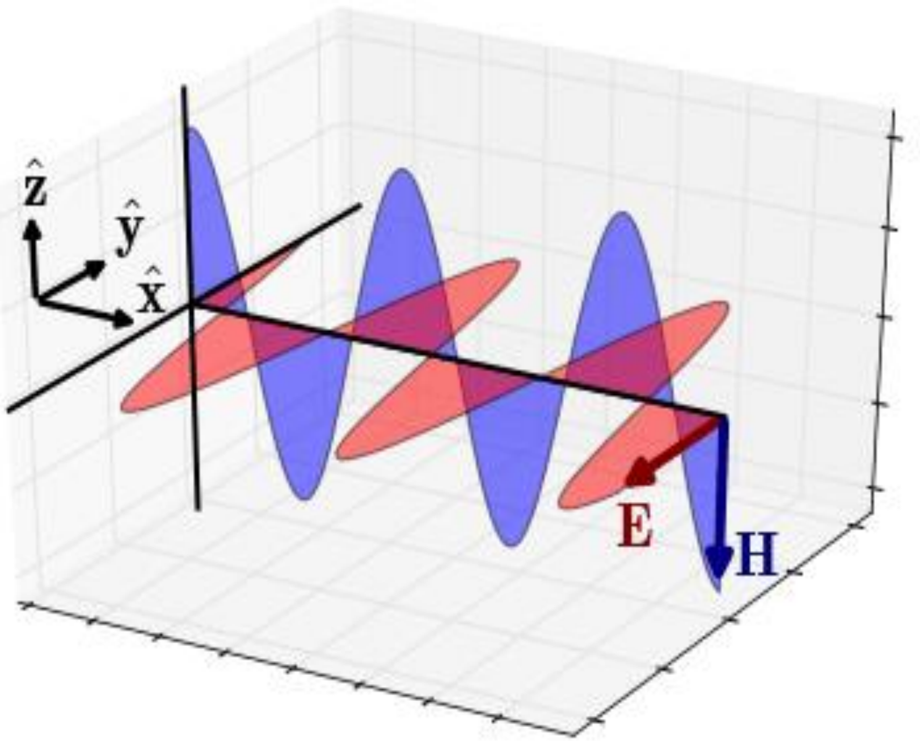
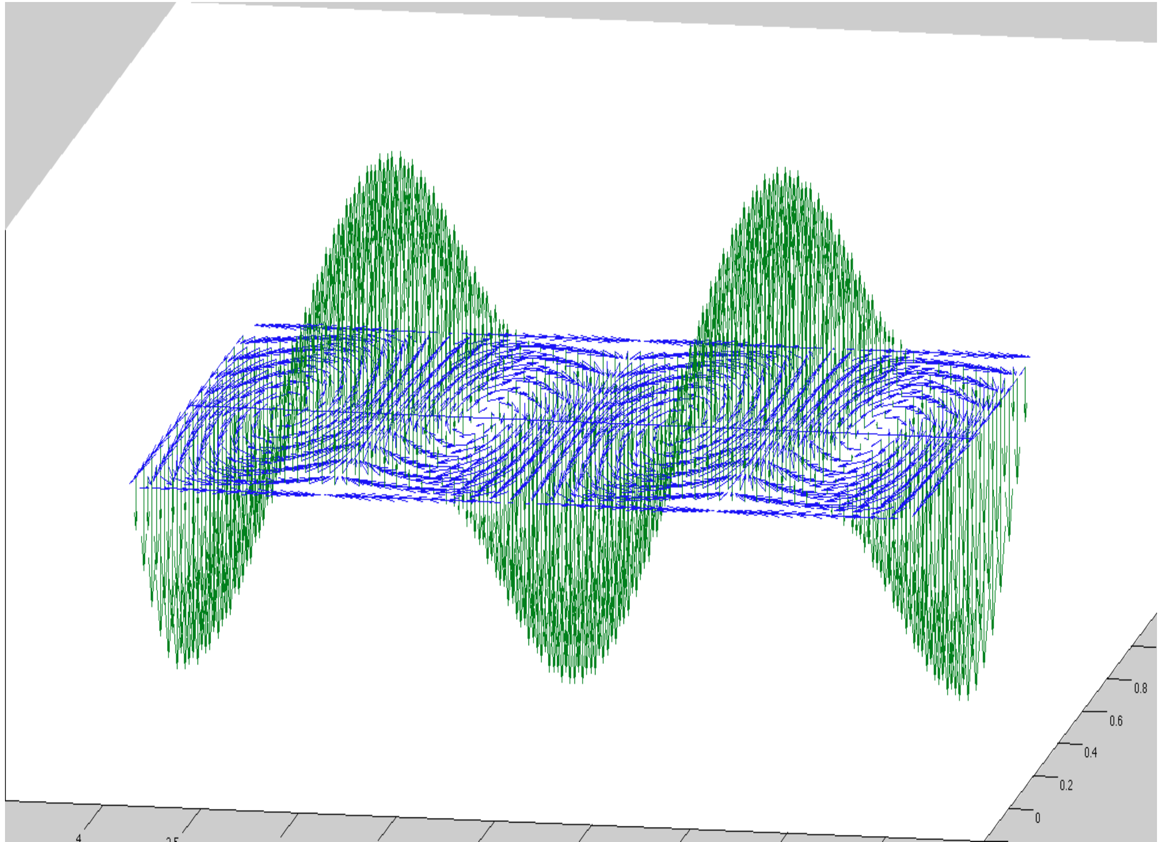
Lecture 10 (Unit-II)

