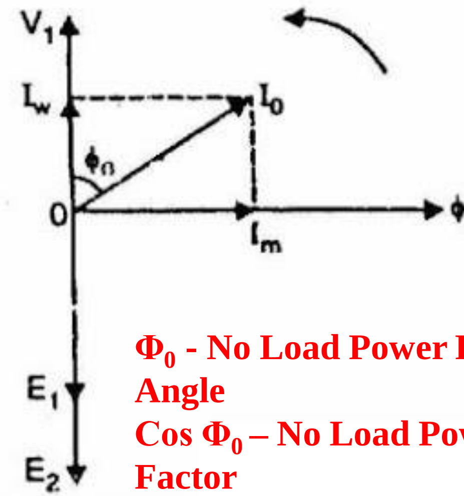


IDEAL TRANSFORMER ON NO LOAD

- For ideal transformer, on no load
- $I_2 = 0$,
- No Core Loss and No Copper Loss
- Primary current draws a current which is just necessary to set up flux in the core.
- $I_1 = I_m$ (Magnetizing Current), sets up flux in the core.
- As, $R_1=0$ and $R_2=0$
- So, $V_1=E_1$ and $E_2=V_2$
- E_1 opposes V_1 (Because of Lenz's Law *i.e* induced EMF)

PRACTICAL TRANSFORMER ON NO LOAD





Φ_0 - No Load Power Factor Angle
Cos Φ_0 – No Load Power Factor

- On no load, In practical transformer has iron losses (hysteresis loss) and eddy current loss as it is subjected to the alternating flux.
- **The no load current I_0 has two components:**

1. **I_m** - Magnetising Component –Sets up flux in the core
2. **I_w** - Active component – Supplies for core loss

No load input power, $W_0 = V_1 I_0 \cos\Phi_0$

As seen from the phasor diagram, the no-load primary current I_0 can be resolved into two rectangular components viz. I_w **and** I_m

(i) The component I_w in phase with the applied voltage V_1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$I_w = I_0 \cos\Phi_0$$

(ii) The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is the component which produces the mutual flux Φ in the core.

$$I_m = I_0 \sin\Phi_0$$

Clearly, I_0 is phasor sum of I_m and I_w.

$$I_0 = \sqrt{(I_m^2 + I_w^2)}$$

No load power factor, $\cos\Phi_0 = I_w/I_0$

It is emphasized here that no load primary copper loss (i.e. $I_0^2R_1$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

No load input power, W_0 = **Iron loss**

***Note*:**

At **no load**, there is **no current** in the **secondary** so that $V_2 = E_2$.

On the primary side, the drops in R_1 and X_1, due to I_0 are also very small because of the smallness of I_0.

Hence, we can say that at no load, $V_1 = E_1$.

Problem:

A 25 kVA, 3300/230 V, 50 Hz, 1-phase transformer draws a no-load current of 15 A when excited on load voltage side and consumes 350 watt. Calculate two components of current.

Solution :

Given : $W_0 = 350\text{Watt},\ I_0 = 15\,\text{A},\ V_0 = 230\,\text{V},\ f = 50\,\text{Hz}$

$$W_0 = V_0 I_0 \operatorname{Cos}\phi_0,$$

$$\operatorname{Cos}\phi_0 = \frac{350}{15*230} = 0.1014\,(\text{lag})$$

$$\operatorname{Sin}\phi_0 = 0.9948,\ I_c = I_0 \operatorname{Cos}\phi_0 = 15*0.1014 = 1.521\,\text{A}.$$

$$I_m = I_0 \operatorname{Sin}\phi_0 = 15*0.9948 = 14.922\,\text{A}.$$

Problem:

A voltage V=200Sin314t is applied to the transformer primary in a no-load test. Te resulting current is found to be i=3Sin(314t-60^0). Determine the core loss and no-load equivalent circuit parameters.

Solution :

Given : $\textit{No Load Current } (I_0) = \frac{I_{max}}{\sqrt{2}} = \frac{3}{\sqrt{2}} = 2.12\,A.$

$$\textit{Core Loss } (W_0) = V_1 I_0 \cos\phi_0 = \frac{200}{\sqrt{2}} * \frac{3}{\sqrt{2}} \cos 60^0 = 150\,\text{Watt}.$$

$$\because V_{1RMS} = \frac{V_{max}}{\sqrt{2}} \text{ and } \phi = 60^0 \text{ as } i \text{ lags } V \text{ by } 60^0$$

$$\textit{Active Component} \Rightarrow I_c = I_0 \cos\phi_0 = \frac{3}{\sqrt{2}} \cos 60^0 = 1.06\,A.$$

$$\textit{Magnetising Component} \Rightarrow I_m = I_0 \sin\phi_0 \text{ or } \sqrt{I_0^2 - I_C^2} = \sqrt{(2.12)^2 - (1.06)^2}$$

$$\Rightarrow I_m = 1.836\,A.,$$

$$\textit{No load resistance} \Rightarrow R_0 = \frac{V_1}{I_c} = \frac{200/\sqrt{2}}{1.06} = 133.42\,\Omega.$$

$$\textit{No load reactance} \Rightarrow X_0 = \frac{V_1}{I_m} = \frac{200/\sqrt{2}}{1.836} = 77\,\Omega.........\text{ANS.}$$

Problem:

The no-load current of a transformer is 4.0 A at 0.25 power factor when supplied at 250 V, 50 Hz. The number of turns on the primary winding is 200. Calculate:

(i) Flux in the core (ii) Core Loss (iii) Magnetising Current

Solution :

$$\text{(i)}\ E_1 = 4.44 f \phi_m N_1 \Rightarrow \phi_m = \frac{E_1}{4.44 f N_1} = \frac{250}{4.44 * 50 * 200}.$$

$$\phi_{max} = 5.63\,\text{mWb}...\text{ANS}$$

$$\text{(ii) Core Loss} = V_1 I_0 \operatorname{Cos}\phi_0 = 250 * 4 * 0.25 = 250\ \text{Watt}$$

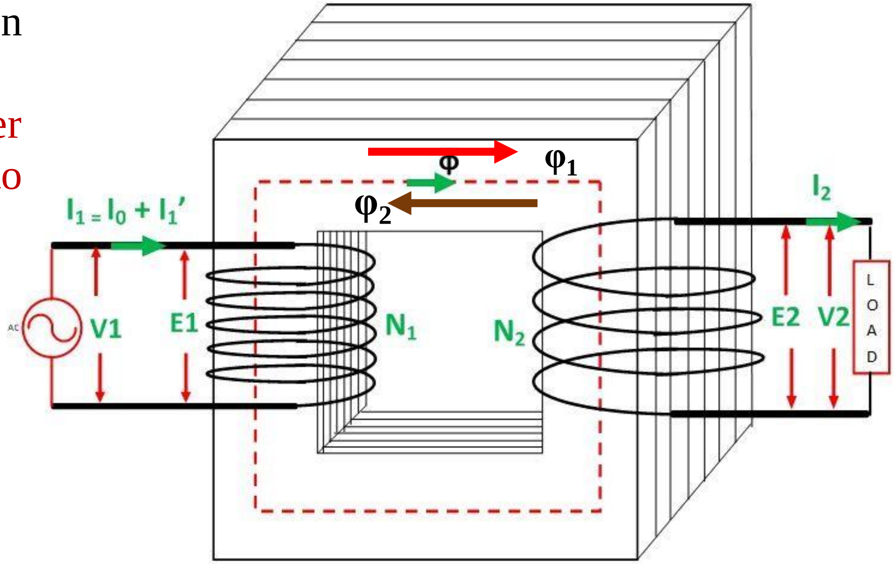
$$\text{(iii)}\ I_m = I_0 \operatorname{Sin}\phi_0 = I_0\sqrt{1 - \operatorname{Cos}^2\phi_0} = 3.873\,\text{A}.$$

$$\text{(iv)}\ I_c = I_0 \operatorname{Cos}\phi_0 = 4 * 0.25 = 1\,\text{A}.......\text{ANS}$$

