Introduction to Mechanics

UNIT-1

SYLLABUS

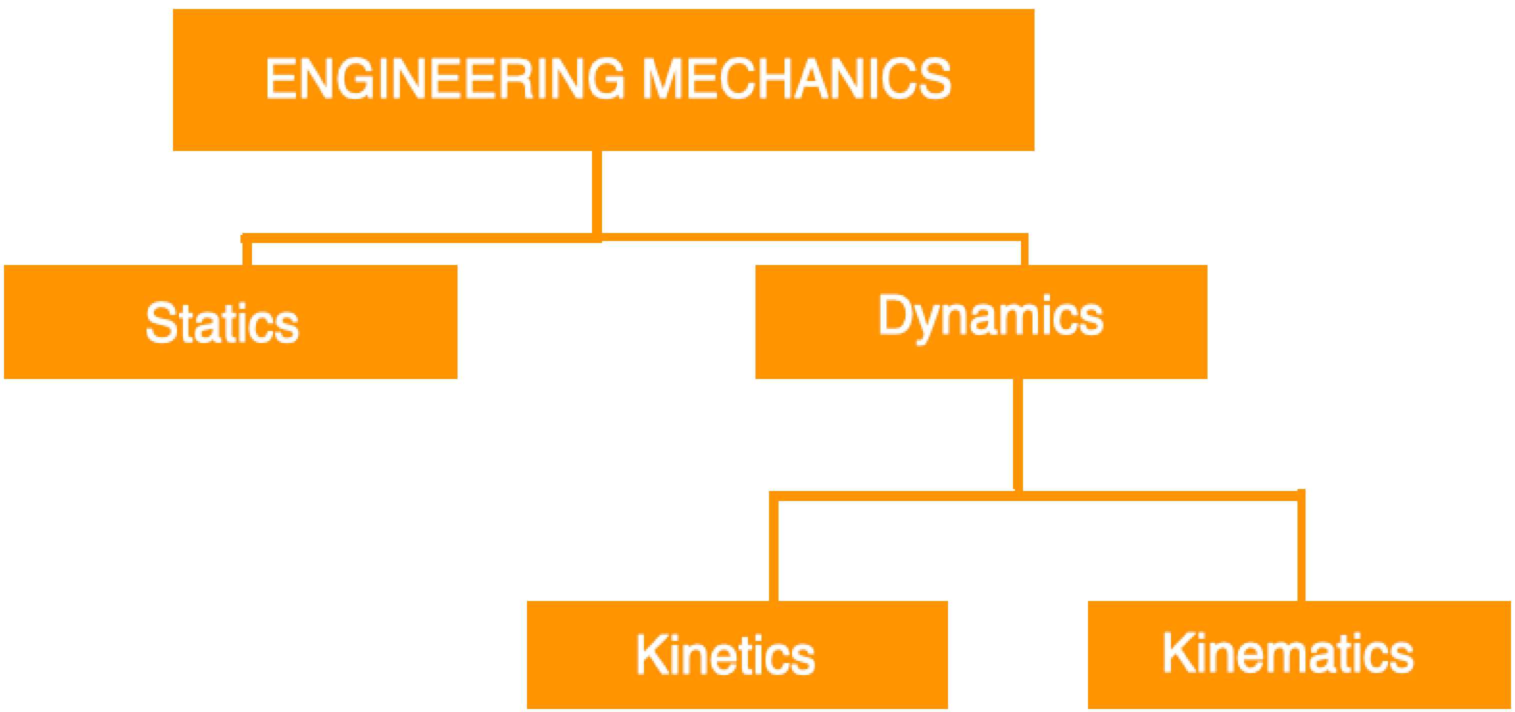
- Force moment and couple, principle of transmissibility, Varignon's theorem. Resultant of force system- concurrent and non-concurrent coplanar forces, Types of supports (Hinge, Roller) and loads (Point, UDL, UVL), free body diagram, equilibrium equations and Support Reactions.

- Normal and shear Stress, strain, Hooke's law, Poisson’s ratio, elastic constants and their relationship, stress-strain diagram for ductile and brittle materials, factor of safety.

Lecture No. 1

Engineering Mechanics

- The branch of science which deals with the forces and their effects on the bodies on which they act is called mechanics.
- Engineering Mechanics can be classified as-



Classification of Engg. Mechanics

1. **Statics**: The branch of applied mechanics which deals with the forces and their effects while acting upon bodies which are at rest is called statics.

2. **Dynamics**: The branch of applied mechanics which deals with the forces and their effects while acting upon bodies which are in motion is called dynamics.

It is further divided into two types:

- ***Kinematics:*** The branch of dynamics which deals with motion of bodies ***without*** considering the forces which cause motion is called kinematics.
- ***Kinetics:*** The branch of dynamics which deals with the relationship between motion of bodies and forces causing motion is called kinetics.

FORCE

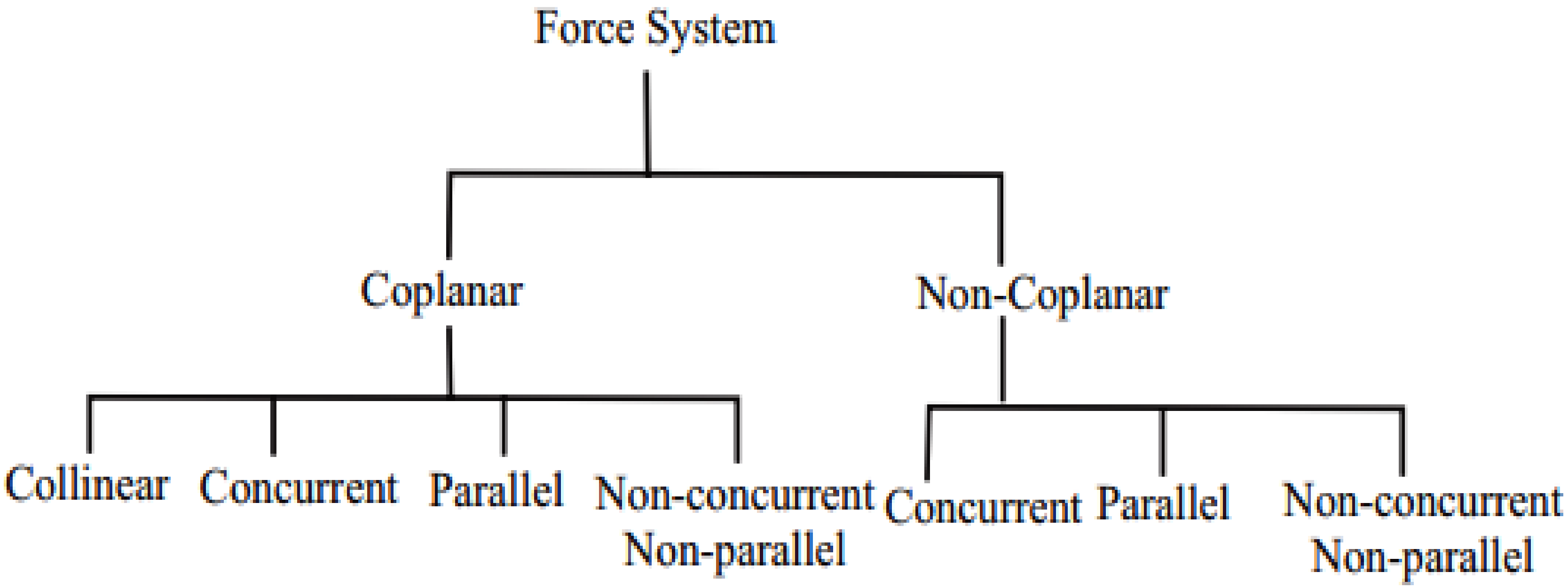
- Force may be defined as a push or pull which either changes or tend to change the state of rest or of uniform motion of a body.
- Force is a vector quantity.
- Unit- Newton (N)
- ***Characteristics of a force:***

The followings are the characteristics of a force:

1. **Magnitude:** The quantity of a force is called its magnitude such as 50 N, 80 N, 25 kg etc.
2. **Direction:** The direction of a force is the direction of the line along which it acts.
3. **Nature of the force or sense:** Nature of the force means whether the force is a push or a pull at the point of application.
4. **Point of application**

Force System

- A force system is a collection of several forces acting simultaneously on a body in one or more plane.
- Classification of Forces System:



Force System

- Coplanar forces: The forces, whose lines of action lie on the same plane.
- Non-coplanar Forces: The forces, whose lines of action lie on the different plane.
- Collinear forces: The forces acting along the same line of action.
- Non-collinear forces: The forces acting along the different lines of action.
- Parallel Forces: The forces line of action are parallel to each other is called parallel forces.

Force System

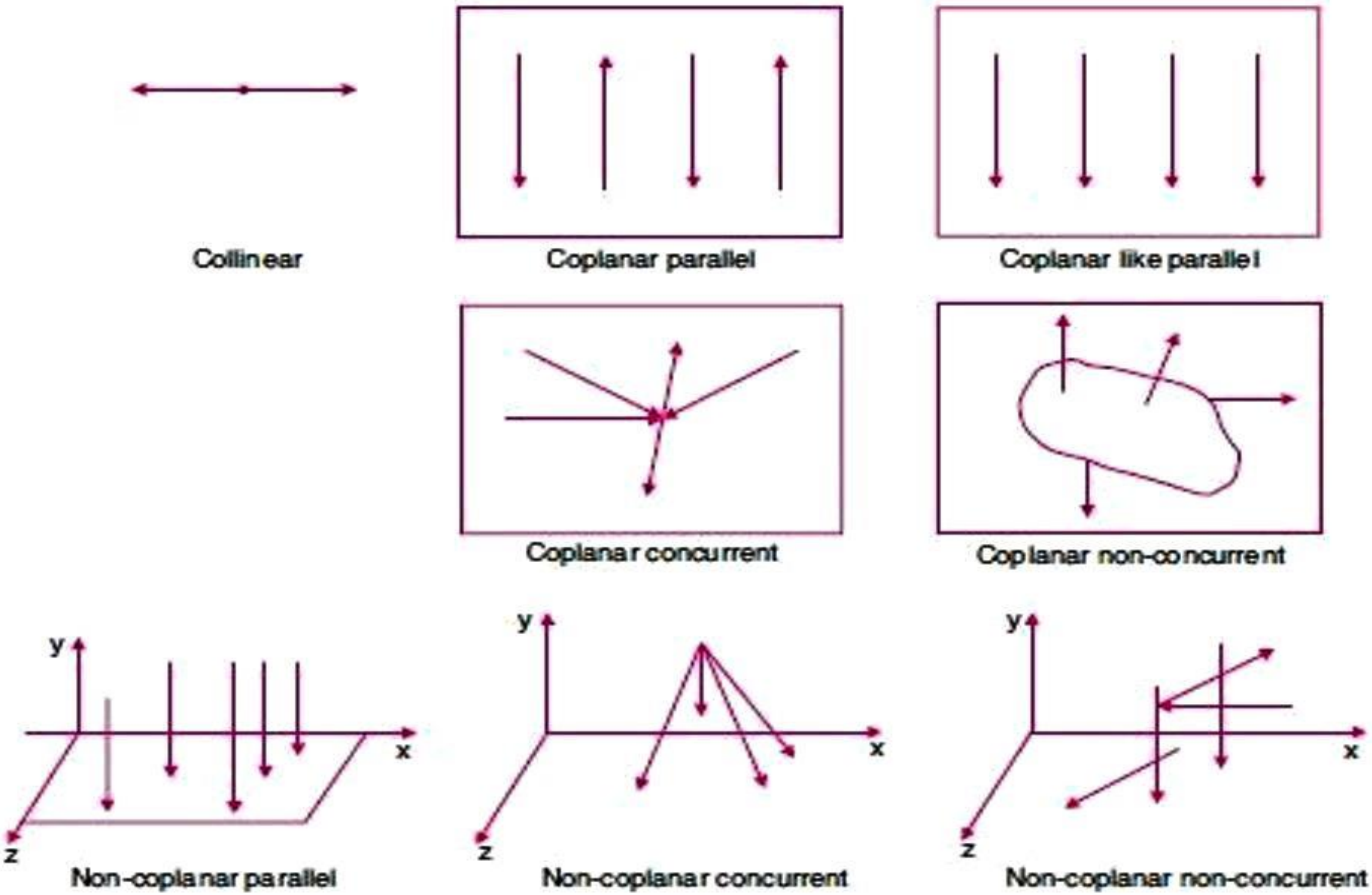
- Concurrent forces: If the forces applied to a body are such that their lines of action meet at a single point, then they are called ***concurrent forces***.

- Non-Concurrent forces: When the forces acting on a body do not intersect at a common point, the system of forces is said to be ***non- concurrent forces.***

Force System

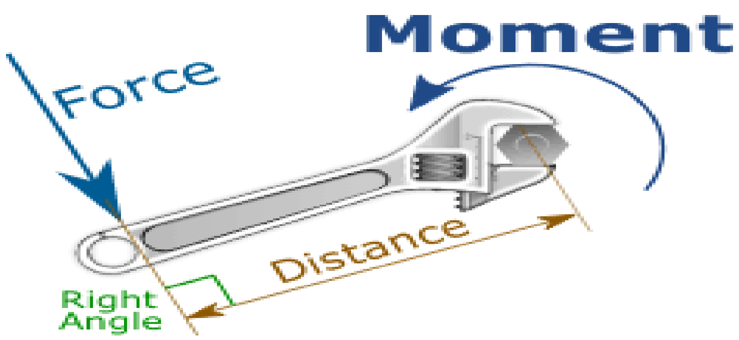
Force System	*Characteristic*	*Examples*
Collinear forces	Line of action of all the forces act along the same line.	Forces on a rope in a tug of war
Coplanar parallel forces	All forces are parallel to each other and lie in a single plane.	System of forces acting on a beam subjected to vertical loads (including reactions)
Coplanar like parallel forces	All forces are parallel to each other, lie in a single plane and are acting in the same direction.	Weight of a stationary train on a rail when the track is straight
Coplanar concurrent forces	Line of action of all forces pass through a single point and forces lie in the same plane.	Forces on a rod resting against a wall
Coplanar non-concurrent forces	All forces do not meet at a point, but lie in a single plane.	Forces on a ladder resting against a wall when a person stands on a rung which is not at its centre of gravity
Non-coplanar parallel forces	All the forces are parallel to each other, but not in the same plane.	The weight of benches in a class room
Non-coplanar concurrent forces	All forces do not lie in the same plane, but their lines of action pass through a single point.	A tripod carrying a camera
Non-coplanar non-concurrent forces	All forces do not lie in the same plane and their lines of action do not pass through a single point.	Forces acting on a moving bus

Force System



Moment of Force

- The tendency of forces is not only to move the body but also to rotate the body. *This rotational tendency of a force is called* ***moment.***
- ***The force multiplied by the*** *perpendicular distance from the point to the line of action of the force is called moment about that point.*
- *Unit of moment* **N-m.**

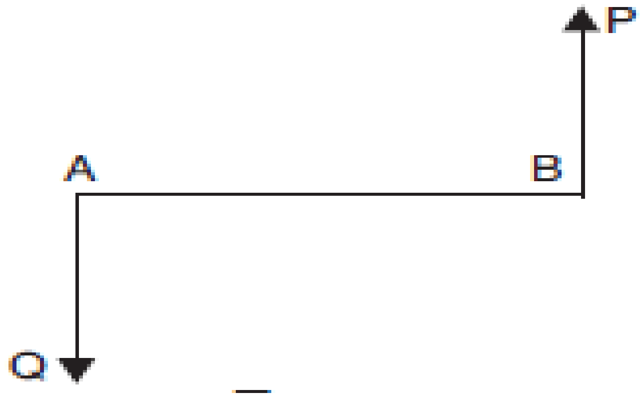


$$\text{Moment of force} = \text{force} \times \text{perpendicular distance}$$
$$= P \times OA$$
$$= P \times l$$

Moment of Couple

- A couple is pair of two equal and opposite forces acting on a body in a such a way that the lines of action of the two forces are not in the same straight line.

- The moment of a couple is known as torque which is equal to one of the forces forming the couple multiplied by arm of the couple.