ELECTROMAGNETIC FIELD THEORY





CONTENTS

INTRODUCTION

- Initially, we studied electricity (when charge is at rest) and magnetism (when charge is in uniform motion).

- In 1820, Oersted $\longrightarrow$ magnetic field can be produced by electric current.

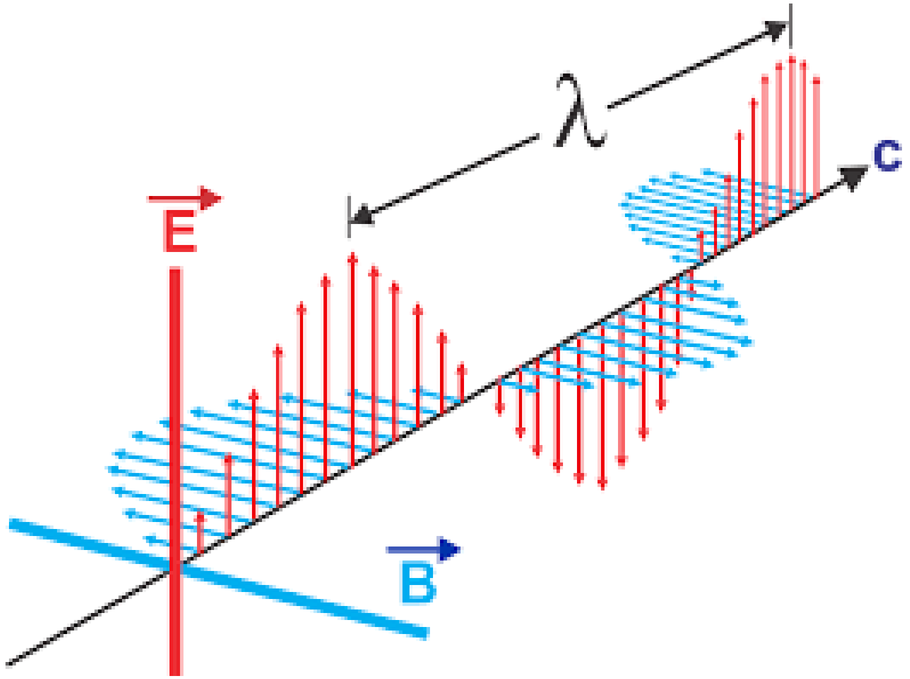
- Faraday $\longrightarrow$ phenomenon of electromagnetic induction and showed that electric current can be produced by a time varying magnetic field.

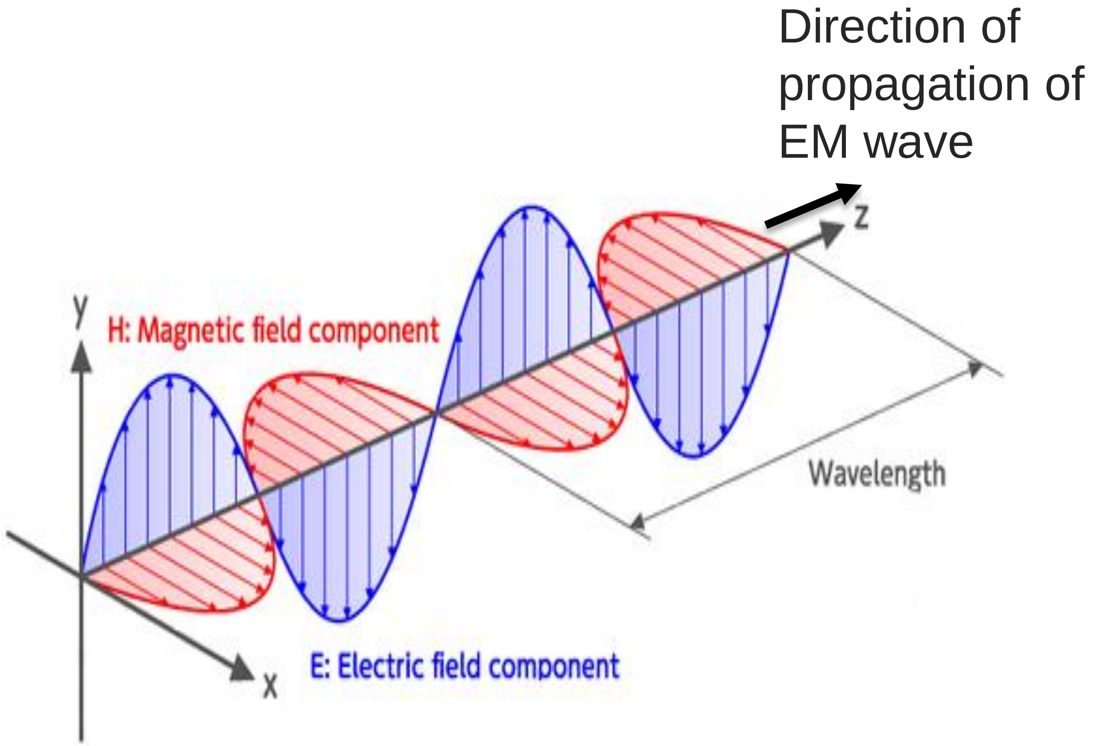
- In 1864, Maxwell unified both electric and magnetic field $\longrightarrow$ An accelerated charge particle generates electromagnetic waves.