TRANSFORMER ON LOAD

(MMF Balancing on Load)

- When the load is connected to the secondary of the transformer, I_2 current flows through their secondary winding.
- The secondary current induces the MMF, N_2I_2 on the secondary winding of the transformer.
- This force set up the flux φ_2 in the transformer core. The flux φ_2 opposes the flux φ, according to **Lenz's law**.
- As the flux φ_2 opposes the flux φ, the resultant flux of the transformer decreases and this flux reduces the induced EMF E_1.
- Thus, the strength of the V_1 is more than E_1 and an additional primary current I'_1 drawn from the main supply.
- The additional current is used for restoring the original value of the flux in the core of the transformer so that $V_1 = E_1$.
- The primary current I'_1 is in phase opposition with the secondary current I_2. Thus, it is called the **primary counter-balancing current**.



TRANSFORMER ON LOAD
(MMF Balancing on Load)

- The additional current I'_1 induces the MMF $N_1I'_1$. And this force set up the flux φ'_1.
- The direction of the flux is the same as that of the φ and it cancels the flux φ_2 which induces because of the **MMF N_2I_2**

Now, $N_1I_1' = N_2I_2$

Therefore,

$$I_1' = \left(\frac{N_2}{N_1}\right) I_2 = KI_2$$

- The phase difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer.
- If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading. The total primary current I_1 is the vector sum of the currents I_0 and I_1'. i.e

$$\vec{I} = \vec{I_0} + \vec{I_1'}$$

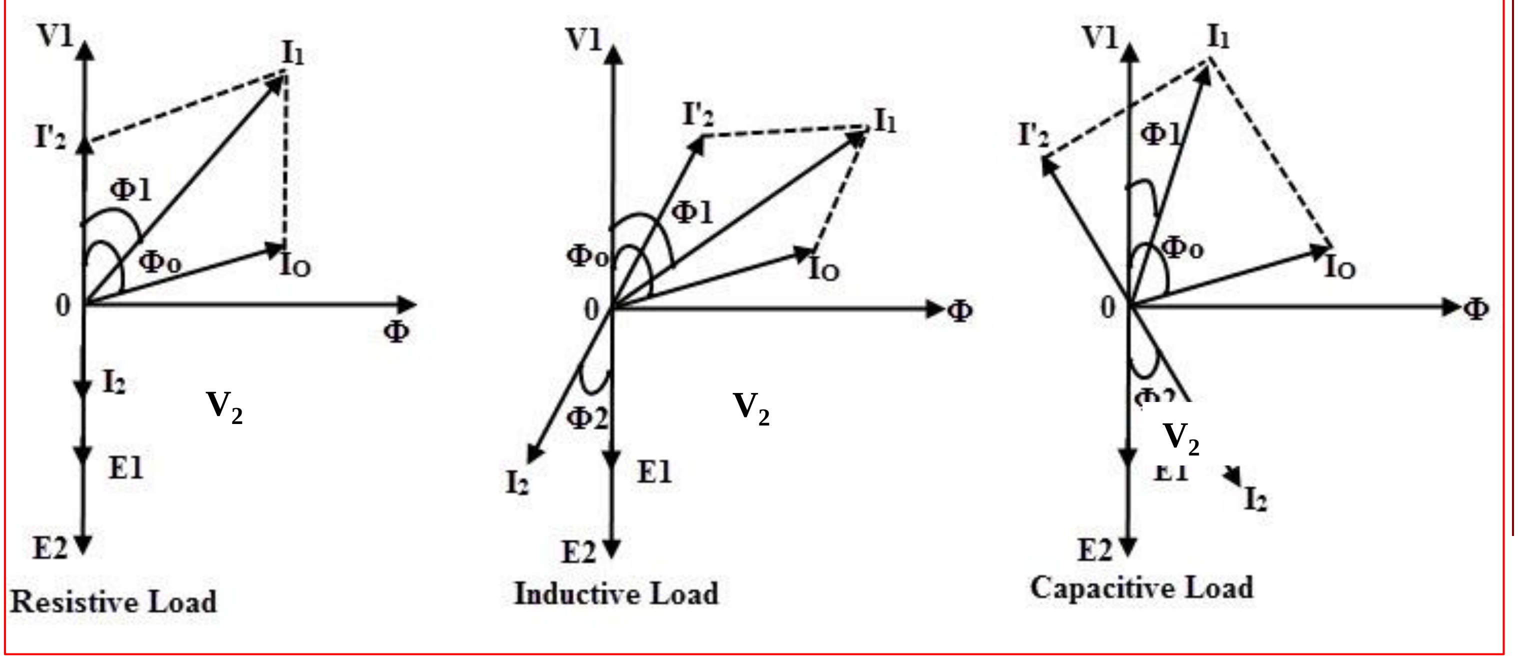
TRANSFORMER ON LOAD

- When the load is connected ; secondary current I_2 flows.
- To counter balance its effect , current I_1' flows in primary.
- Hence resultant current in primary; $I_1 = I_o + I_1'$
- Current I_o is hardly 2 to 5 % of full load primary current.
- Copper losses also occur in addition with iron losses.

STEPS TO DRAW THE PHASOR DIAGRAM

- Take flux ϕ, a reference
- Induces EMF E_1 and E_2 lags the flux by 90 degrees.
- The component of the applied voltage to the primary equal and opposite to induced EMF in the primary winding. E_1 is represented by V_1'.
- Current I_0 lags the voltage V_1' by 90 degrees.
- The power factor of the load is lagging. Therefore current I_2 is drawn lagging E_2 by an angle ϕ_2.
- The resistance and the leakage reactance of the windings result in a voltage drop, and hence secondary terminal voltage V_2 is the phase difference of E_2 and voltage drop.
- $V_2 = E_2$ – voltage drops $I_2 R_2$ is in phase with I_2 and I_2X_2 is in quadrature with I_2.
- The total current flowing in the primary winding is the phasor sum of I_1' and I_0.
- Primary applied voltage V_1 is the phasor sum of V_1' and the voltage drop in the primary winding.
- Current I_1' is drawn equal and opposite to the current I_2
- $V_1 = V_1$' + voltage drop I_1R_1 is in phase with I_1 and I_1X_I is in quadrature with I_1.
- The phasor difference between V_1 and I_1 gives the power factor angle ϕ_1 of the primary side of the transformer.
- The power factor of the secondary side depends upon the type of load connected to the transformer.
- If the load is inductive as shown in the above phasor diagram, the power factor will be lagging, and if the load is capacitive, the power factor will be leading.
- Where I_1R_1 is the resistive drop in the primary windings I_2X_2 is the reactive drop in the secondary winding

PHASOR DIAGRAM OF TRANSFORMER ON LOAD



Problem: **A single phase 440/110V, transformer takes a no-load current of 4 A at 0.2 power factor. If the secondary supplies a current of 100 A at a power factor of 0.8 lagging. Determine:-**

(i) The current drawn by the primary winding (ii) The magnetising reactance and resistance representing core loss.