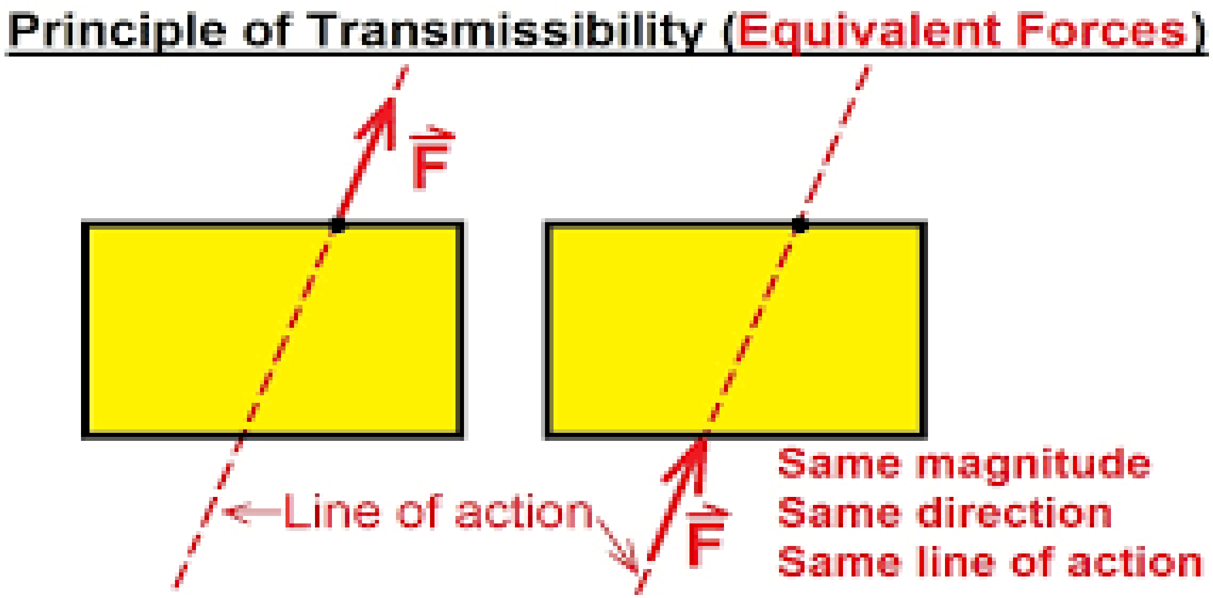
Moment of couple

The following are the examples of couples in every day life.

1. Opening or closing a water tap. The two forces constitute a couple.
2. Turning the cap of a pen.
3. Unscrewing the cap of an ink bottle.
4. Twisting a screw driver.
5. Steering a motor-car.

Principal of transmissibility of forces

It states that the point of application of a force may be changed to any other point along the line of action of the force without any change in the external effects produced by the force.



Law of superposition of forces

"The **principle of superposition of forces** states that the resultant force has the same effect as the sum of the individual forces acting on an object."

Lecture No. 2

Varignon's Theorem

The Principle of Moments, also known as Varignon's Theorem, states that the moment of any force is equal to the algebraic sum of the moments of the components of that force.

OR

If many coplanar forces are acting on a body, then algebraic sum of moments of all the forces about a point in the plane of the forces is equal to the moment of their resultant about the same point.

Varignon's Theorem Proof

The algebraic sum of the moments of a system of coplanar forces about a moment centre in their plane is equal to the moment of their resultant force about the same moment centre.

Proof: Referring to Fig. 2.6 let R be the resultant of forces F_1 and F_2 and B the moment centre. Let d, d_1 and d_2 be the moment arms of the forces, R, F_1 and F_2, respectively from the moment centre B. Then in this case, we have to prove that:

$$Rd = F_1 d_1 + F_2 d_2$$

Join AB and consider it as y axis and draw x axis at right angles to it at A [Fig. 2.6(b)]. Denoting by θ the angle that R makes with x axis and noting that the same angle is fomed by perpendicular to R at B with AB_1, we can write:

$$Rd = R \times AB \cos\theta$$

$$= AB \times (R \cos\theta)$$

$$= AB \times R_x \qquad ...(a)$$

where R_x denotes the component of R in x direction.

Varignon's Theorem Proof

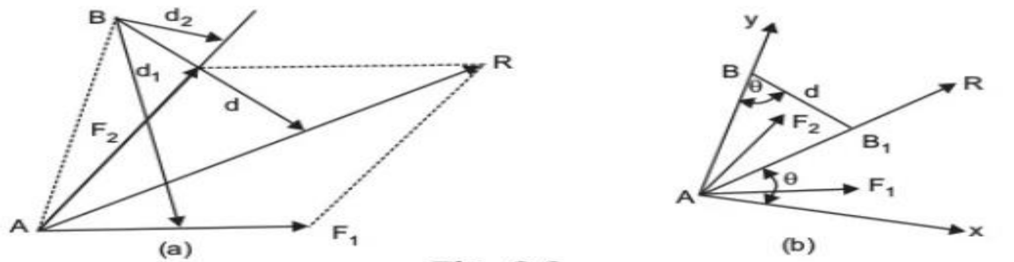


Fig. 2.6

Similarly, if F_{1x} and F_{2x} are the components of F_1 and F_2, in x direction, respectively, then

$$F_1\,d_1 = AB\,F_{1x} \quad \text{...}(b)$$

and

$$F_2\,d_2 = AB\,F_{2x} \quad \text{...}(c)$$

From Eqns. (b) and (c)

$$F_1\,d_1 + F_2\,d_2 = AB\,(F_{1x} + F_{2x})$$
$$= AB \times R_x \quad \text{...}(d)$$

From equation (a) and (d), we get

$$Rd = F_1\,d_1 + F_2\,d_2$$

If a system of forces consists of more than two forces, the above result can be extended as given below:

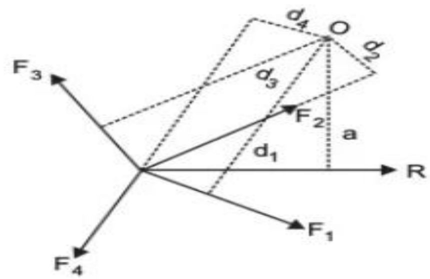
Let F_1, F_2, F_3 and F_4 be four concurrent forces and R be their resultant. Let d_1, d_2, d_3, d_4 and a be the distances of line of action of forces F_1, F_2, F_3, F_4 and R, respectively from the moment centre O, [Ref. Fig 2.7].

If R_1 is the resultant of F_1 and F_2 and its distance from O is a_1, then applying Varignon's theorem:

$$R_1\,a_1 = F_1\,d_1 + F_2\,d_2$$

If R_2 is the resultant of R_1 and F_3 (and hence of F_1, F_2 and F_3) and its distance from O is a_2, then applying Varignon's theorem:

$$R_2 a_2 = R_1 a_1 + F_3 d_3$$
$$= F_1\,d_1 + F_2\,d_2 + F_3\,d_3$$



Varignon's Theorem Proof

Now considering R_2 and F_4, we can write:

$$Ra = R_2 a_2 + F_4 d_4$$

Since R is the resultant of R_2 and F_4 (i.e. F_1, F_2, F_3 and F_4).

$$\therefore \quad Ra = F_1d_1 + F_2d_2 + F_3d_3 + F_4d_4 \quad \text{...(2.5)}$$

Thus, the moment of the resultant of a number of forces about a moment centre is equal to the sum of the moments of its component forces about the same moment centre.

Varignon's Theorem Problem

Example 2. *Find the moment of 100 N force acting at B about point A as shown in Fig. 2.8.*

Solution: 100 N force may be resolved into its horizontal components as 100 cos 60° and vertical component 100 sin 60°. From Varignon's theorem, moment of 100 N force about the point A is equal to sum of the moments of its components about A.

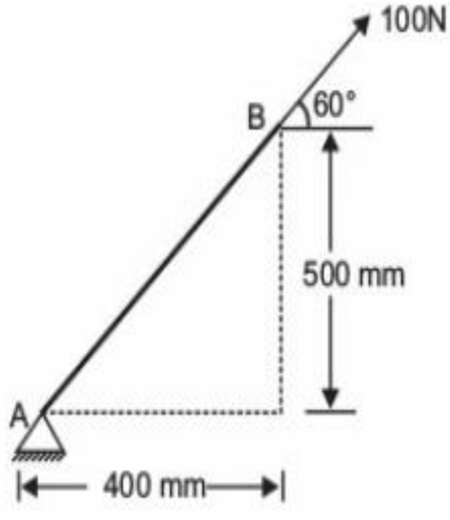


Fig. 2.8

Taking clockwise moment as positive,

$$
\begin{aligned}
M_A &= 100 \cos 60° \times 500 - 100 \sin 60° \times 400 \\
&= 25,000 - 34,641.02 \\
&= -9641.02 \text{ N-mm} \\
&= \mathbf{9641.016 \text{ N-mm Anticlockwise.}}
\end{aligned}
$$

Resultant of Forces

- A resultant force is a single force which can replace two or more forces and produce the same effect on the body as the forces.
- **LAWS OF FORCES**

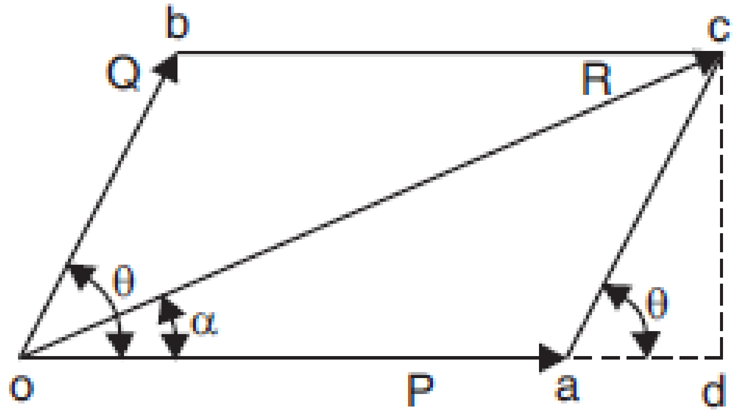
  The method of determination of the resultant of some forces acting simultaneously on a particle called composition of forces.

The various laws used for the composition of forces are given as under :

1. Parallelogram law of forces,
2. Triangle law of forces,
3. Polygon law of forces.

Parallelogram law of forces

- If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant may be represented in magnitude and direction by the diagonal of the parallelogram which passes through their point of intersection.”
- Let two forces P and Q acting simultaneously on a particle be represented in magnitude and direction by the adjacent sides oa and ob of a parallelogram oacb drawn from a point o, their resultant R will be represented in magnitude and direction by the diagonal oc of the parallelogram.



Parallelogram law of forces

The value of R can be determined either graphically or analytically as explained below :

As shown in Fig. in the parallelogram oacb, from c drop a perpendicular cd to oa at d when produced. Now from the geometry of the figure

$$\angle cad = \theta,\ ac = Q$$

$$\therefore \quad cd = Q \sin \theta$$

$$ad = Q \cos \theta$$

From right-angled triangle, *odc*

$$oc = \sqrt{(od)^2 + (cd)^2}$$

$$= \sqrt{(oa + ad)^2 + (cd)^2}$$

or

$$R = \sqrt{(P + Q \cos\theta)^2 + (Q \sin \theta)^2}$$

$$= \sqrt{P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta}$$

$$= \sqrt{P^2 + Q^2 (\sin^2 \theta + \cos^2 \theta) + 2PQ \cos \theta}$$

$$= \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\therefore \quad R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Let the resultant makes an angle α with P as shown in figure.

Then

$$\tan \alpha = \frac{cd}{od} = \frac{cd}{oa + ad}$$

$$= \frac{Q \sin \theta}{P + Q \cos \theta}$$

Parallelogram law of forces

Case 1. If $\theta = 0°$, *i.e.*, when the forces P and Q act along the same straight line then quation (2.3) reduces to

$$R = P + Q \qquad (\because \cos 0° = 1)$$

Case 2. If $\theta = 90°$, *i.e.*, when the forces P and Q act at right angles to each other, then

$$R = \sqrt{P^2 + Q^2} \qquad (\because \cos 90° = 0)$$

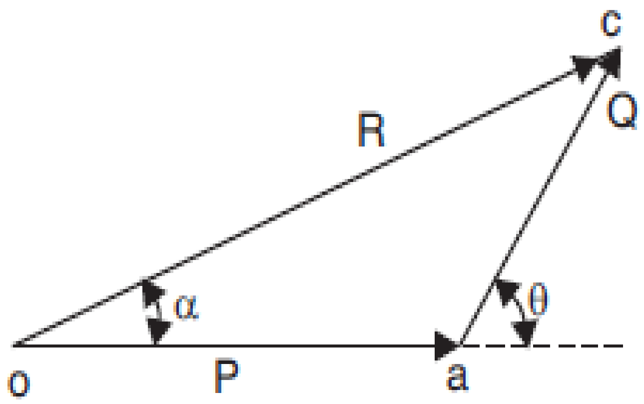
Case 3. If $\theta = 180°$, *i.e.*, the forces P and Q act along the same straight line but in opposite lirections, then

$$R = P - Q \qquad (\because \cos 180° = -1)$$

The resultant will act in the direction of the greater force.

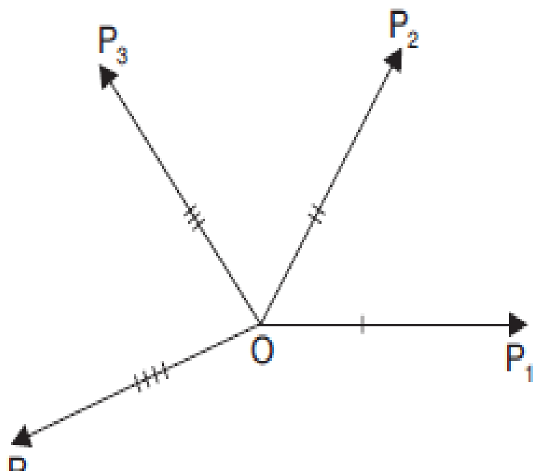
Triangle law of forces

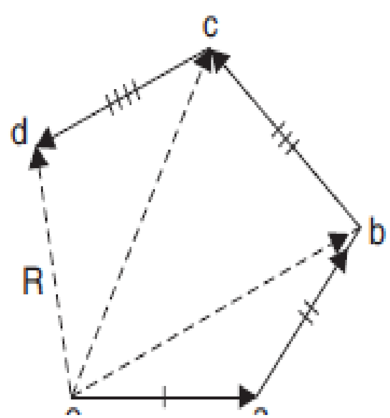
- "If two forces acting simultaneously on a body are represented in magnitude and direction by the two sides of triangle taken in order then their resultant may be represented in magnitude and direction by the third side taken in opposite order."
- Let *P and Q be the two coplanar concurrent forces. The resultant force R in this case can be obtained with the help of the* triangle law of forces both graphically and analytically as given below :



Polygon law of forces

- "If a number of coplanar concurrent forces, acting simultaneously on a body are represented in magnitude and direction by the sides of a polygon taken in order, then their resultant may be represented in magnitude and direction by the closing side of a polygon, taken in the opposite order".
- If the forces P1, P2, P3, and P4 acting simultaneously on a particle be represented in magnitude and direction by the sides oa, ab, bc and cd of a polygon respectively, their resultant is represented by the closing side do in the opposite direction as shown in Fig.



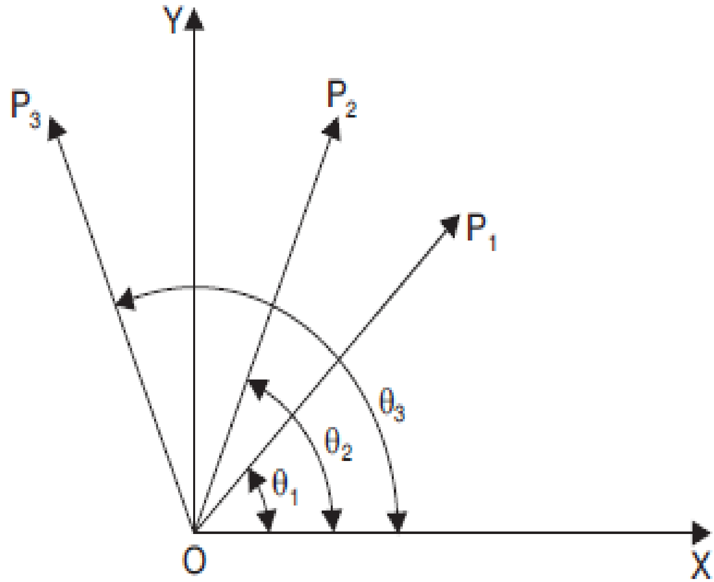


RESULTANT OF SEVERAL COPLANAR CONCURRENT FORCES

To determine the resultant of a number of coplanar concurrent forces any of the following two methods may be used :

1. Graphical method (Polygon law of forces),
2. Analytical method (Principle of resolved parts).

Resultant by analytical method



Resultant by analytical method

The resolved parts in the direction OX and OY of

P_1 are $P_1 \cos \theta_1$ and $P_1 \sin \theta_1$, respectively,

P_2 are $P_2 \cos \theta_2$ and $P_2 \sin \theta_2$ respectively

and P_3 and $P_3 \cos \theta_3$ and $P_3 \sin \theta_3$ respectively.