Solution :

$$I_0 = 4A,\ Cos\,\phi_0 = 0.2,\ \phi_0 = Cos^{-1}(0.2) = 78.463^0 \Rightarrow I_0 = 4\angle -78.463^0\,A.$$

$$I_2 = 100\,A, Cos\,\phi_2 = 0.8,\ \phi_2 = Cos^{-1}(0.8) = 36.86^0 \Rightarrow I_2 = 100\angle -36.86^0\,A.$$

$$K = \frac{N_2}{N_1} = \frac{110}{440} = 0.25$$

$$I_2^{`} = KI_2 = \frac{1}{4} * 100 = 25\,A \Rightarrow I_2^{`} = 25\angle -36.86^0\,A.$$

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2^{`} = 4\angle -78.463^0 + 25\angle -36.86^0\,A \Rightarrow (0.80 - j3.919) + (20 - j15)$$

$$\bar{I}_1 = 28.1170\angle -42.28^0\,A.$$

$$I_c = I_0\,Cos\,\phi_0 = 4 * 0.2 = 0.8\,A.$$

$$I_m = I_0\,Sin\,\phi_0 = 4\sqrt{1 - Cos^2\,\phi_0} = 3.9191\,A.$$

$$R_0 = \frac{V_1}{I_c} \Rightarrow \frac{440}{0.8} = 550\,\Omega, \qquad X_0 = \frac{V_1}{I_m} = \frac{440}{3.9191} \Rightarrow 112.2706\ \Omega$$ .... ANS

Problem: A single phase transformer has 100 turns on the primary and 200 turns on the secondary. The no load current is 3 Ampere at a power factor of 0.2 lagging. Calculate the primary current and power factor when the secondary current is 280 A at a power factor of 0.8 lagging.

Solution :

$$\text{Cos}\,\phi_2 = 0.8(\text{lag}) \Rightarrow \phi_2 = \text{Cos}^{-1}(0.8) = 36.86^0 \Rightarrow I_2 = 280\angle -36.86^0\,A.$$

$$I_2^{`} = KI_2 \Rightarrow \frac{N_2}{N_1} * I_2 \Rightarrow \frac{200}{1000} * 280 \Rightarrow I_2^{`} = 56\angle -36.86^0\,A.$$

$$I_0 = 3A,\ \text{Cos}\,\phi_0 = 0.2,\ \phi_0 = \text{Cos}^{-1}(0.2) = 78.5^0 \Rightarrow \text{Sin}\,\phi_0 = 0.98$$

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2^{`} = [3\angle -78.5^0 + 56\angle -36.86^0\,A] \Rightarrow [3(0.20 - j0.98) + 56(0.80 - j0.60)]$$

$$\Rightarrow 45.4 - j36.54$$

$$\bar{I}_1 = 58.3\angle -38.86\,A.$$

$$\Phi_1 = +38.86^0(\text{Lag}) \Rightarrow \text{Cos}\,\Phi_1 = 0.778(\text{Lag}).......\ \text{ANS}$$

Lecture 24 & 25

EQUIVALENT CIRCUIT

- Equivalent impedance of transformer is essential to be calculated because the electrical power transformer is an electrical power system equipment for estimating different parameters of the electrical power system which may be required to calculate the total internal impedance of an electrical power transformer, viewing from primary side or secondary side as per requirement.

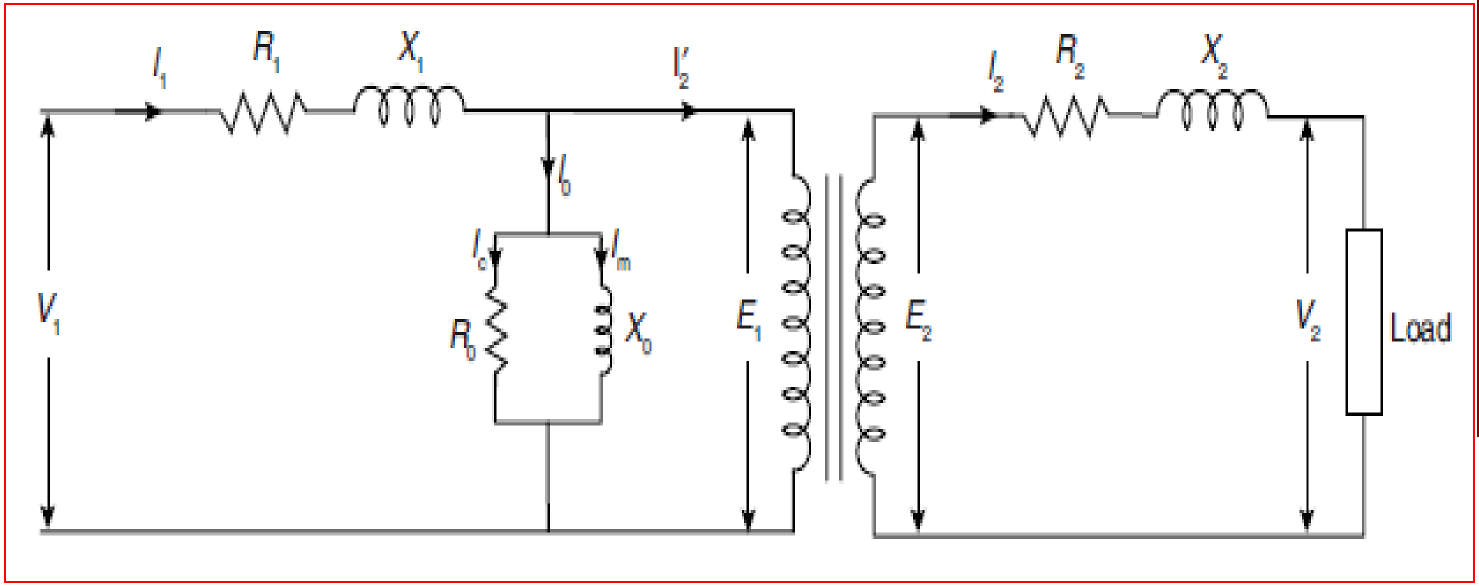
- This calculation requires equivalent circuit of transformer referred to the primary or equivalent circuit of transformer referred to secondary sides respectively. Percentage impedance is also a very essential parameter of the transformer.

- Special attention is to be given to this parameter during installing a transformer in an existing electrical power system. Percentage impedance of different power transformers should be properly matched during parallel operation of power transformers.

- The percentage impedance can be derived from the equivalent impedance of the transformer so, it can be said that the equivalent circuit of the transformer is also required during the calculation of the % impedance.