If the resultant R makes an angle θ with OX then by the principle of resolved parts :

$$R \cos \theta = P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3$$

$$= \Sigma H \qquad \ldots(i)$$

and

$$R \sin \theta = P_1 \sin \theta_1 + P_2 \sin \theta_2 + P_3 \sin \theta_3$$

$$= \Sigma V \qquad \ldots(ii)$$

Now, by squaring and adding eqns. (i) and (ii), we get

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

and by dividing eqn. (ii) by eqn. (i), we get

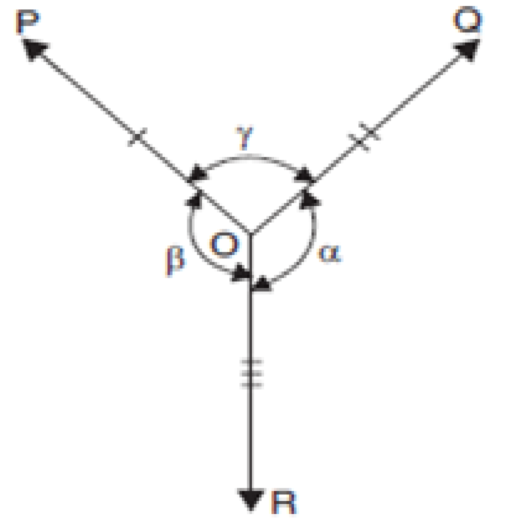
$$\frac{R \sin \theta}{R \cos \theta} = \tan \theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

LAMI'S THEOREM

It states that "If three coplanar forces acting on a point in a body keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces."

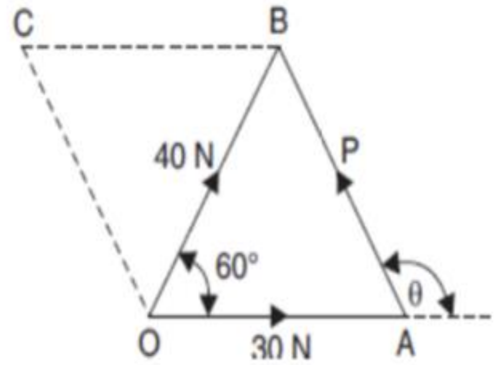
Fig. shows three forces P, Q and R acting at a point O. Let the angle between P and Q be α, between Q and R be β and between R and P be γ . If these forces are in equilibrium then according to Lam's theorem-



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Numerical

Example 2.. The resultant of two forces P and 30 N is 40 N inclined at 60° to the 30 N force. Find the magnitude and direction of *P*.



From the knowledge of trigonometry, we know that in ΔOAB

From the knowledge of trigonometry, we know that in ΔOAB,

$$AB^2 = OA^2 + OB^2 - 2.OA \,.\, OB \cos 60°$$

$$P^2 = (30)^2 + (40)^2 - 2 \times 30 \times 40 \cos 60°$$

$$P = \sqrt{900 + 1600 - 2400 \times 0.5}$$

$$= \mathbf{36.06\ N.\ (Ans.)}$$

Now applying sine rule,

$$\frac{P}{\sin 60°} = \frac{40}{\sin (180° - \theta)}$$

$$\frac{36.06}{0.866} = \frac{40}{\sin (180° - \theta)}$$

$$\sin (180° - \theta) = \frac{40 \times 0.866}{36.06} = 0.96$$

$$180 - \theta = 73.74° \quad \text{or} \quad 73° \, 44'$$

$$= \mathbf{106° \, 16'.\ (Ans.)}$$

Lecture No. 3

Types of Supports

When a number of forces are acting on a beam then support of the beam will provide the reactions called Support Reactions.

- **Following are types of supports:**

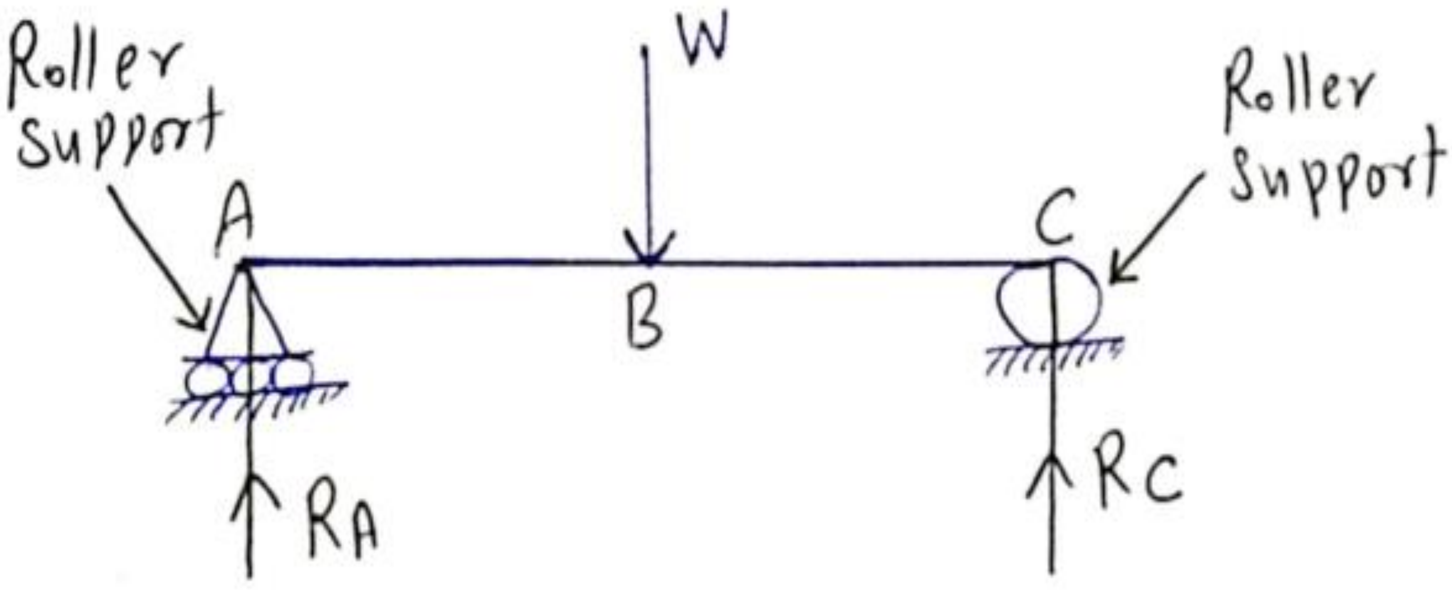
1. **Roller support**

2. **Hinge(Pin) Support**

3. **Fixed (Built-in) Support**

1. Roller support

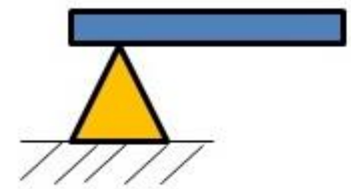
- Axial motion is permitted
- Rotation is permitted
- Only vertical motion is restricted
- No of reaction = 1, R_V



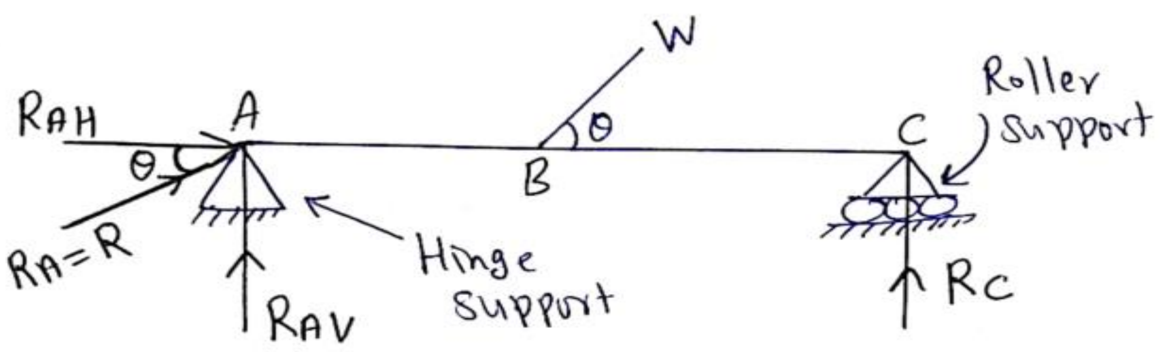
Roller support real life example

2. Pin(Hinged) support

- Vertical motion is restricted
- Axial motion is restricted
- Rotation is permitted
- No of reaction = 2, R_{AV} and R_{AH}



Pinned Support



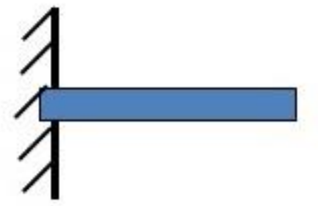
$$R_A = R = \sqrt{R_{AV}^2 + R_{AH}^2}$$

$$\tan\theta = \frac{R_{AV}}{R_{AH}}$$

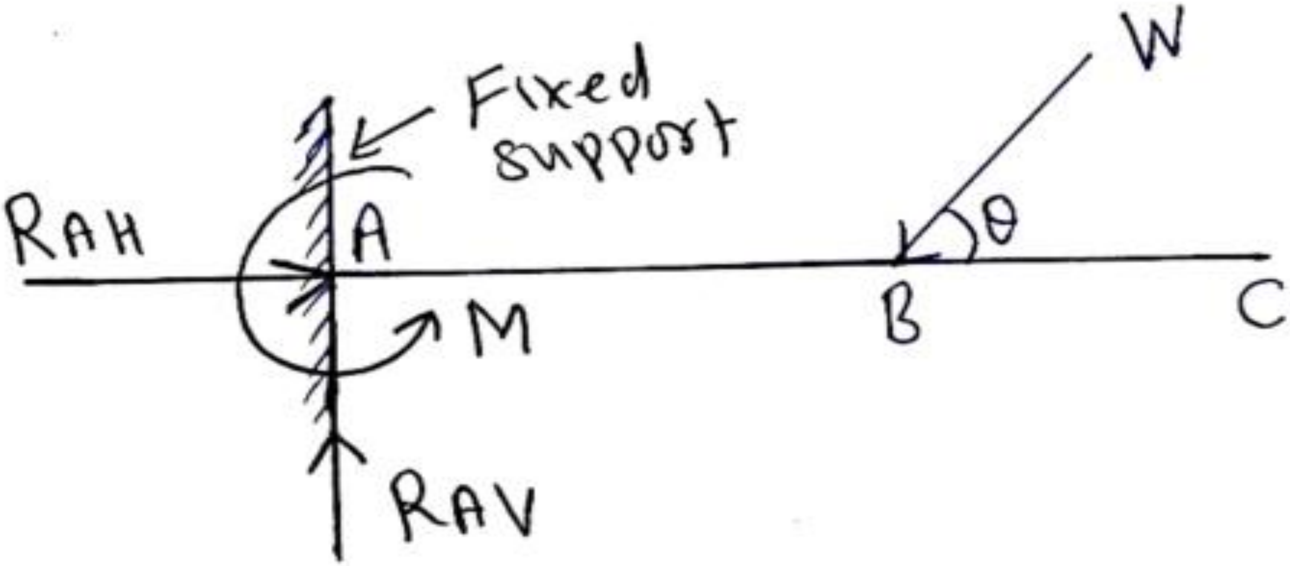
Pin support real life example

3. Fixed support

- Vertical motion is restricted.
- Axial motion is restricted.
- Rotation is restricted.
- No of reaction = 3,
- R_{AV}, R_{AH} and M_z



Fixed Support

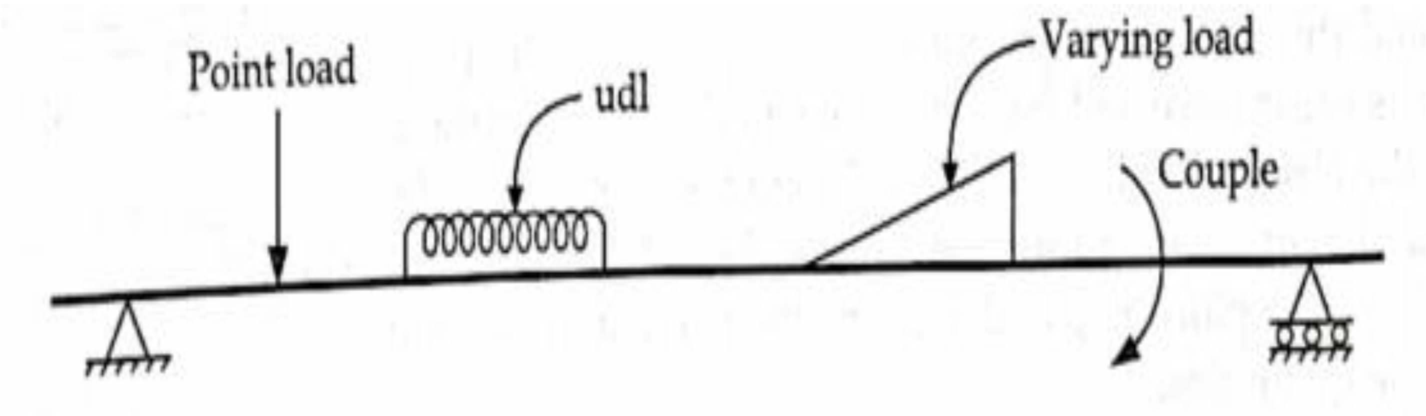


Fixed support real life example

Type of Loading on the beam

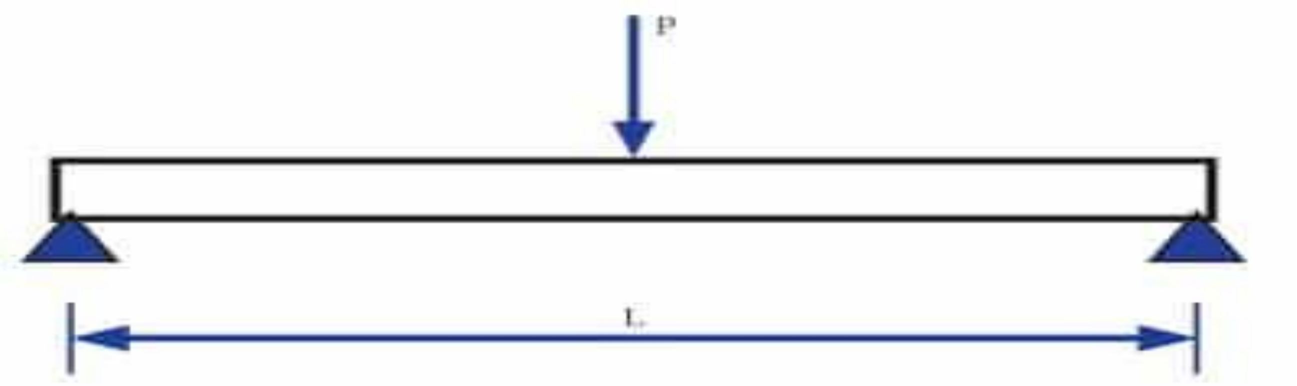
Following are the types of load which are subjected to a beam

1. Concentrated load(point load)
2. Uniformly Distributed load(UDL)
3. Uniformly Varying load(UVL)
4. Moment (M)



1. Concentrated load(point load)

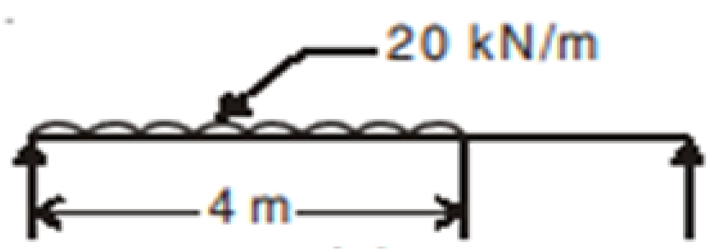
- This load acts at a point.
- It is represented by an arrow as shown in Fig.