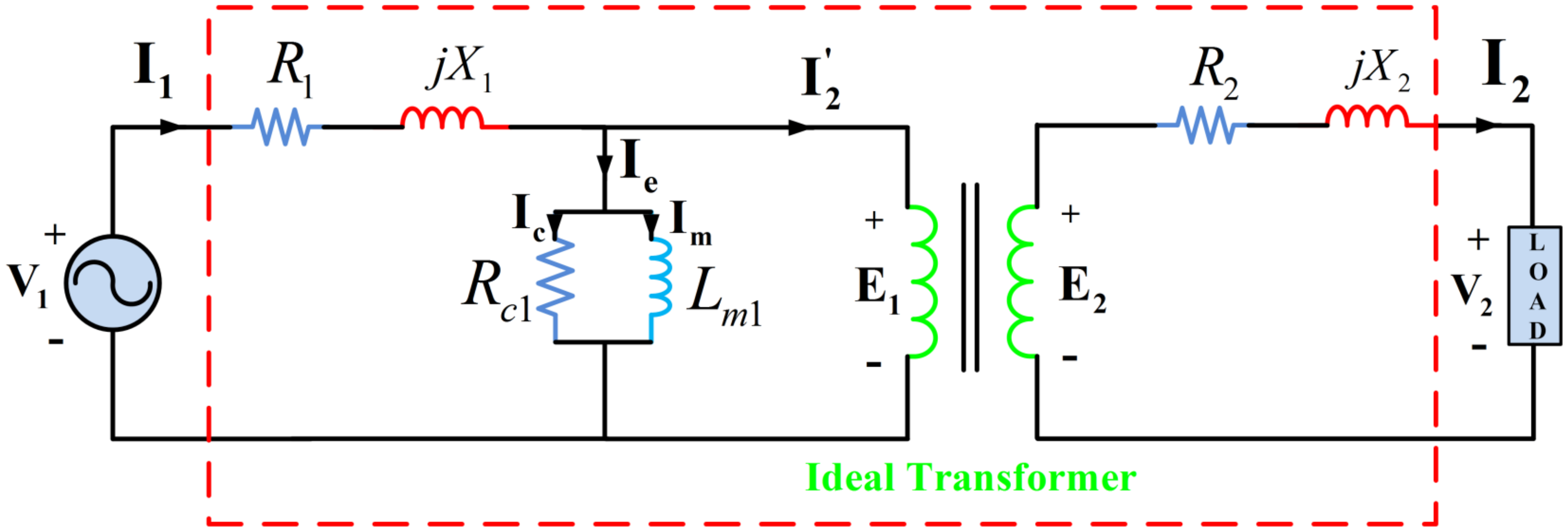
EQUIVALENT CIRCUIT OF TRANSFORMER

Exact Equivalent Circuit



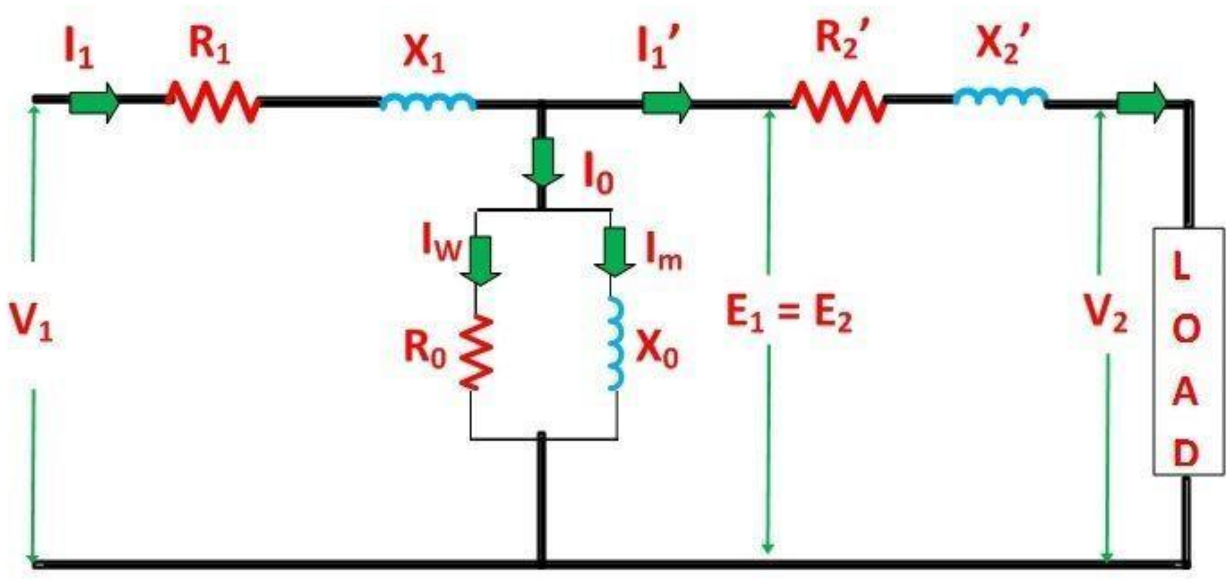
EQUIVALENT CIRCUIT

- Exact equivalent circuit is drawn as shown in figure below.
- Primary & secondary windings are electrically separated.
- For analysis purpose it is electrically connected by referring one side to another side.
- Generally primary is referred to secondary.
- Secondary values get changed when referred to primary.



EQUIVALENT CIRCUIT OF TRANSFORMER

In this case, to draw the equivalent circuit of the transformer all the quantities are to be referred to the primary as shown in the figure below:



Circuit Diagram of Transformer when all the Secondary Quantities are Referred to Primary Side

Exact Equivalent Circuit Referred to Primary

The following are the values of resistance and reactance given below

- **Secondary resistance referred to the primary side is given as:**

$$R_2' = \frac{R_2}{K^2}$$

- **The equivalent resistance referred to the primary side is given as:**

$$R_{ep} = R_1 + R_2'$$

- **Secondary reactance referred to the primary side is given as:**

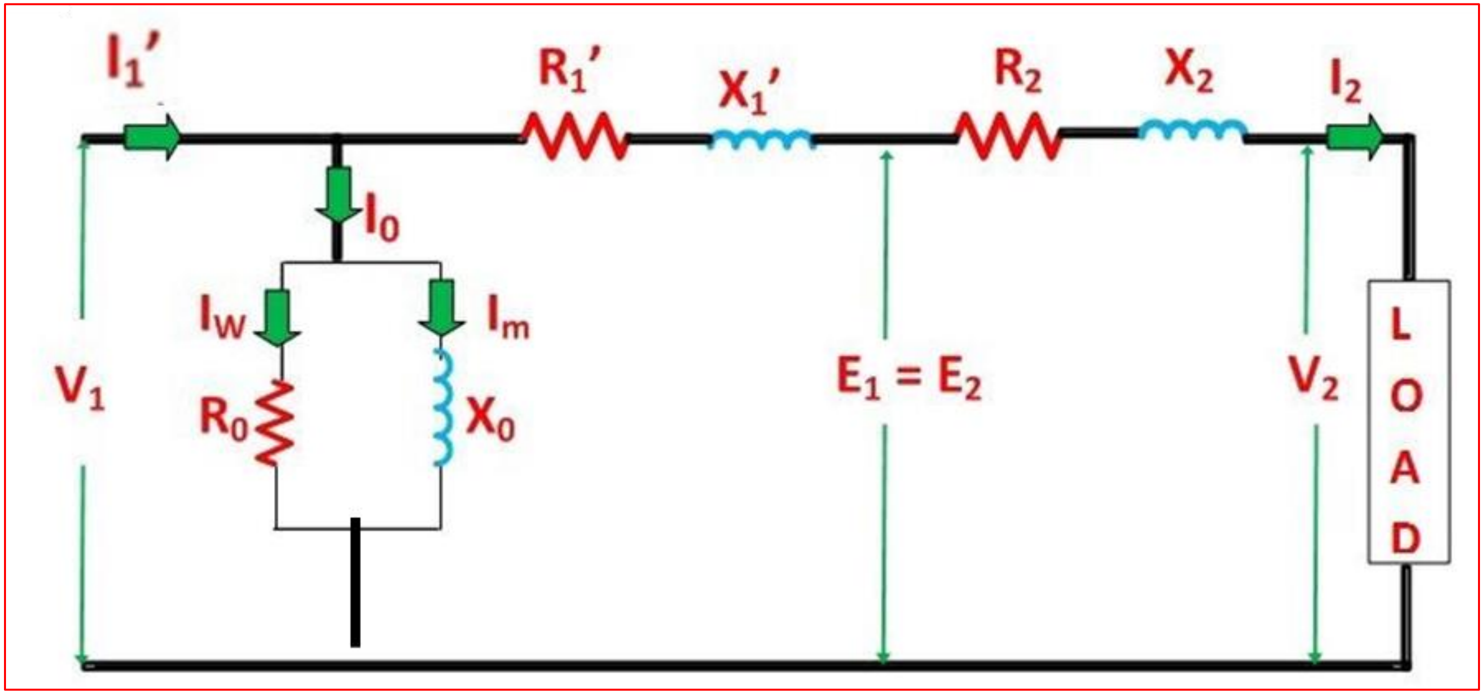
$$X_2' = \frac{X_2}{K^2}$$

- **The equivalent reactance referred to the primary side is given as:**

$$X_{ep} = X_1 + X_2'$$

EQUIVALENT CIRCUIT OF TRANSFORMER

Exact Equivalent Circuit Referred to Secondary



Circuit Diagram of Transformer, When All the Primary Quantities are Referred to Secondary Side

Exact Equivalent Circuit Referred to Secondary

The following are the values of resistance and reactance given below:

- Primary resistance referred to the secondary side is given as:

$$R_1' = K^2 R_1$$

- The equivalent resistance referred to the secondary side is given as:

$$R_{2e} = R_2 + R_1'$$

- Primary reactance referred to the secondary side is given as:

$$X_1' = K^2 X_1$$

- The equivalent reactance referred to the secondary side is given as

$$X_{eq} = X_2 + X_1'$$

Note: *No-load current I_0 is hardly* ***3 to 5%*** *of full load rated current, the parallel branch consisting of resistance R_0 and reactance X_0 can be omitted without introducing any appreciable error in the behavior of the transformer under the loaded condition.*

Problem: A 15 kVA, 220/110V Single phase transformer has R_{1}=1.75 Ω, R_2=0.0045 Ω. The leakage reactances are X_1=2.6 Ω, X_2=0.0075 Ω.

Calculate:

a) Equivalent resistance and reactance referred to primary.

b) Equivalent resistance and reactance referred to secondary.

c) Equivalent impedance referred to primary and secondary

d) Total copper loss

Solution :

Given : $V_1 = 2200V, V_2 = 110V, R_1 = 1.75\Omega, R_2 = 0.0045\Omega, X_1 = 2.6\Omega, X_2 = 0.0075\Omega$

$$K = \frac{V_2}{V_1} = \frac{110}{2200} = 0.05$$

$$R_{1e} = R_1 + R_2^{'} = R_1 + \frac{R_2}{K^2} = 1.75 + \frac{0.0045}{(0.05)^2} = 3.55\ \Omega$$

$$X_{1e} = X_1 + X_2^{'} = X_1 + \frac{X_2}{K^2} = 2.6 + \frac{0.0075}{(0.05)^2} = 5.6\ \Omega$$

$$R_{2e} = R_2 + R_1^{'} = R_2 + K^2 R_1 = 0.0045 + (0.05)^2 * 1.75 = 0.00887\ \Omega$$

$$X_{2e} = X_2 + X_1^{'} = X_2 + K^2 X_1 = 0.0075 + (0.05)^2 * 2.6 = 0.014\ \Omega$$

$$Z_{1e} = R_{1e} + jX_{1e} = (3.55 + j5.6) \Rightarrow |Z_{1e}| = 6.6304\ \Omega$$

$$Z_{2e} = R_{2e} + jX_{2e} = (0.00887 + j0.014) \Rightarrow |Z_{2e}| = 0.01657\ \Omega$$

$$(P_{cu})_{Total} = I_1^2 R_{1e} + I_2^2 R_2 = (6.81)^2 * 1.75 + (136.36)^2 * 0.0045 = 165.027\,\text{Watt}$$

$$\text{or} \Rightarrow I_1^2 R_{1e} = (6.81)^2 * 3.55 = 165.027\,\text{Watt}$$

$$\text{or} \Rightarrow I_2^2 R_{2e} = (136.36)^2 * 0.00887 = 165.027\,\text{Watt}$$

$$(I_1)_{F.L.} = \frac{kVA * 1000}{V_1} = \frac{15 * 10^3}{2200} \Rightarrow 6.8181\,A.$$

$$(I_2)_{F.L.} = \frac{kVA * 1000}{V_2} = \frac{15 * 10^3}{110} \Rightarrow 136.3636\,A.$$

Problem: **The ohmic values of the circuit parameters of a transformer having a turn ratio of 5 are R_1=0.5 A Ω, R_2=0.021 Ω, X_1=3.2 Ω, X_2=0.12 Ω, R_0=350 Ω, referred to primary and $X_0 = 98$ Ω referred to primary. Draw the approximate equivalent circuit of the transformer referred to secondary. Draw the approximate equivalent circuit of the transformer referred to secondary. Show the numerical values of the circuit parameters.**

Solution :

$$\text{Turns ratio} \Rightarrow \frac{N_1}{N_2} = 5,\ K = \frac{N_2}{N_1} = \frac{1}{5} = 0.2$$

$$R_1' = K^2 R_1 = (0.2)^2 * 0.5 = 0.02\ \Omega$$

$$X_1' = K^2 X_1 = (0.2)^2 * 3.2 = 0.128\ \Omega$$

$$R_{2e}' = R_2 + R_1' \Rightarrow R_2 + K^2 R_1 = 0.041\ \Omega$$

$$X_{2e} = X_2 + X_1' = X_2 + K^2 X_1 = 0.248\ \Omega$$

$$R_0' = K^2 R_0 \Rightarrow (0.2)^2 * 350 = 14\ \Omega$$

$$X_0' = K^2 X_0 \Rightarrow (0.2)^2 * 98 = 3.92\ \Omega$$

Problem: A 20 kVA, 2000/200V, single phase 50Hz transformer has a primary resistance of 1.5 Ω, and reactance of 2 Ω. The secondary resistance and reactance are 0.015 Ω and 0.02 Ω respectively. The no load current of transformer is 1 A at 0.2 power factor. Find:

i) Equivalent resistance, reactance, Impedance referred to primary
ii) Supply current
iii) Total copper loss
iv) Draw approximate equivalent circuit referred to primary