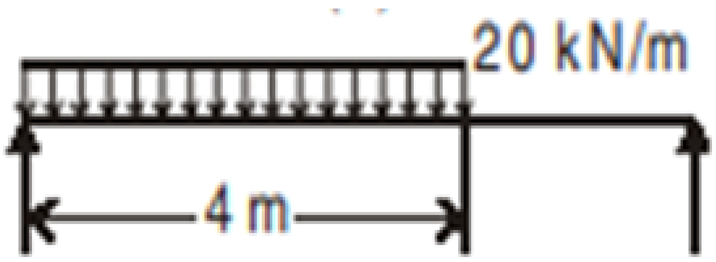


2. Uniformly Distributed load(UDL)

- Load acts over a certain length
- Intensity of load is uniform
- It is represented as shown in Fig.
- Total load = Area of plane fig (rectangle)
- Total load acts at middle of the loaded length.

- Given load may be replaced by a 20×4 = 80 kN, concentrated load acting at a distance 2 m from the left support.

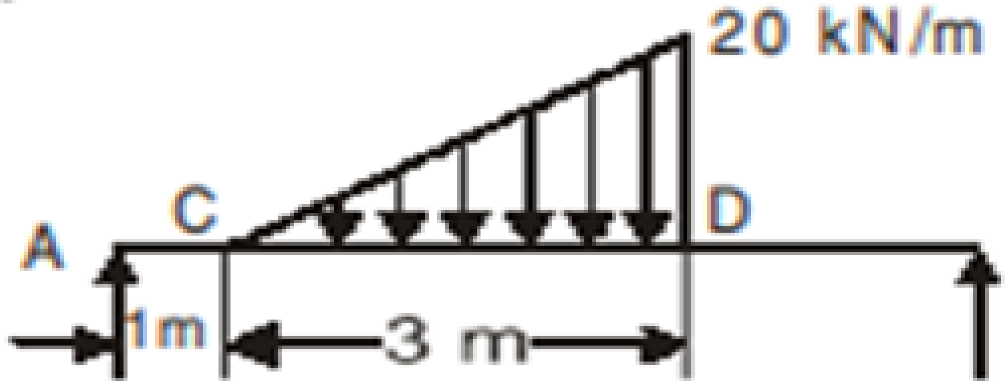




3. Uniformly Varying load(UVL)

- The load varies Uniformly from C to D.
- Total load = Area of plane fig (triangle)
- Centroid of the triangle represents the center of gravity of the load. (1/3rd from **D** or 2/3rd from **C**)

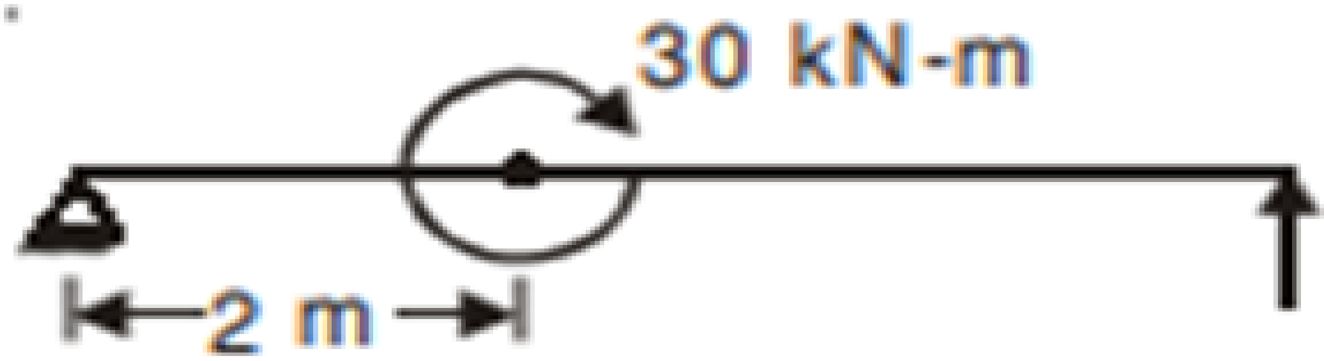
- E.g.
- Its intensity is zero at C and is 20 kN/m (maximum) at D.
- Total load is 1 /2 × 3 × 20 = **30 kN**
- This load is equivalent to 30 kN acting at 3 m from A.



4. Moment

- A beam may be subjected to external moment at certain points.

- In Fig. the beam is subjected to clockwise moment of 30 kN-m at a distance of 2 m from the left support.



Equilibrium Equations

- The body is said to be in equilibrium if the resultant of all forces acting on it is zero.
- There are two major types of static equilibrium, namely, translational equilibrium and rotational equilibrium.

Concurrent force system

$\Sigma Fx=0$

$\Sigma Fy=0$

Parallel Force System

$\Sigma F=0$

$\Sigma M=0$

Non-Concurrent Non-Parallel Force System

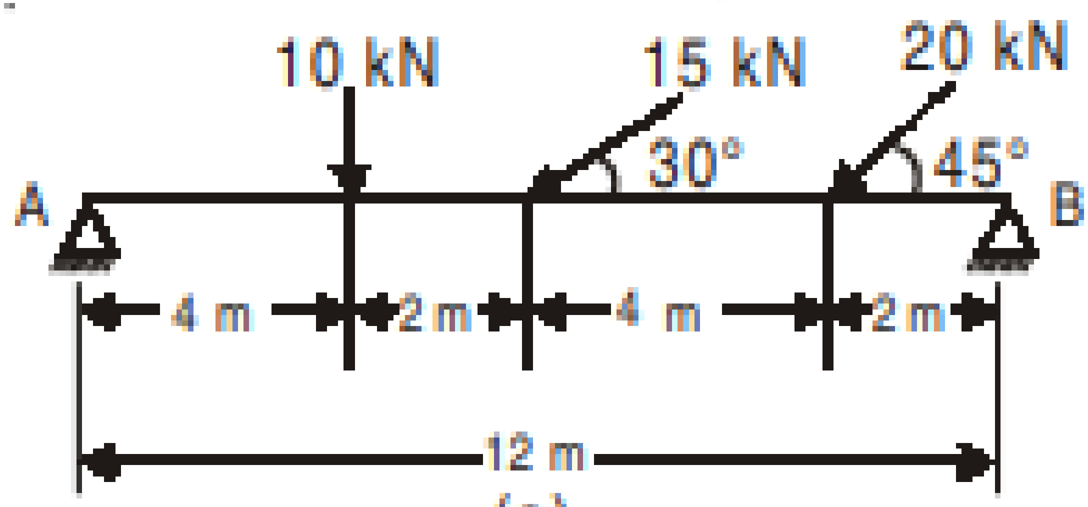
$\Sigma Fx=0$

$\Sigma Fy=0$

$\Sigma M=0$

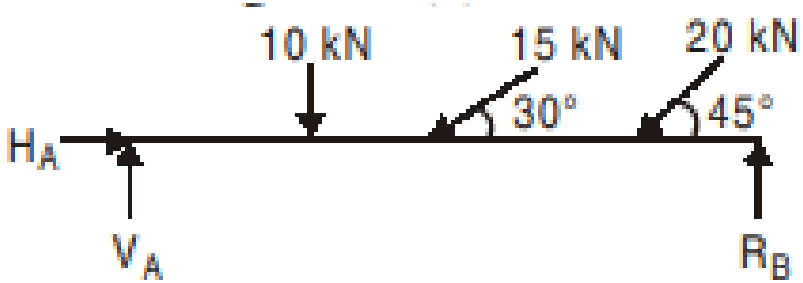
Example - 1

The beam AB of span 12 m shown in Fig. is hinged at A and is on rollers at B. Determine the reactions at A and B for the loading shown in the Fig.



(a)

Solution:



(b)

Now, $\sum H = 0$, gives

$$H_A - 15 \cos 30° - 20 \cos 45° = 0$$

$$H_A = 27.1325 \text{ kN}$$

[**Note:** For finding moments, inclined loads are split into their vertical and horizontal components. Horizontal components do not produce moment about *A*.]

$\sum M_A = 0$, gives

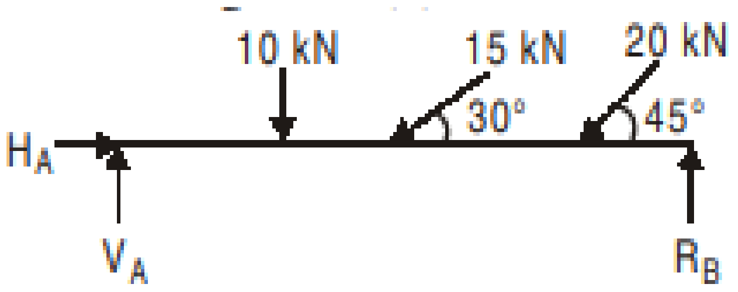
$$R_B \times 12 - 10 \times 4 - 15 \sin 30° \times 6 - 20 \sin 45° \times 10 = 0$$

$$\mathbf{R_B = 18.8684 \text{ kN.}}$$

$$\sum V = 0, \text{ gives}$$

$$V_A + 18.8684 - 10 - 15 \sin 30° - 20 \sin 45° = 0$$

$$V_A = 12.7737 \text{ kN}$$

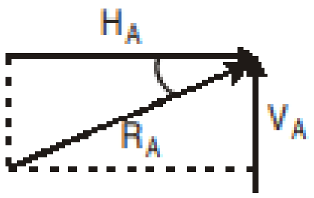


(b)

$$R_A = \sqrt{H_A^2 + V_A^2} = \sqrt{27.1325^2 + 12.7737^2}$$

$$\mathbf{R_A = 29.989 \text{ kN.}}$$

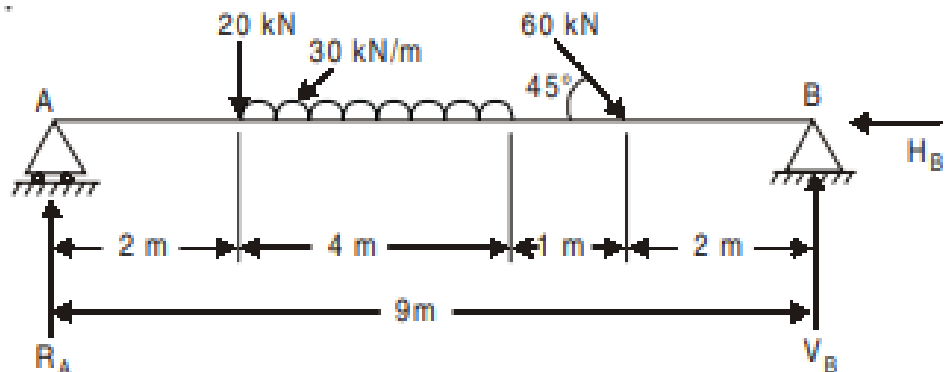
$$\alpha = \tan^{-1}\frac{(12.7737)}{(27.1325)} = \mathbf{25.21°.}$$



(c)

Example - 2

Find the reactions at supports A and B of the loaded beam shown in Fig.



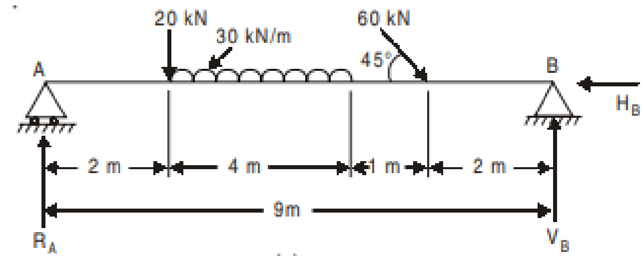
(a)

Solution: The reaction at A is vertical. Let H_B and V_B be the components of the reaction at B.

$$\sum M_B = 0, \text{ gives}$$

$$R_A \times 9 - 20 \times 7 - 30 \times 4 \times 5 - 60 \sin 45^\circ \times 2 = 0$$

$$\therefore \quad \mathbf{R_A = 91.6503\ kN.}$$

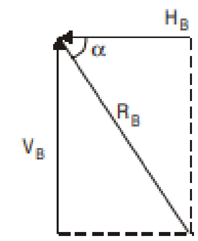


(a)

$$\sum H_A = 0, \text{ gives}$$

$$H_B - 60 \cos 45^\circ = 0$$

$$\mathbf{H_B = 42.4264\ kN.}$$



(b)

$$\sum V_A = 0$$

$$91.6503 + V_B - 20 - 30 \times 4 - 60 \sin 45^\circ = 0$$

$$\mathbf{V_B = 90.7761\ kN.}$$

$$R_B = \sqrt{42.4264^2 + 90.7761^2}$$

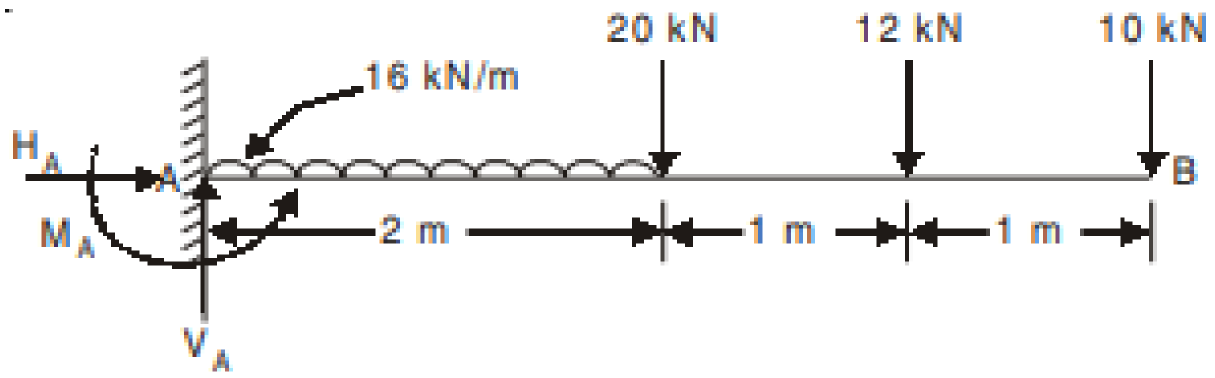
$$\mathbf{R_B = 100.2013\ kN.}$$

$$\alpha = \tan^{-1}\frac{90.7761}{42.4264}$$

$\boldsymbol{\alpha} = \mathbf{64.95^\circ}$, as shown in Fig.

Example - 3

The cantilever shown in Fig. is fixed at A and is free at B. Determine the reactions when it is loaded as shown in the Figure.

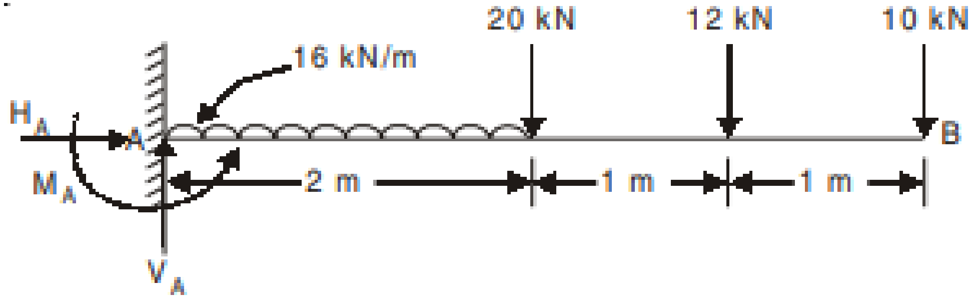


Solution

Let the reactions at A be H_A, V_A and M_A as shown in the figure

$$\Sigma H = 0, \text{ gives}$$

$$\boldsymbol{H_A = 0.}$$



$$\Sigma V = 0, \text{ gives}$$

$$V_A - 16 \times 2 - 20 - 12 - 10 = 0$$

$$\boldsymbol{V_A = 74 \text{ kN.}}$$

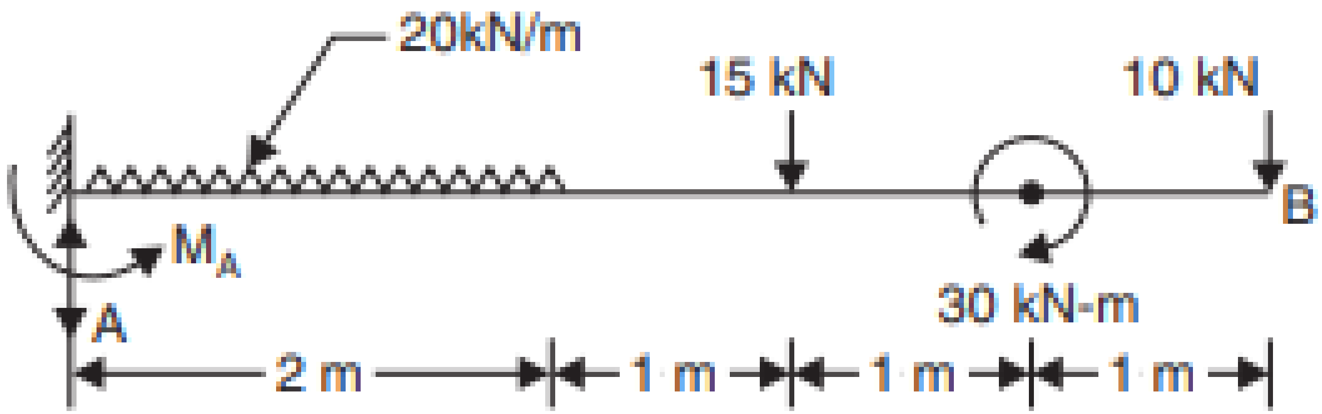
$$\Sigma M = 0, \text{ gives}$$

$$M_A - 16 \times 2 \times 1 - 20 \times 2 - 12 \times 3 - 10 \times 4 = 0$$

$$\boldsymbol{M_A = 148 \text{ kN-m.}}$$

Example - 4

Compute the reaction developed at support in the cantilever beam shown in Fig.