Solution :

$$\text{(i)}\ R_{1e} = R_1 + \frac{R_2}{K^2} = 0.1 + \frac{0.004}{(0.02)^2} = 0.2\ \Omega,$$

$$R_{2e} = R_2 + K^2 R_1 = 0.004 + (0.2)^2 * 0.1 = 0.008\ \Omega$$

$$X_{1e} = X_1 + \frac{X_2}{K^2} = 0.3 + \frac{0.012}{(0.2)^2} = 0.6\ \Omega$$

$$X_{2e} = X_2 + K^2 X_1 = 0.012 + (0.2)^2 * 0.3 = 0.024\ \Omega$$

$$\text{(ii)}\ Z_{1e} = (R_{1e} + jX_{1e}) = (0.2 + j0.6) \Rightarrow |Z_{1e}| = 0.6325\ \Omega$$

$$Z_{2e} = (R_{2e} + jX_{2e}) = (0.008 + j0.024) \Rightarrow |Z_{2e}| = 0.0253\ \Omega$$

$$\text{(iii)}\ Z_1 = R_1 + jX_1 = (0.1 + j0.3) = \_\_\_\_\_\_\_\_\_\_.$$

$$Z_2 = R_2 + jX_2 = (0.004 + j0.012) = \_\_\_\_\_\_\_\_\_\_.$$

$$(I_1)_{F.L.} = \frac{kVA * 1000}{V_1} = \frac{100 * 10^3}{1100} \Rightarrow \frac{1000}{11} A.$$

$$\text{Equivalent resistance drop referred to primary} = I_1 R_{1e} = \frac{1000}{11} * 0.2 = 18.18V$$

$$\text{Equivalent reactance drop referred to primary} = I_1 X_{1e} = \frac{1000}{11} * 0.6 = 54.54V$$

$$\% \text{Resistance drop} = \frac{I_1 R_{1e}}{V_1} * 100 = \frac{18.18}{1100} * 100 = 1.653\%$$

$$\% \text{Reactance drop} = \frac{I_1 X_{1e}}{V_1} * 100 = \frac{54.54}{1100} * 100 = 4.96\%$$

Problem: A 100 kVA, 1100/220V, single phase 50Hz transformer has a leakage impedance (0.1+j0.4) Ω for H.V. winding and (0.006+j0.015) Ω for L.V. winding. Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.

Solution :

$$\text{Turn ratio} = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{1100}{220} = 5,$$

$$\text{Transformation ratio} \Rightarrow K = \frac{N_2}{N_1} = \frac{1}{5} = 0.2$$

$$[R_1 = 0.1\ \Omega,\ R_2 = 0.006\ \Omega,\ X_1 = 0.4\Omega,\ X_2 = 0.015\Omega]$$

$$\because Z_1 = (0.1 + j0.4)\Omega \quad Z_2 = (0.006 + j0.015)\Omega$$

Referred to H.V. Side

$$R_{1e} = R_1 + \frac{R_2}{K^2} = 0.1 + \frac{0.006}{(0.2)^2} = 0.25\ \Omega,\ \ X_{1e} = X_1 + \frac{X_2}{K^2} = 0.4 + \frac{0.015}{(0.02)^2} = 0.775\ \Omega$$

$$Z_{1e} = R_{1e} + jX_{1e} = (0.25 + j0.775) = 0.8143\Omega$$

Referred to L.V. Side

$$R_{2e} = R_2 + K^2R_1 = 0.006 + (0.1) * (0.2)^2 = 0.01\ \Omega$$

$$X_{2e} = X_2 + K^2X_1 = 0.015 + (0.2)^2 * (0.4) = 0.031\ \Omega$$

$$Z_{2e} = R_{2e} + jX_{2e} = (0.01 + j0.031) = 0.0326\ \Omega$$

NUMERICALS (Please Refer to Notes)

1. 30 KVA, 2000/200 V, 1-Ø transformer has R_1 = 3.5 Ω, X_1 = 4.5 Ω and R_2 = 0.015 Ω, X_2 = 0.02 Ω . Find (i) equivalent impedance referred to pr. (ii) total cu loss. [UPTU 2012, 2014]

2. Draw the exact equivalent circuit of single phase transformer. [UPTU 2010, 2013]

3. 100 KVA, 2400/240 V, transformer has no-load current of 0.64A and core loss of 700W, when it is energized at rated HV voltage. Find the (i) components of no-load current (ii) no-load parameters. [UPTU 2010, 2014]

TRANSFORMER VOLTAGE REGULATION

- The **output voltage** of a transformer **varies** with the load even if the input voltage remains constant.
- This is **because** a real transformer has **series impedance** within it.
- Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load, defined by the given equation:

$$Regulation\ up = \frac{V_{2,nl} - V_{2,fl}}{V_{2,fl}} \times 100\%$$

$$Regulation\ down = \frac{V_{2,nl} - V_{2,fl}}{V_{2,nl}} \times 100\%$$

$$At\ no\ load\ k = \frac{V_2}{V_1}$$

$$Regulation\ up = \frac{\left(\frac{V_1}{k}\right) - V_{2,fl}}{V_{2,fl}} \text{ x } 100\%$$

$$Regulation\ down = \frac{\left(\frac{V_1}{k}\right) - V_{2,fl}}{V_{2,nl}} \text{ x } 100\%$$

- **Ideal transformer have voltage regulation = 0%.**

TRANSFORMER VOLTAGE REGULATION

$$\text{Voltage regulation} = \frac{\text{no-load voltage} - \text{full-load voltage}}{\text{no-load voltage}}$$

Recall,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\text{Secondary voltage on no load, } V_2 = V_1\left(\frac{N_2}{N_1}\right)$$

V_2 is a secondary terminal voltage on full load

Substituting, we have

$$\text{Voltage regulation} = \frac{V_1\left(\frac{N_2}{N_1}\right) - V_2}{V_1\left(\frac{N_2}{N_1}\right)}$$

Problem: **A 5 kVA, 250/125V, single phase 50Hz transformer has a primary resistance of 0.2 Ω and reactance of 0.75 Ω. The secondary resistance is 0.05 Ω and reactance is 0.2 Ω.**

Determine:

(i) It's regulation while supplying full load on 0.8 leading power factor

(ii) The secondary terminal voltage on full load and 0.8 leading power factor

(iii) Determine % resistive and reactive drops.

Solution :

Given : $R_1 = 0.2\,\Omega, R_2 = 0.05\,\Omega, X_1 = 7.5\ \Omega, X_2 = 0.02\ \Omega$

$\text{Cos}\,\phi = 0.8\,(\text{lead})$

$$(I_2)_{F.L.} = \frac{kVA}{V_2} = \frac{5*10^3}{125} = 40\,A$$

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{125}{250} = 0.5$$

For leading P.F. $E_2 < V_2$

$$R_{2e} = R_2 + K^2 R_1 = 0.05 + (0.5)^2 * 0.2 = 0.1\ \Omega$$

$$X_{2e} = X_2 + K^2 X_1 = 0.2 + (0.5)^2 * 0.75 = 0.3875\ \Omega$$

(i) $\cos\phi = 0.8$

$$\% \text{ of V.R.} = \frac{I_2[R_{2e}\cos\phi - X_{2e}\sin\phi]}{V_2} * 100 = \frac{40[0.1*0.8 - 0.3875*0.6]}{125} * 100$$

$$\% \text{ of V.R.} = -4.88\%$$

(ii) For leading P.F.

$$E_2 = V_2 + I_2 R_{2e}\cos\phi - I_2 X_{2e}\sin\phi$$

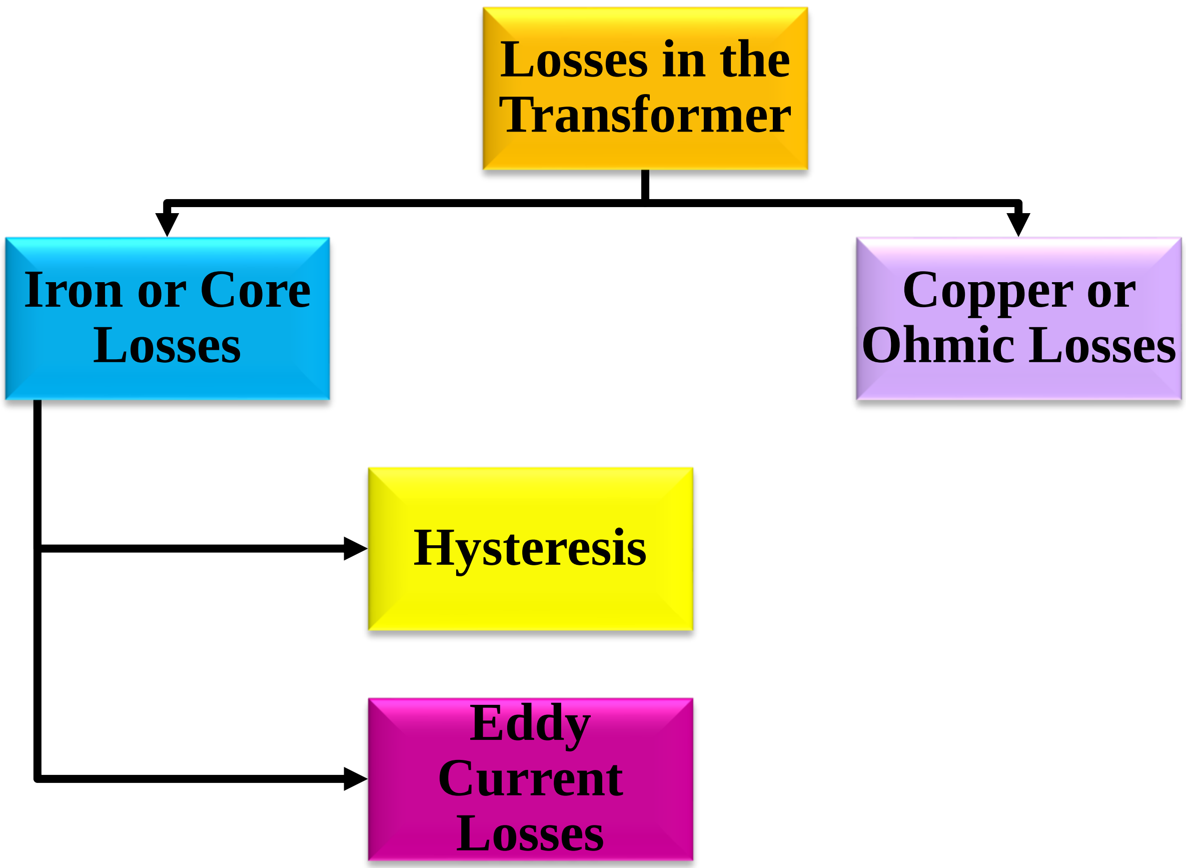
$$V_2 = E_2 - I_2 R_{2e}\cos\phi + I_2 X_{2e}\sin\phi$$

$$V_2 = 125 - 40*[0.1*0.8 - 0.3875*0.6]$$

$$V_2 = 125 - 40*[-6.1] \Rightarrow V_2 = 131.1\text{V}$$

Lecture 26

TRANSFORMER LOSSES



TRANSFORMER LOSSES

GROUP OF INSTITUTIONS

❖ **Iron Losses**

- Iron losses are caused by the alternating flux in the core of the transformer as this loss occurs in the core it is also known as ***Core Loss***. Iron loss is further divided into ***Hysteresis* and *Eddy Current Loss***. These losses are **fixed**.

❖ **Hysteresis Loss**

- The core of the transformer is subjected to an alternating magnetizing force, and for each cycle of EMF, a hysteresis loop is traced out.
- Power is dissipated in the form of heat known as hysteresis loss and given by the equation shown below:

$$P_h = K_h B_{max}^{1.6} f V \quad Watts$$

Where,

- K_h is a **proportionality constant** which depends upon the volume and quality of the material of the core used in the transformer,
- f is the supply **frequency**,
- B_{max} is the **maximum or peak value** of the flux density.
- V is the **volume** of magnetic material in m^3

NOTE: The iron or core losses can be minimized by using silicon steel material for the construction of the core of the transformer.

TRANSFORMER LOSSES

GROUP OF INSTITUTIONS

❖ Eddy Current Loss

- When the flux links with a closed circuit, an EMF is induced in the circuit and the current flows, the value of the current depends upon the amount of EMF around the circuit and the resistance of the circuit.
- Since, the core is made of conducting material, these EMFs circulate currents within the body of the material. These circulating currents are called **Eddy Currents**.
- They will occur when the conductor experiences a changing magnetic field. As these currents are not responsible for doing any useful work, and it produces a loss (***I^2R loss***) in the magnetic material known as an **Eddy Current Loss**.
- The eddy current loss is minimized by making the core with thin laminations.
- The equation of the eddy current loss is given as:

$$P_e = K_e B_{max}^2 t^2 f^2 V \quad Watts$$

Where,

- ***K_e*** – coefficient of eddy current. Its value depends upon the nature of magnetic material like volume and resistivity of core material, the thickness of laminations
- ***B_{max}*** – maximum value of flux density in wb/m^2
- ***t*** – thickness of lamination in meters
- ***f*** – frequency of reversal of the magnetic field in Hz
- ***V*** – the volume of magnetic material in m^3

COPPER LOSS OR OHMIC LOSS

- These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and the secondary current. R_1 and R_2 are the resistance of primary and secondary winding then the copper losses occurring in the primary and secondary winding will be ***$I_1^2R_1$*** and ***$I_2^2R_2$*** respectively.

Therefore, the total copper losses will be

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} + I_2^2 R_{02}$$

- These **losses varied** according to the load and known hence it is also known as **variable losses**.
- Copper losses **vary** as the **square** of the load current.

Lecture 27 & 28

EFFICIENCY OF TRANSFORMER

$$\text{Efficiency}\,(\eta) = \frac{\text{Output Power}}{\text{Input Power}} \times 100 = \frac{\text{Output Power}}{\text{Output Power} + \text{Loss}} \times 100$$

Where,

$$\text{Output Power} = V_2 I_2 \cos\theta_2$$

$$\text{Loss} = \text{Iron loss} + \text{copper loss}$$

$$\text{Iron loss}\,(P_i) = \text{Hysteresis loss} + \text{Eddy current loss}$$

The efficiency of the transformer at a load x times full load will be

$$\eta = \frac{x V_2 I_2 \cos\theta_2}{x V_2 I_2 \cos\theta_2 + P_i + x^2 P_c}$$

MAXIMUM EFFICIENCY

The efficiency at a given load is given as :

$$\eta = \frac{xS\cos\phi}{(xS\cos\phi) + P_i + x^2 P_{cu}}$$

$$\eta = \frac{S\cos\phi}{(S\cos\phi) + \frac{P_i}{x} + xP_{cu}}$$

Now for maximum efficiency, denominator should be minimum;

$$\frac{d}{dx}(S\cos\phi + \frac{P_i}{x} + xP_{cu}) = 0$$

Hence finally $x^2 P_{cu} = P_i$; i.e ; copper loss = iron loss.

Also corresponding load for max. efficiency; $x = \sqrt{\frac{P_i}{P_{cu}}}$

Problem: The efficiency of a 400kVA, single phase transformer is 98.77% at full load 0.8 power factor lagging and 99.13% at half load unity power factor.