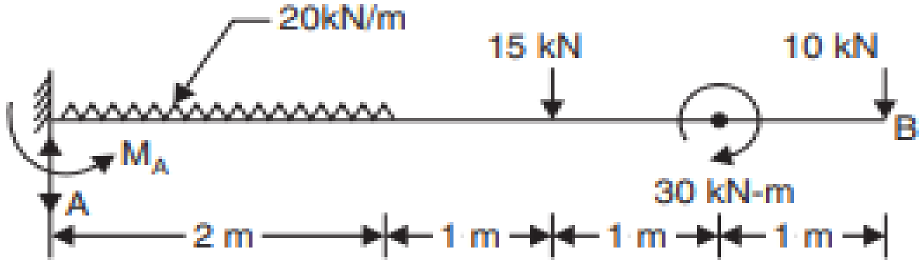


Solution: Let the vertical reaction be V_A and moment be M_A. There is no horizontal component of reactions, since no load is having horizontal component

$$\Sigma V = 0, \text{ gives}$$

$$V_A - 20 \times 2 - 15 \times 10 = 0$$

$$\mathbf{V_A = 65\ kN.}$$



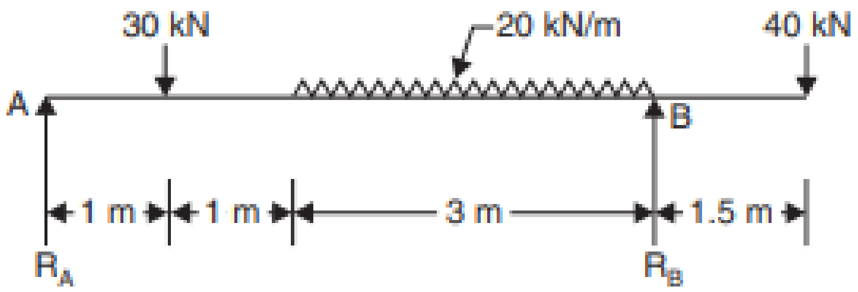
$$\Sigma M = 0, \text{ gives}$$

$$M_A - 20 \times 2 \times 1 - 15 \times 3 - 30 - 10 \times 5 = 0$$

$$\mathbf{M_A = 165\ kN\text{-}m}$$

Example - 5

Determine the reactions at supports A and B of the overhanging beam shown in Fig



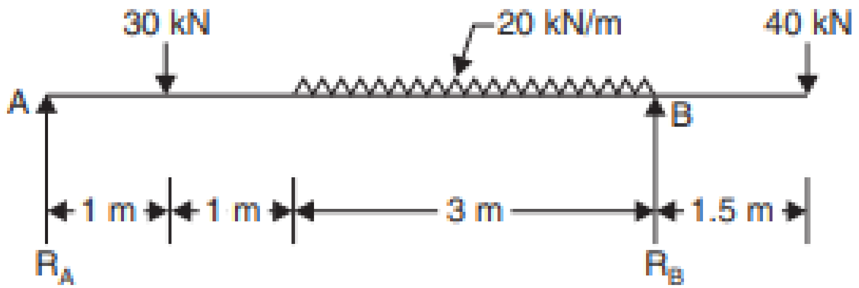
Solution:

As supports A and B are simple supports and loading is only in vertical direction, the reactions R_A and R_B are in vertical directions only

$$\Sigma M_A = 0, \text{ gives}$$

$$R_B \times 5 - 30 \times 1 - 20 \times 3 \times (2 + 1.5) - 40 \times 6.5 = 0$$

$$\mathbf{R_B = 100 \text{ kN.}}$$



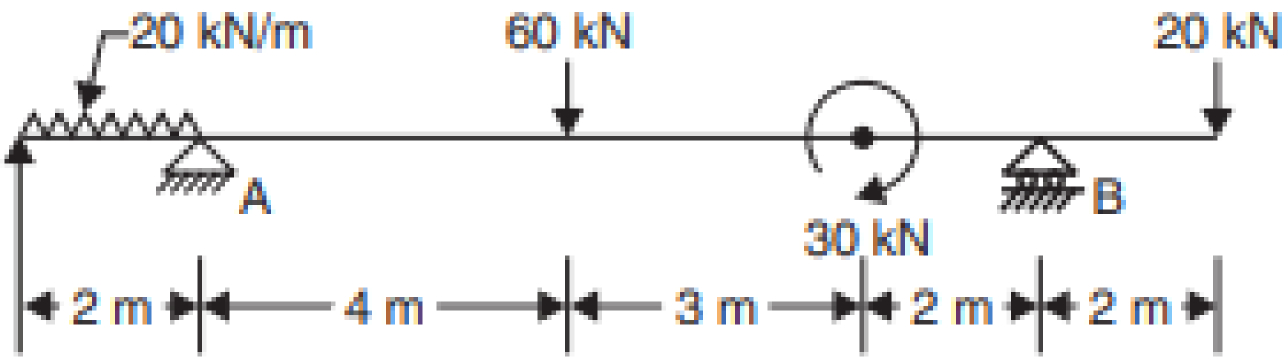
$$\Sigma V = 0, \text{ gives}$$

$$R_A + R_B - 30 - 20 \times 3 - 40 = 0$$

$$R_A = 130 - R_B = 130 - 100 = \mathbf{30 \text{ kN.}}$$

Example - 6

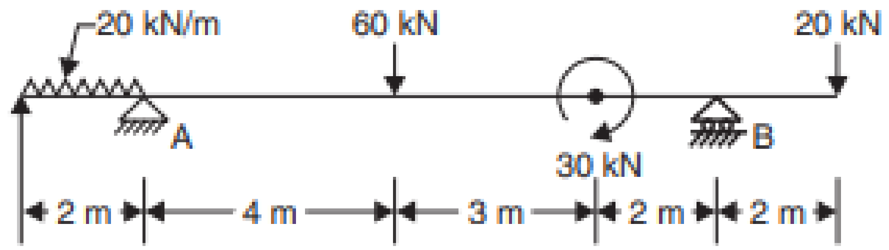
Find the reactions at supports A and B of the beam as shown in Fig.



Solution: Let V_A and H_A be the vertical and the horizontal reactions at A and V_B be vertical reaction at B.

$\Sigma H = 0$, gives

$\mathbf{H_A = 0.}$



$\Sigma M_A = 0$, gives

$$-20 \times 2 \times 1 + 60 \times 4 + 30 + 20 \times 11 - V_B \times 9 = 0$$

$$\mathbf{V_B = 50\ kN.}$$

$\Sigma V = 0$, gives

$$-20 \times 2 + V_A - 60 + V_B - 20 = 0$$

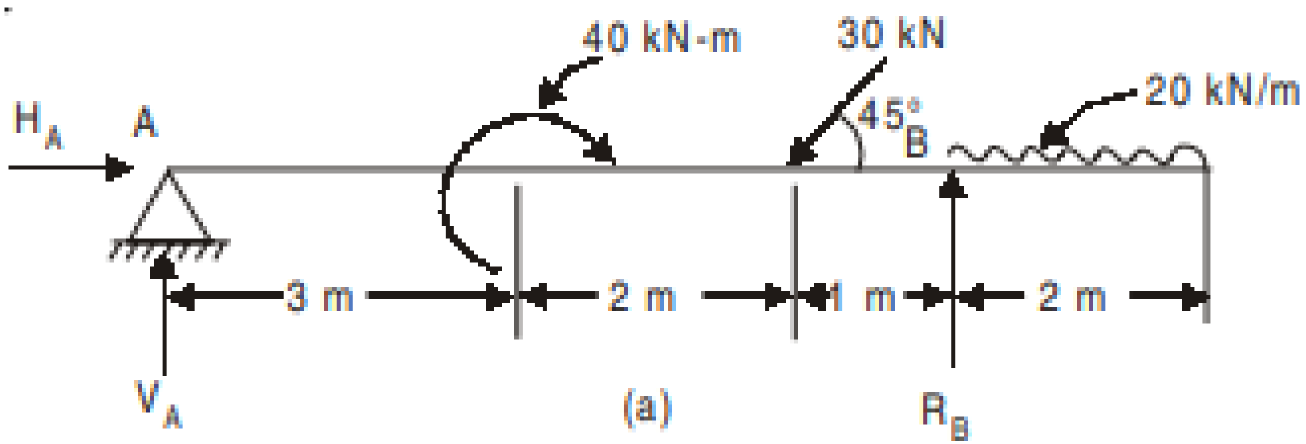
$$V_A = 120 - V_B = 120 - 50$$

$$\mathbf{V_A = 70\ kN.}$$

Lecture No. 4

Example - 7

Determine the reactions at A and B of the overhanging beam shown in Fig.



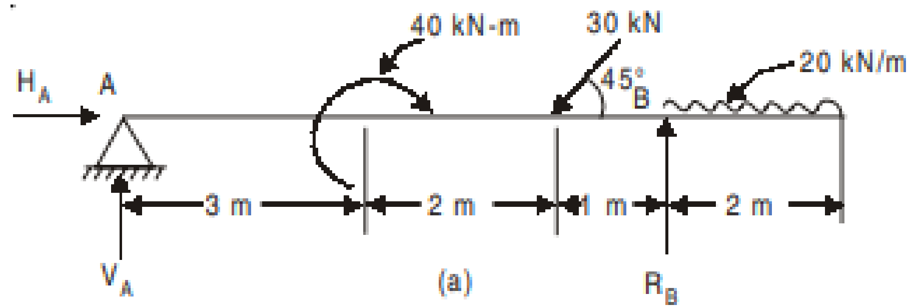
(a)

Solution

$$\sum M_A = 0$$

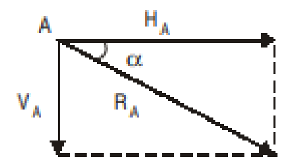
$$R_B \times 6 - 40 - 30 \sin 45° \times 5 - 20 \times 2 \times 7 = 0$$

$$\mathbf{R_B = 71.0110\ kN.}$$



$$\sum H = 0$$

$$H_A = 30 \cos 45° = 21.2132\ kN$$



$$\sum V = 0$$

$$V_A - 30 \sin 45° + R_B - 20 \times 2 = 0$$

$$V_A = 30 \sin 45° - R_B + 40$$

$$V_A = -9.7978$$

(Negative sign show that the assumed direction of V_A is wrong. In other words, V_A is acting vertically downwards).

$$R_A = \sqrt{V_A^2 + H_A^2}$$

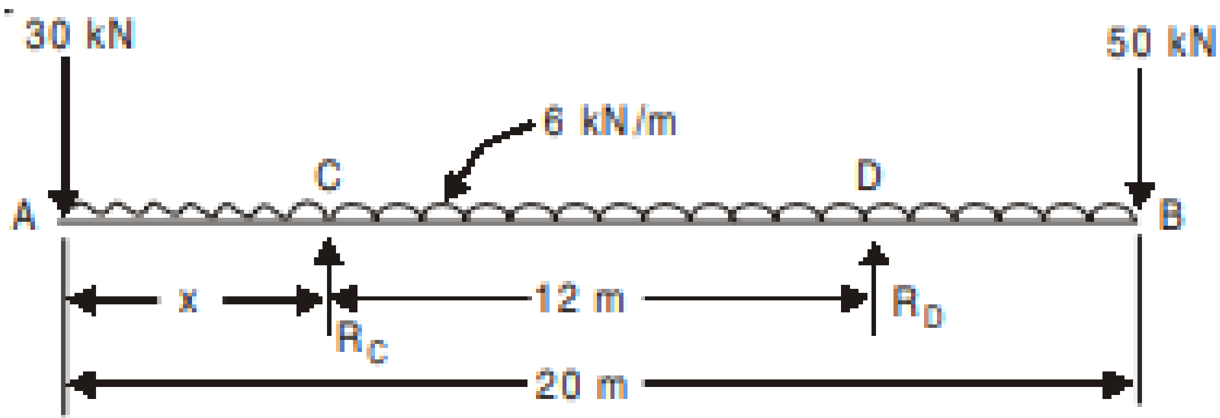
$$\mathbf{R_A = 23.3666\ kN.}$$

$$\alpha = \tan^{-1} \frac{V_A}{H_A}$$

$\boldsymbol{\alpha} = \mathbf{24.79°}$, as shown in Fig.

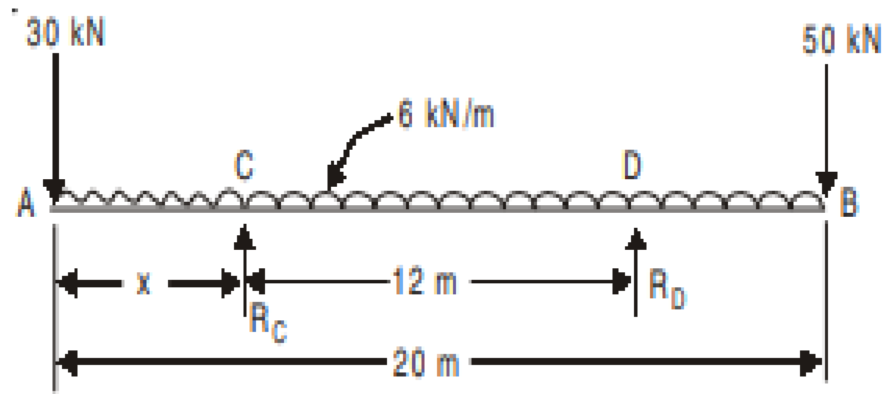
Example - 8

A beam AB 20 m long supported on two intermediate supports 12 m apart, carries a uniformly distributed load of 6 kN/m and two concentrated loads of 30 kN at left end A and 50 kN at the right end B as shown in Fig. How far away should the first support C be located from the end A so that the reactions at both the supports are equal ?



Solution: Let the left support C be at a distance x metres from A.

Now, $R_C = R_D$ (given)



$\Sigma V = 0$, gives

$$R_C + R_D - 30 - 6 \times 20 - 50 = 0$$

$$2R_C = 30 + 120 + 50 \quad \text{since } R_C = R_D$$

$$R_C = 100 \text{ kN}$$

$$R_D = 100 \text{ kN}$$

$\Sigma M_A = 0$, gives

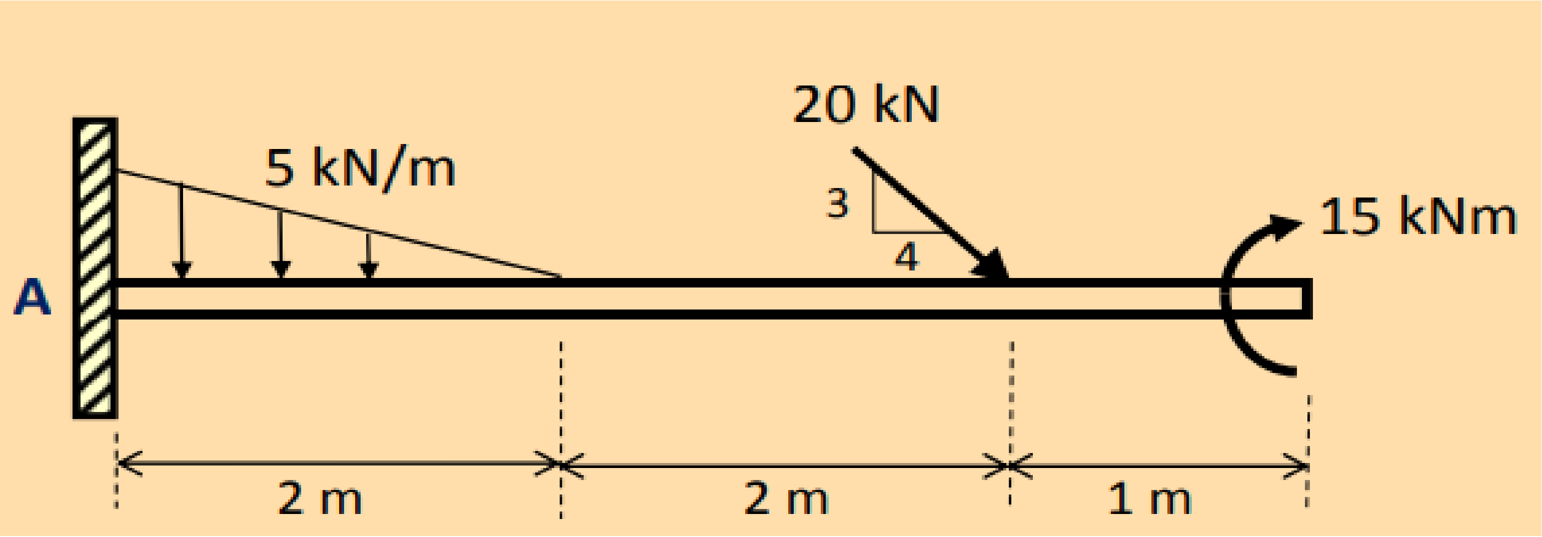
$$100x + 100\,(12 + x) - 6 \times 20 \times 10 - 50 \times 20 = 0$$

$$200x = 1000$$

$$\mathbf{x = 5\ m.}$$

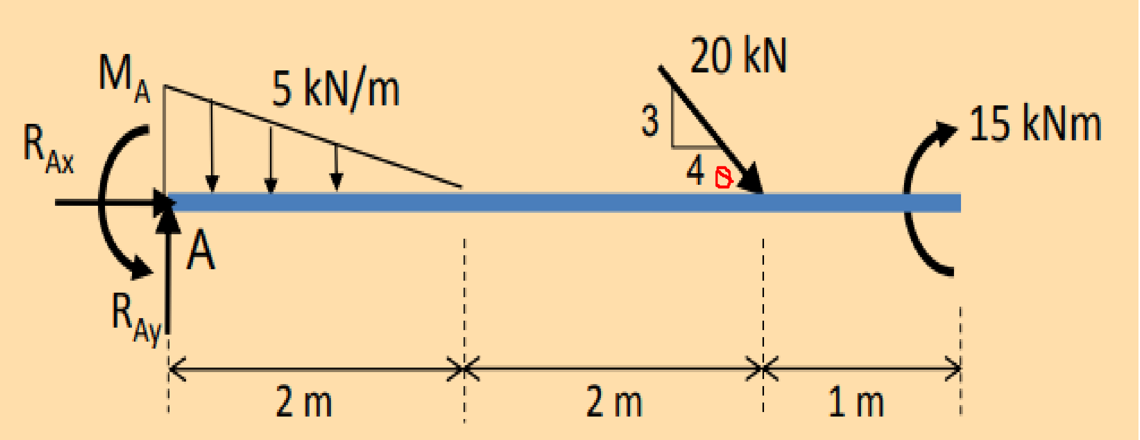
Example – 9

A cantilever beam is loaded as shown. Determine all reactions at support A.



SOLUTION

$\sin \theta = 3/5 \qquad \cos \theta = 4/5$



$$\Sigma F_X = 0$$

$R_{AX} + 20 \cos \theta = 0$

$R_{AX} + 20 \times 4/5 = 0$

R_{AX} = - 16 kN **Answer**

$$\Sigma F_Y = 0$$

$R_{AY} - 20 \sin \theta - ½ \times 5 \times 2 = 0$

$R_{AY} - 20 \times 3/5 - ½ \times 5 \times 2 = 0$

R_{AY} = 17 kN **Answer**

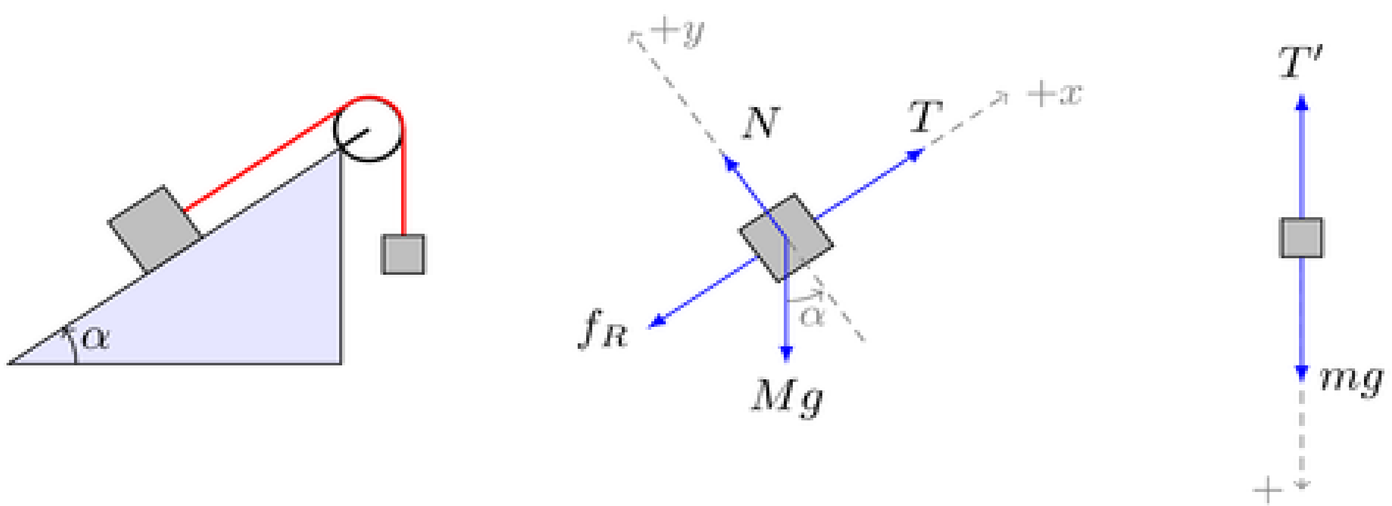
$$\Sigma M_A = 0$$

$-M_A + \frac{1}{2} \times 5 \times 2 \times \frac{1}{3} \times 2 + 20 \sin \theta \times 4 + 15 = 0$

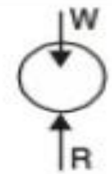
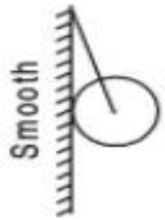
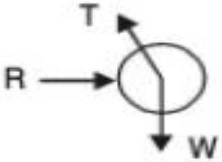
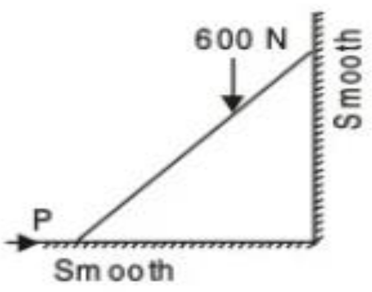
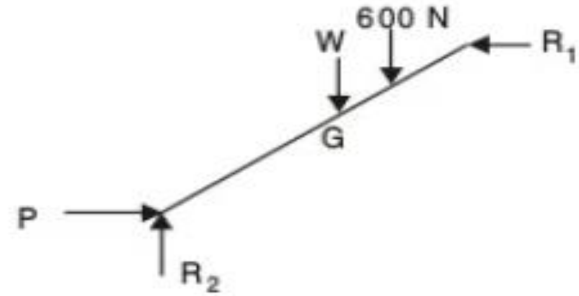
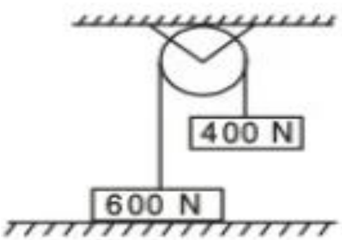
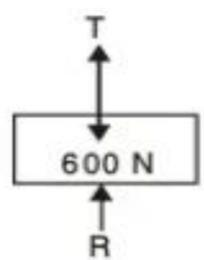
$\boldsymbol{M_A = 66.33\ kNm}$ **Answer**

Free Body Diagram(FBD)

A free body diagram consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.



FBD Standard cases

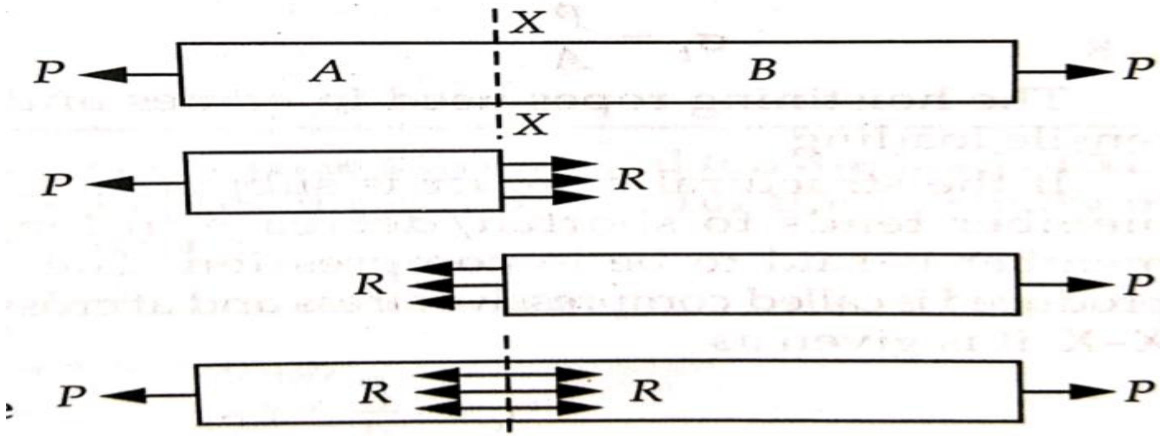
Reacting Bodies	FBD required for	FBD
	Ball	
	Ball	
	Ladder	
	Block weighing 600 N	

Lecture No. 5

Stress(σ)

- Stress is defined as the internal resisting force developed at a point offered by a body against deformation when it is subjected to external load.

- $\sigma = \frac{R}{A} = \frac{P}{A}$



- **Unit-** **N/m², N/mm²**

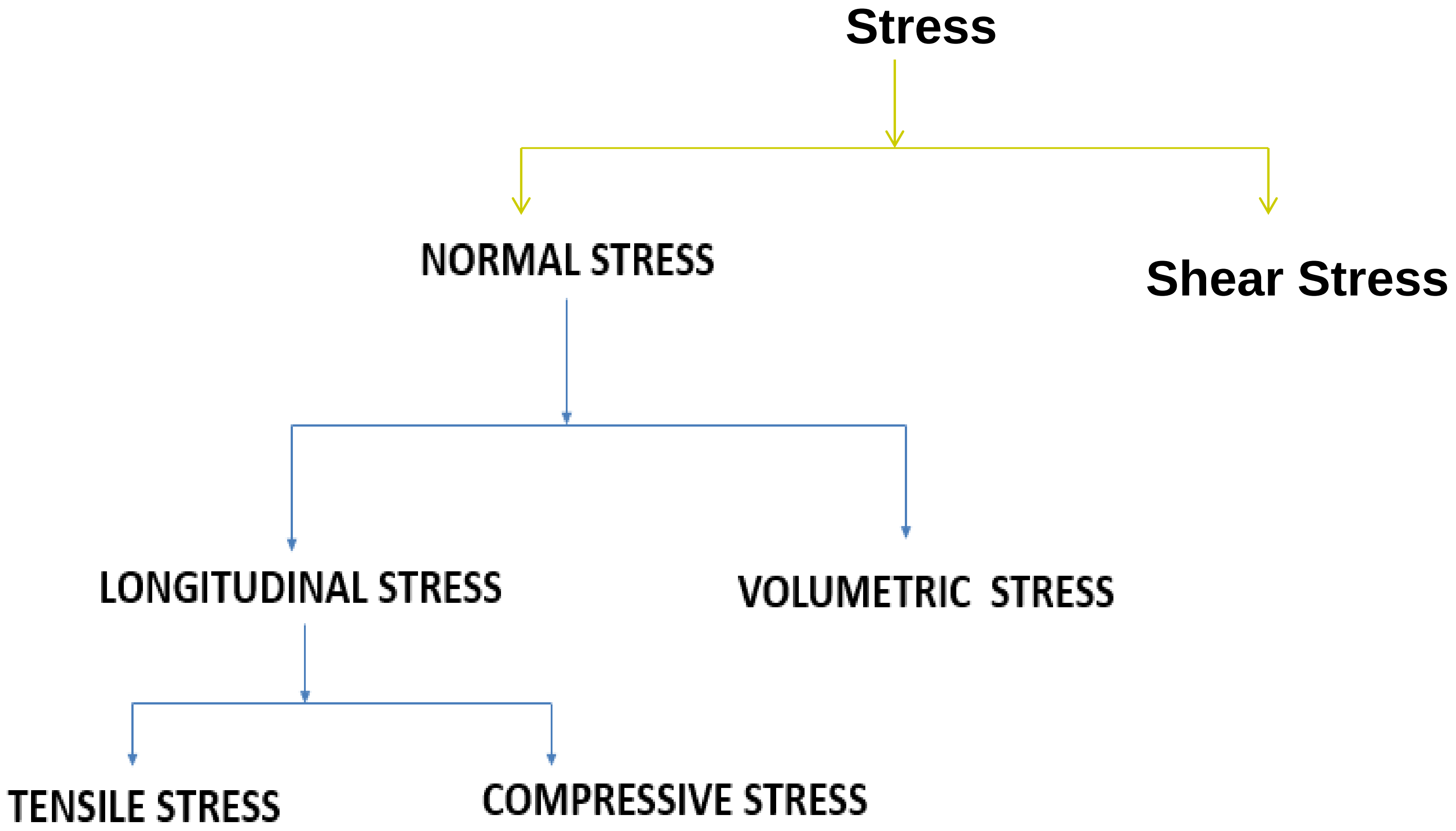
1 Pascal = N/m^2 and 1MPa = N/mm^2

Stress Vs Pressure

Stress	Pressure
Mag. Of IRF developed at a point.	Mag. Of External Applied Force at a point.
Stress acts normal or parallel to surface.	Always acts normal to the surface.
Mag. Of stress at a point in different direction is different.	Mag. Of Pressure at a point in all the direction remains same.
Vector quantity.	Scalar quantity.
It can not be measured.	It can be measured.
Due to Stress, pressure will not be developed.	Due to pressure, Stress will be developed.