Find:

(i) Iron losses at half full load and at half load.

(ii) Cu loss at full load and at half load.

$$\eta_{F.L.} = \frac{(VA\,rating)\,\mathrm{Cos}\,\phi_2}{(VA\,rating)\,\mathrm{Cos}\,\phi_2 + P_i + (P_{cu})_{F.L.}}$$

$$0.9877 = \frac{400\ X\ 10^3\ X\ 0.8}{400 * 10^3 * 0.8 + P_i + (P_{cu})_{F.L.}}$$

Hence, $P_i + (P_{cu})_{F.L.} = 3985.01569$ (1)

$$\eta_{H.L.} = \frac{0.5(VA\,rating)\cos\phi_2}{0.5(VA\,rating)\cos\phi_2 + P_i + (0.5)^2 + (P_{cu})_{F.L.}}$$

Since, $(\eta = \frac{1}{2} * 0.5)$

$$0.9913 = \frac{0.5(400 * 10^3) * 1}{0.5(400 * 10^3) * \ 1 + P_i + 0.25(P_{cu})_{F.L.}}$$

Hence,

$$P_i + (P_{cu})_{F.L.} = 1755.27085 \quad (2)$$

On solving Eq. (1) and Eq. (2), we get,

$$P_i = 1012.022\, watt$$

$$(P_{cu})_{F.L.} = 2972.99\, watt$$

(i) Iron loss remain same for all load conditions

$$(P_i)_{F.L.} + (P_i)_{H.L.} = 1012.022\, watt$$

(i) $(P_{cu})_{F.L.} = 2972.99\, watt$

$$(P_{cu})_{H.L.} = \eta^2 (P_{cu})_{F.L.}$$

Since, ($\eta = \frac{1}{2} * 0.5$)

$$= \frac{1}{2} * 2972.99$$

$$(P_{cu})_{H.L.} = 743.2482\, watt$$

Problem: A 250 kVA single phase transformer has Iron loss of 1.8 kW. The full load copper loss is 2000 Watt. Calculate

(i) Efficiency at full load, 0.8 lagging P.F.

(ii) Efficiency at half load, 0.8 lagging P.F.

(iii) kVA supplied at maximum efficiency

(iv) % of load at which maximum efficiency occurs.

Solution:

Given: Iron loss = P_i = 1800 watt, Copper loss $(P_{cu})_{F.L.} = 2000\ watt$

(i)

$$\eta_{F.L.} = \frac{(VA\ rating)\ \text{Cos}\ \phi_2}{(VA\ rating)\ \text{Cos}\ \phi_2 + P_i + (P_{cu})_{F.L.}}$$

$$= \frac{250\ X\ 10^3\ X\ 0.8}{(250\ X\ 10^3\ X\ 0.8) + 1800 + 2000}$$

$$= 0.98135$$

$$\%\eta_{F.L.} = 98.135\ \%$$

(ii) $\eta = 0.5$

$$\eta_{H.L.} = \frac{0.5(VA\ rating)\cos\phi_2}{0.5(VA\ rating)\cos\phi_2 + P_i + \eta^2 (P_{cu})_{F.L.}}$$

$$\%\ \eta_{H.L.} = \frac{0.5(250\ X\ 10^3)\ X\ 0.8}{(250\ X\ 10^3\ X\ 0.8) + 1800 + \frac{1}{4}(2000)}\ X\ 100$$

$$\%\ \eta_{H.L.} = 97.75$$

(iii) $(kVA)_{\eta_{max}} = (kVA)rating * \sqrt{\frac{P_i}{(P_{cu})_{F.L.}}} = 250 * \sqrt{\frac{1800}{2000}}$

$$(kVA)_{\eta_{max}} = 237.1708\ KVA$$

(iv)

$$\eta_{max} = \frac{(KVA)_{\eta_{max}} \cos\phi_2}{(KVA)_{\eta_{max}} \cos\phi_2 + 2P_i}$$

$$= \frac{237.17\ X\ 10^3\ X\ 0.8}{(237.17\ X\ 10^3\ X\ 0.8) + 2\ \text{X}\ 1800}$$

$$= 0.98137$$

$$\%\ \eta_{max} = 98.137$$

(v)

$$Loading = \frac{(KVA)_{\eta_{max}}}{(KVA)} = \sqrt{\frac{P_i}{(P_{cu})_{F.L.}}} = \sqrt{\frac{1800}{2000}} = 0.9486$$

$$\%\ Loading = 94.86\ \%$$

Problem: In a 25 kVA, 2000/200 V, transformer, the iron and copper losses are 200 Watt and 400 Watt respectively. Calculate the efficiency at half-full load and 0.8 power factor lagging. Determine also the maximum efficiency and the corresponding load.

Solution :

$$\text{Full load Copper loss, } P_c = 400\,W = 0.4\,kW, \quad \text{Iron loss, } P_i = 200\,W = 0.2\,kW$$

$$\text{Output at half load and 0.8 P.F.} = \frac{1}{2} * 25 * 0.8 = 10\,kW$$

$$\text{Copper losses at half load} = \left(\frac{1}{2}\right)^2 * (0.4)\,kW = 0.1\,kW$$

$$\text{So, efficiency at half-full load and 0.8 P.F.}$$

$$\eta = \frac{P_{out}}{P_{in}} * 100 = \frac{10}{10 + 0.1 + 0.2} * 100 \Rightarrow 97.087\,\% \text{.......ANS.}$$

Load corresponding to maximum efficiency

$$\text{Rated kVA} * \sqrt{\frac{P_i}{P_c}} = 25 * \sqrt{\frac{0.2}{0.4}} \Rightarrow 17.68\ \text{kVA........ANS.}$$

Maximum efficiency, assuming power factor unity

$$\frac{17.68 * 10}{17.68 + 0.2 + 0.2} * 100 = 97.788\,\% \text{........ANS.}$$

Problem: A transformer is rated at 100kVA, at full load its copper loss, 1400 Watt and iron losses are 940 Watt. Calculate:

(i) The efficiency at full load, unity power factor.

(ii) The efficiency at half-full load, the same power factor and 0.8 power factor lagging.

(iii) The load kVA at which maximum efficiency will occur.

Solution:

Given: Copper loss at full load, P_c =1400 W =1.4 kW

(i) At full load unity power factor

Transformer output = Rated kVA $*$ $(\cos\phi)$ = 100 $*$1.0 = 100 kW

Transformer input = output+ P_i + P_c = 100 + 0.94 +1.4 = 102.34 kW

Transformer efficiency $\eta = \frac{Output}{Input} * 100$ = 97.714 %

(ii) At half load unity power factor

Transformer output = $\frac{1}{2} *$Rated kVA $*$ $(\cos\phi)$ = 100 $*$1.0 = $\frac{1}{2} *$100W

=50KW

Transformer input = output + iron loss + copper loss

$$= 50 + 0.94 + \left(\frac{1}{2}\right)^2 * 1.4$$

= 51.29 kW

Transformer efficiency $\eta = \frac{Output}{Input} * 100$ = 97.485 %

(iii) Load at which maximum efficiency will occur

$$= (kVA)_{rating} * \sqrt{\frac{P_i}{(P_{cu})}} = 100 * \sqrt{\frac{0.94}{1.4}} = 81.94 \ kVA$$

Questions?