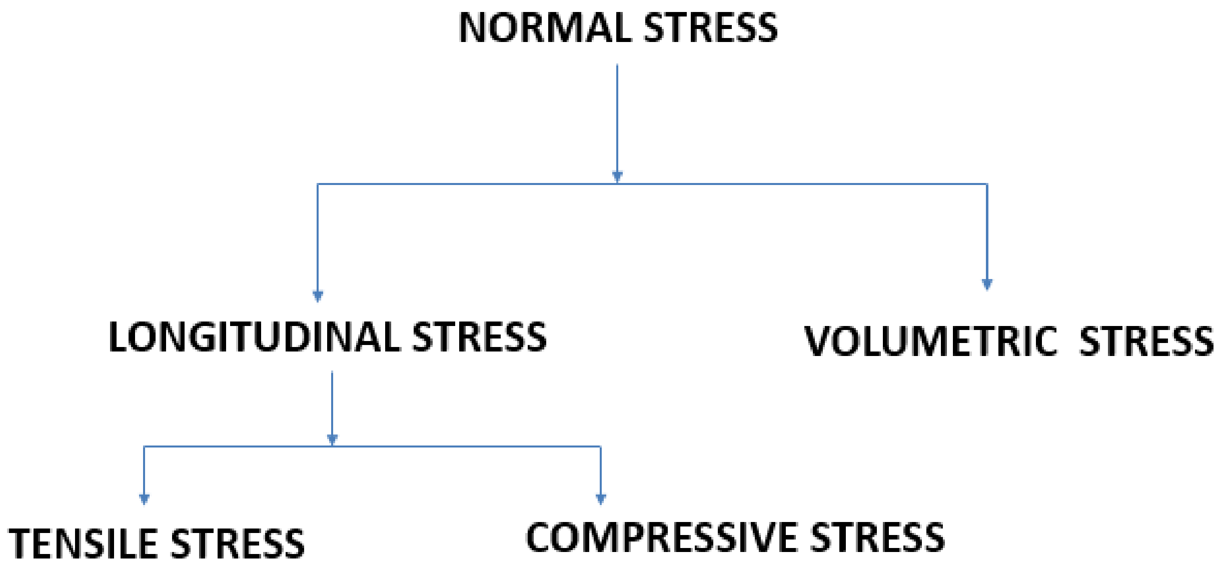
Types of Stress

Stress is of following types:



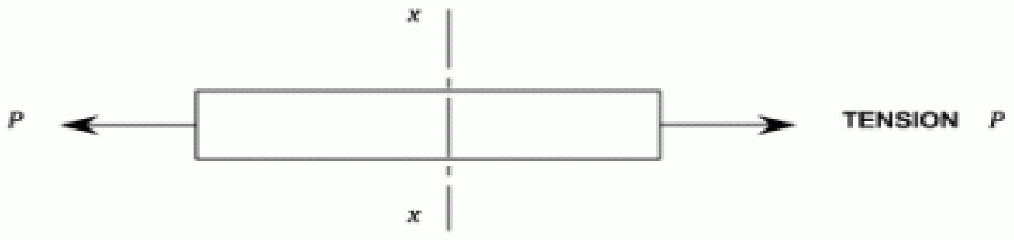
Normal Stress

- Stress is said to be Normal stress when the direction of the deforming force is perpendicular to the cross-sectional area of the body.



Tensile Stress(σ_t)

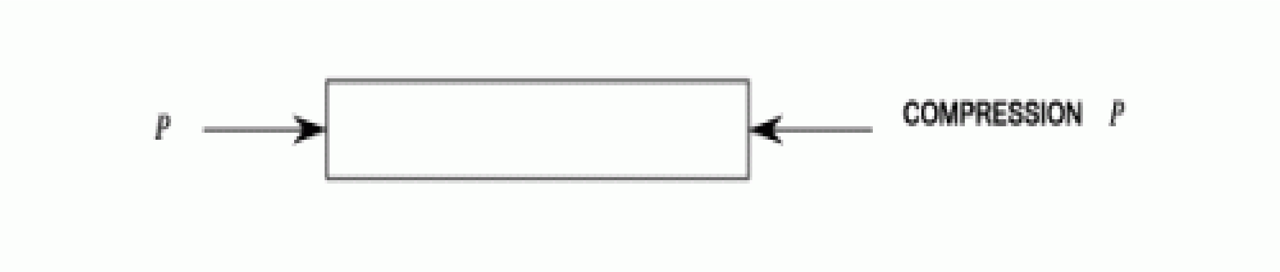
- When a structural member is subjected to two equal and opposite **tensile forces**, the stress produced is called tensile stress.



- The tensile stress at any cross-section X-X is given as $\sigma_t = \frac{P}{A}$

Compressive Stress(σ_c)

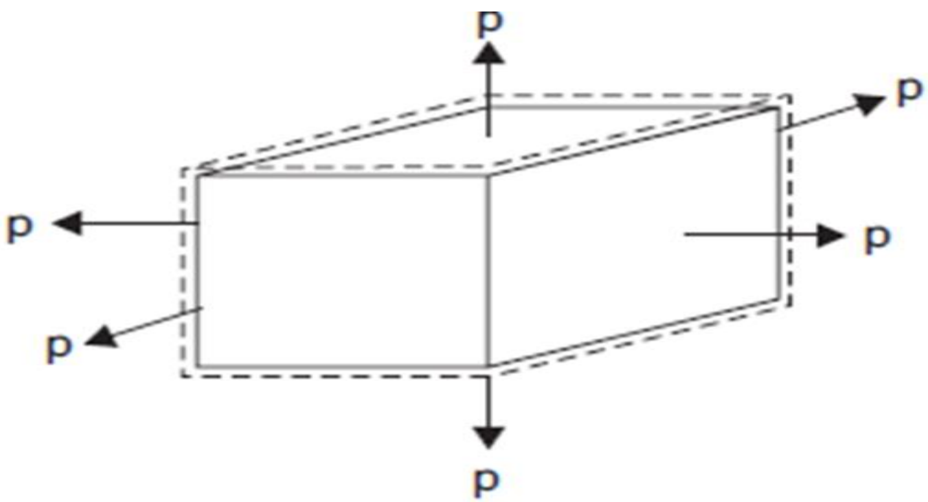
- When a structural member is subjected to two equal and opposite **compressive forces**, the stress produced is called compressive stress.



- The compressive stress at any cross-section X-X is given as $\sigma_c = \frac{P}{A}$

Volumetric Stress(σ_v)

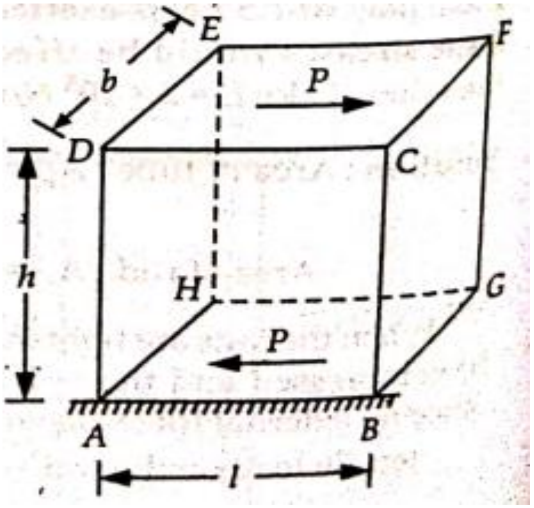
- When the deforming force or applied force acts from all dimension resulting in the change of volume of the object then such stress in called volumetric stress or Bulk stress.
- In short, When the volume of body changes due to the deforming force it is termed as Volume stress.



Shear Stress(τ)

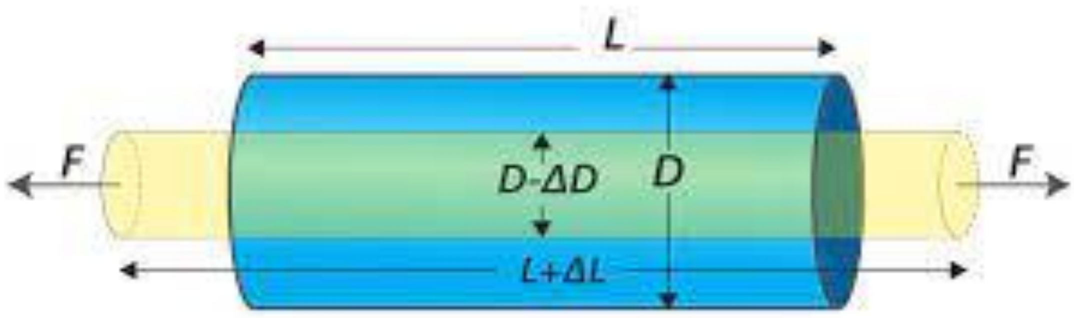
- Stress produced by a force tangential to the surface of a body is known as shear stress.
- It is represented by τ.
- Consider a rectangular block ABCD fixed at the bottom plane and subjected to tangential force P at the upper plane.

$$\text{shear stress } \tau = \frac{\text{tangential force}}{\text{area of face } DCFE} = \frac{P}{bl}$$



Strain(ε)

- When a body is subjected to tensile or compressive load, its dimension will increase or decrease along the line of action of load applied.



- It can be defined as the ratio of change in dimension to the original dimension.
- $$\boldsymbol{Strain = \frac{change\ in\ dimension}{original\ dimension}}$$
- Strain is dimensionless quantity.

Types of Strain

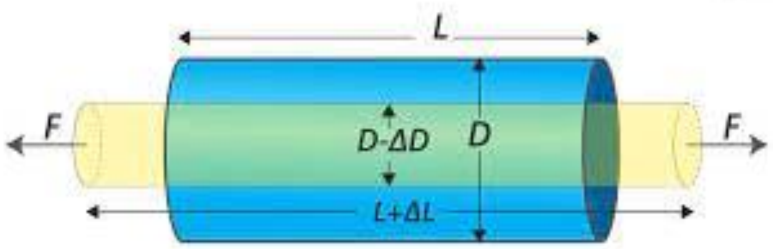
- Strain
  - Normal Strain
    - Longitudinal Strain
    - Lateral Strain
    - Volumetric Strain
  - Shear Strain

Types of Strain

- Strain in the direction of applied load is called longitudinal or primary or linear strain.

- Strain in the perpendicular direction of applied load is called lateral strain.

- Longitudinal and lateral strain are always opposite in nature.

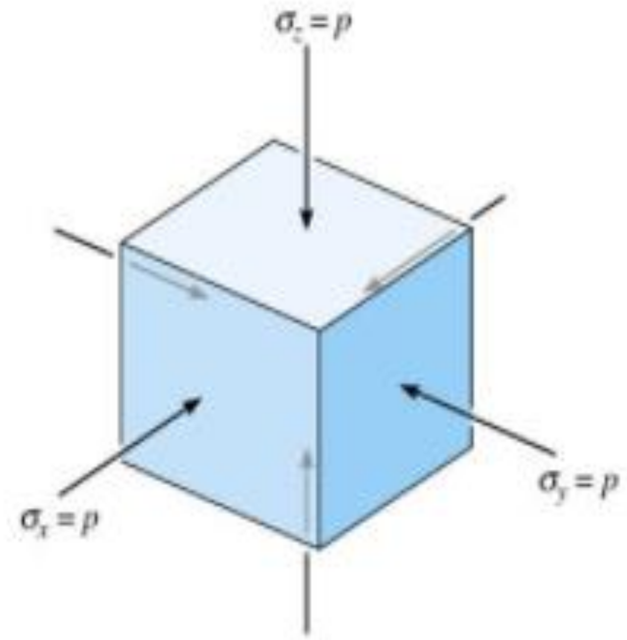


$$Longitudinal\ strain = \frac{\delta L}{L}$$

$$Lateral\ Strain = \frac{\delta b}{b} = \frac{\delta d}{d}$$

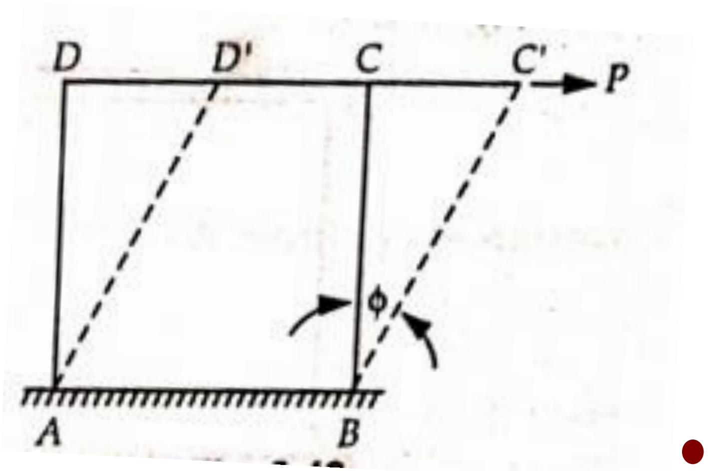
Volumetric Strain(ε_v)

- When a body is immersed in a fluid to a large depth, the body is subjected to equal external pressure at all points on the body.
- Due to this external pressure, stress is produced within the body which is called hydrostatic stress.
- This external pressure causes change in volume of the body.
- This change in volume per unit volume is called volumetric strain, ε_v.



Shear Strain(φ)

- Strain produced by a force tangential to the surface of a body is known as Shear strain.
- Under the action of tangential force, the block ABCD gets distorted and takes the shape ABC'D' by deforming through an angle φ.



- **tan φ = φ =** $\frac{CC'}{BC}$
- The angular deformation φ in radians represents the shear strain.

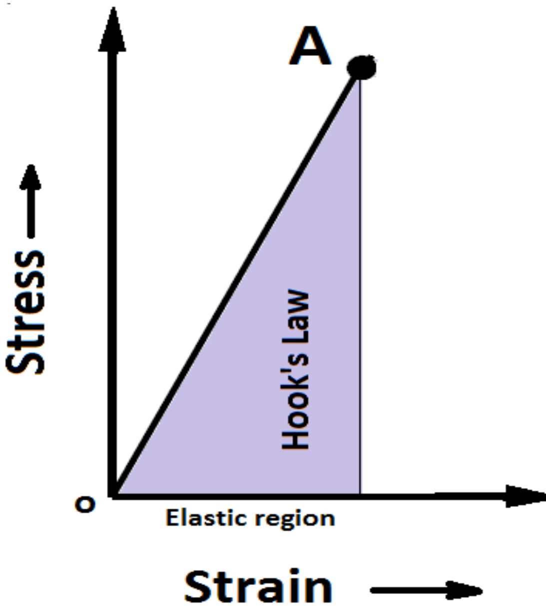
Hooke's Law

- Hooke's law states that when a material is loaded **within proportional limit**, stress is directly proportional to strain.

  Mathematically,

$$\sigma \, \alpha \, \varepsilon$$

$$\sigma = E \, \varepsilon$$



Where, E = constant of proportionality and called as Young's Modulus or Modulus of Elasticity

Now, $E = \frac{\sigma}{\varepsilon}$; $\quad E = \frac{stress}{strain}$

- We know that, Stress (σ) = $\frac{P}{A}$ and Strain (ε) = $\frac{\Delta L}{L}$

$$\Delta L = \frac{PL}{AE}$$

- **Young Modulus,** $$E=\frac{stress}{strain}$$

- Unit of Young's modulus is same as unit of stress because strain is dimensionless quantity.

- **N/m^2, Pa, Kpa, Mpa, Gpa**

- E is a property of the material

Material	Steel	Cast iron	Aluminium	Brass	Bronze
E, GPa	200–210	100–110	68–70	100–110	110–120

Poisson's Ratio(μ)

- It is defined as the ratio of **lateral strain** to **longitudinal strain.**

- **Poisson's Ratio(μ) =** $\frac{\textit{Lateral strain}}{\textit{Longitudinal strain}}$

- **Poisson's ratio is dimensionless.**

- The value of **μ** lies between **0.25 to 0.33** for most of the **engineering materials**.

Lecture No. 6

Elastic Constants

- Different types of stresses and their corresponding strains within elastic limit are related which are referred to as elastic constants.

- For homogenous and isotropic material no. of elastic constants are four.(**μ, E, G, K).**

1) Poisson's Ratio(μ)
2) Modulus Of Elasticity or Young's modulus(E)
3) Modulus Of Rigidity or Shear Modulus(G)
4) Bulk Modulus(K)

- Elastic constants are used to-

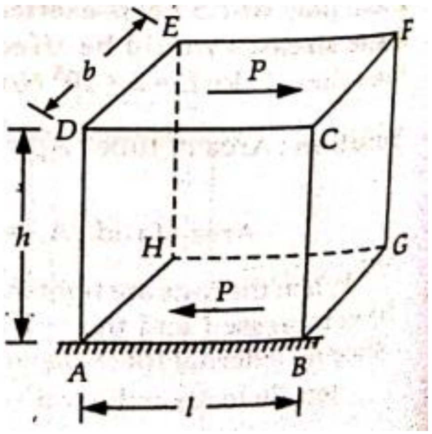
i. determine strain theoretically
ii. obtain relationship between stress and strain

Modulus Of Elasticity or Young's Modulus(E)

- It is defined as the ratio of **normal stress** and **normal strain**, when material is loaded within elastic limit.
- $$E = \frac{\sigma}{\varepsilon} = \frac{stress}{strain}$$
- Unit of Young's modulus is same as unit of stress because strain is dimensionless quantity.
- **N/m^2, Pa, Kpa, Mpa, Gpa**
- E is a property of the material.

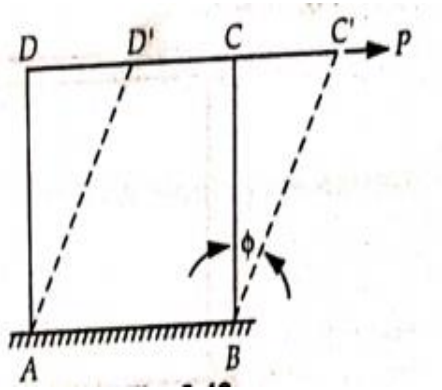
Modulus of Rigidity or Shear Modulus(G)

- It is defined as the ratio of **shear stress** and **shear strain**, when material is loaded within elastic limit.



$$G = \frac{Shear\ Stress}{Shear\ Strain} = \frac{\tau}{\varphi}$$

- Unit of shear modulus is same as unit of stress because strain is dimensionless quantity.



- **N/m², Pa, KPa, MPa, GPa**

Bulk Modulus(K)

- It is defined as the ratio of **volumetric stress** and **volumetric strain**, when material is loaded within elastic limit.

$$\mathbf{G} = \frac{Volumetric\ Stress}{Volumetric\ Strain} = \frac{\sigma_v}{\varepsilon_v}$$

- Unit of Bulk modulus is same as unit of stress because strain is dimensionless quantity.

- **N/m^2, Pa, KPa, MPa, GPa**

Relationship between E, G and μ

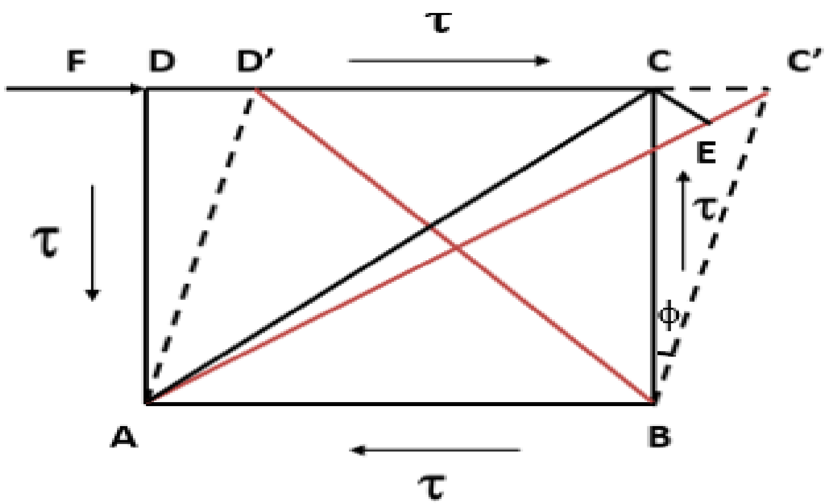
Consider a cubic element ABCD

When the block is subjected to tangential force it distorts to a new shape ABC'D'.

Longitudinal stain in diagonal AC

$$= \frac{AC' - AC}{AC} = \frac{AC' - AE}{AC}$$

$$\frac{EC'}{AC}$$



Relationship between E, G and μ

Extension CC’ is very small , ∠AC’B is assumed to be equal to ∠ACB =45°.

EC’ = CC’ COS 45

= CC’/ √2

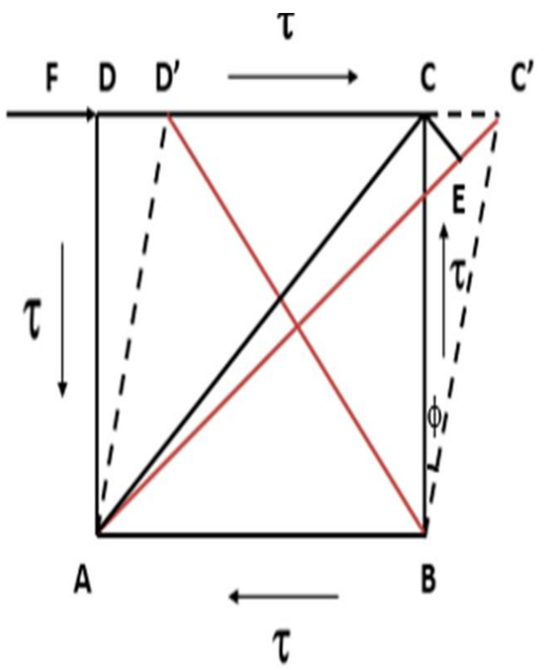
$$\text{Longitudinal Strain} = \frac{CC'}{\sqrt{2}\,AC} = \frac{CC'}{\sqrt{2}\sqrt{2}\,BC} = \frac{CC'}{2\,BC}$$

From triangle BCC’

$$\tan\phi = \frac{CC'}{BC}$$

$$\text{Longitudinal Strain} = \frac{\tan\phi}{2} = \frac{\phi}{2}$$

Where ϕ represents the shear strain.



Relationship between E, G and μ

In terms of shear stress τ and modulus of rigidity G

Shear strain(ϕ) = τ/G

Longitudinal stain for diagonal AC = τ/2G

The strain in diagonal AC is also given by =

Strain due to tensile stress in AC – strain due to compressive stress in BD

Note: In case of pure shear stress ($\sigma = \tau$)

$$= \frac{\tau}{E} - \left(-\mu \frac{\tau}{E}\right) = \frac{\tau}{E}(1 + \mu)$$

$$\tau/2G \quad = \frac{\tau}{E}(1 + \mu)$$

$$E = 2G(1 + \mu)$$

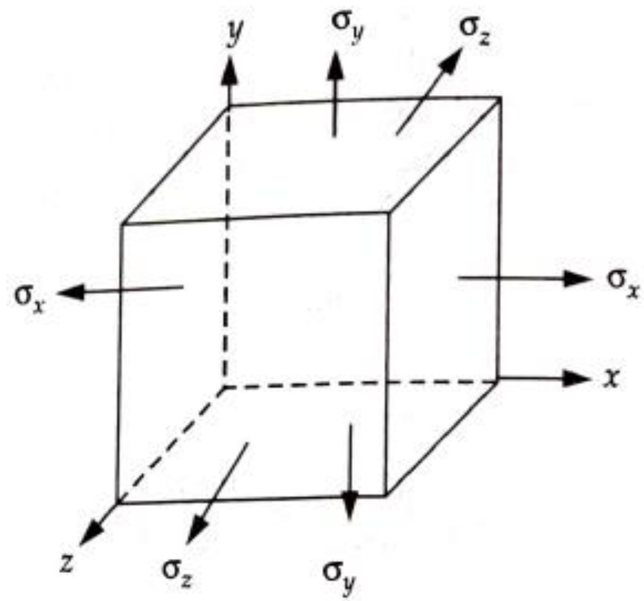
Relationship between E, K and μ

- Consider a cubical element subjected to volumetric stress σ which acts simultaneously along mutually perpendicular x , y and z direction.

- $\boldsymbol{\varepsilon}_{\mathrm{x}} =$ strain in x-direction due to $\boldsymbol{\sigma}_x$ - strain in x direction due to $\boldsymbol{\sigma}_y$ - Strain in x-direction due to $\boldsymbol{\sigma}_z$

- Strain in x direction-

$$= \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$



Relationship between E, K and μ

But $\sigma_x = \sigma_y = \sigma_z = \sigma$

$$\therefore \quad \varepsilon_x = \frac{\sigma}{E} - \mu \frac{\sigma}{E} - \mu \frac{\sigma}{E} = \frac{\sigma}{E}(1 - 2\mu)$$

Likewise $\varepsilon_y = \frac{\sigma}{E}(1 - 2\mu)$ and $\varepsilon_z = \frac{\sigma}{E}(1 - 2\mu)$

Volumetric strain $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3\sigma}{E}(1 - 2\mu)$

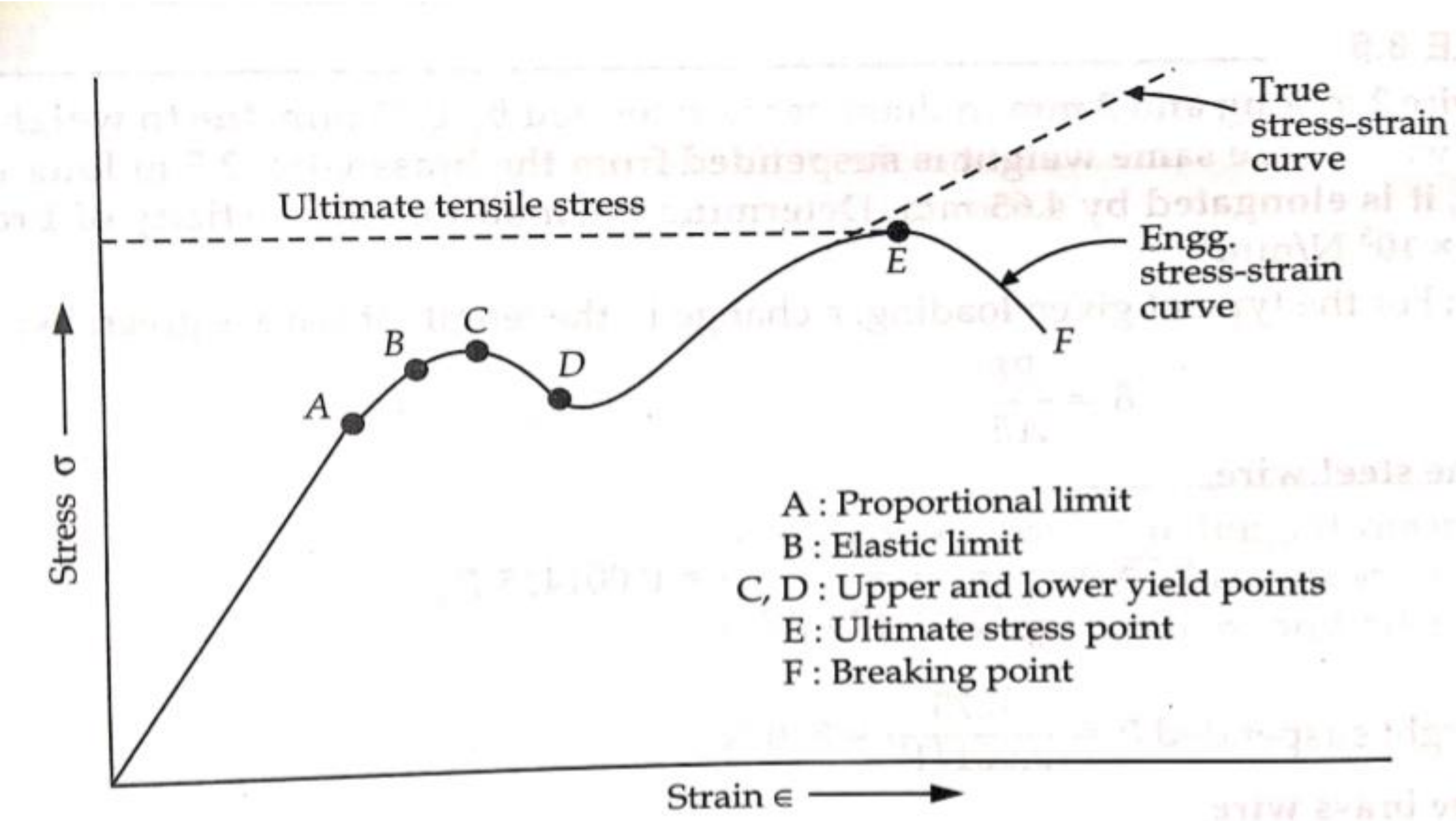
Now, bulk modulus $K = \frac{\text{volumetric stress}}{\text{volumetric strain}} = \frac{\sigma}{\frac{3\sigma}{E}(1 - 2\mu)} = \frac{E}{3(1 - 2\mu)}$

$$\therefore \quad E = 3K(1 - 2\mu)$$

Lecture No. 7

Stress-Strain diagram

A. For Ductile material(e.g. Mild steel)



❖ Proportional limit(**A**):

- Upto this limit, stress is a linear function of strain and material obeys Hook's law.
- **0-A** is a straight line of the curve and its slope represents the value of modulus of elasticity.

❖ Elastic limit(**B**):

- It represents maximum stress upto which material is still able to regain its original shape and size after removal of load i.e. upto this point deformation is recoverable.

- **Upper yield point(C) and Lower yield point(D):**
  - Beyond elastic limit, the material shows considerable strain even though there is no increase in load or stress.
  - **Deformation is not fully recoverable i.e. the behaviour of material is inelastic.**
  - This phenomenon from C to D is called yielding.
- **Ultimate stress point(E):**
  - After yielding has taken place, the material becomes hardened and increase in load is required to take the material to its maximum stress at point E.
  - Point E represents the maximum stress of this curve and this point is known as ultimate stress point.

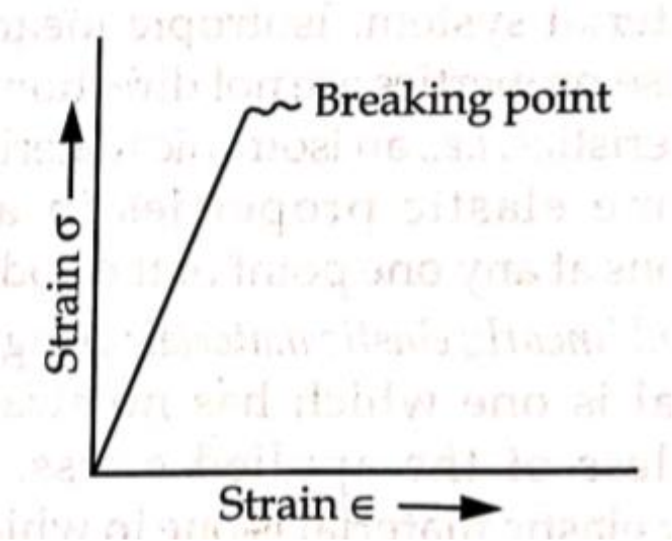
❖ Breaking point(F):

- In the portion **EF**, there is falling off the load(stress) from the maximum until fracture takes place at **F**.
- The point **F** is known as fracture or breaking point and corresponding stress is called the breaking stress.

Stress-Strain diagram

B. For Brittle material (e.g. Cast Iron)

❖ For brittle materials, like cast Iron, no appreciable deformation is obtained and the failure occurs without yielding.



❖ Brittle material fails suddenly with out any indication.

❖ Ductile materials give indication before failure.

Factor of safety(FoS)

- FoS is defined as Ultimate stress to working stress.

$$\text{FoS} = \frac{\textit{Ultimate stress}}{\textit{Working stress}}$$

- A **factor of safety** increases the **safety** of people and reduces the risk of failure of a product.
- If a structure fails there is a risk of injury and death as well as a company's financial loss.
- How much FoS is required depends on the materials and its applications.
- As the FoS increases, the cost of the product also increases therefore cost is also a considerable parameter.
- For safe design FoS must be greater than unity.

Factor of safety(FoS)

- **Ultimate Stress: It** is the maximum stress that a material can withstand while being stretched or pulled before breaking.

- **Working Stress: working stress** is known as the maximum allowable **stress** that a material or object will be subjected to when it is in service. This stress is always lower than the **Yield stress** and the ultimate tensile stress.

Lecture No. 8

Question 1.1

A steel bar of 1.5 m long, 50 mm wide and 20 mm thick is subjected to an axial tensile load of 120 kN. If the extension in the length of the bar is 0.9 mm, make calculation for intensity of stress, strain and modulus of elasticity of the bar material.

Ans: $\sigma = 120\ N/mm^2$, $\varepsilon = 0.0006$, $E = 200\ GN/m^2$

Question 1.2

A hollow right circular cylinder is made of cast iron and has a outside diameter of 75 mm and an inside diameter of 60 mm. The cylinder measures 600 mm in length and is subjected to axial compressive load of 50kN.Determine normal stress and shortening in length of the cylinder under this load. Take the modulus of elasticity of cast iron to be 100 GPa.

Ans: $\sigma = 31.45\ N/mm^2$, $d\!l = 0.1887$ mm

Question 1.3

A tensile load of 56 kN was applied to a bar of 30 mm diameter with 300 mm gauge length. Measurements showed 0.12 mm increase in length and the corresponding 0.0036 mm contraction in diameter. Make calculations for the Poisson's ratio and the values of three moduli(elastic constants).

Ans: $\mu = 0.3$, $E = 1.9815 \times 10^5$ N/mm^2 , $G = 0.762 \times 10^5$ N/mm^2,K = 1.65x105 N/mm^2

Thank You