- The coupled oscillating electric and magnetic field that moves with the speed of light and exhibit wave behavior is called **Electromagnetic Wave**.

  The microwaves, infrared rays, ultraviolet rays, X-rays and γ- rays are few examples of electromagnetic waves.

➢ **NOTE:** The electric field and magnetic field of an electromagnetic wave are perpendicular to each other and also perpendicular to the direction of propagation of EM waves.





Some important tools

- Dell Operator

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k} \qquad \text{(Vector quantity)}$$

- Laplacian Operator

$$\nabla^2 = \vec{\nabla}.\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad \text{(Scalar quantity)}$$

- $Gradiant\ of\ \phi = grad\ \phi = \vec{\boldsymbol{\nabla}}\phi$
- $Divergence\ of\ \vec{\boldsymbol{A}} = div\ \vec{\boldsymbol{A}} = \vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{A}}$
- $Curl\ of\ \vec{\boldsymbol{A}} = Curl\ \vec{\boldsymbol{A}} = \vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{A}}$

Some important tools

State Gauss divergence and Stoke's theorem.

Gauss Divergence theorem – Surface integral of a vector over a closed surface area is equal to volume integral of divergence of the same vector over the volume enclosed by that surface area, i.e.

$$\oint \vec{\boldsymbol{A}}.\overrightarrow{\boldsymbol{dS}} = \int (\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{A}})dV$$

Some important tools

Stoke's theorem – Line integral of a vector over a closed loop is equal to surface integral of curl of the same vector over the surface area enclosed by that loop, i.e.

$$\oint \vec{\boldsymbol{A}}.\overrightarrow{\boldsymbol{dl}} = \int (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{A}}).\overrightarrow{\boldsymbol{dS}}$$

What is the equation of continuity? Obtain the required expression for it. Also, give its physical significance. [**2016-17, 2018-19**]

CONTINUITY EQUATION

Electric current through a closed surface area is given by

$$I = \oint \vec{J}.d\vec{S} \qquad \ldots\ldots\ldots (1)$$

where $\vec{J}$ is current density

Consider, a closed surface S enclosing a volume V. If ρ is the volume density inside the volume, then

$$I = -\frac{dq}{dt} = -\frac{d}{dt}\int \rho dV \qquad \ldots\ldots(2)$$

-ve sign indicate that charge decreases inside the volume with time.

If there is neither source nor sink inside the volume, then according to the law of conservation of charge value of electric current given by equations (1) and (2) will be same.Therefore

$$\oint \vec{\boldsymbol{J}}.\boldsymbol{d}\vec{\boldsymbol{S}} = -\frac{d}{dt}\int \rho dV \qquad \dots\dots\dots(3)$$

Applying Gauss divergence theorem on left hand side we get

$$\int (\vec{\nabla}.\overrightarrow{\boldsymbol{J}})dV = -\frac{d}{dt}\int \rho dV$$

$$\int (\vec{\nabla}.\vec{\boldsymbol{J}})dV = -\int \frac{\partial \rho}{\partial t} dV$$

$$\int \left(\vec{\nabla}.\vec{\boldsymbol{J}} + \frac{\partial \rho}{\partial t}\right) dV = 0$$

For any arbitrary volume, the integral must be zero. Thus,

$$\vec{\nabla}.\vec{\boldsymbol{J}} + \frac{\partial \rho}{\partial t} = 0$$

This is called equation of continuity.

CONTINUITY EQUATION

- **Physical significance:**

Law of conservation of charge: It states that the total current flowing out of some volume must be equal to the rate of decrease of charge within the volume, assuming that charge can neither be created nor be destroyed.

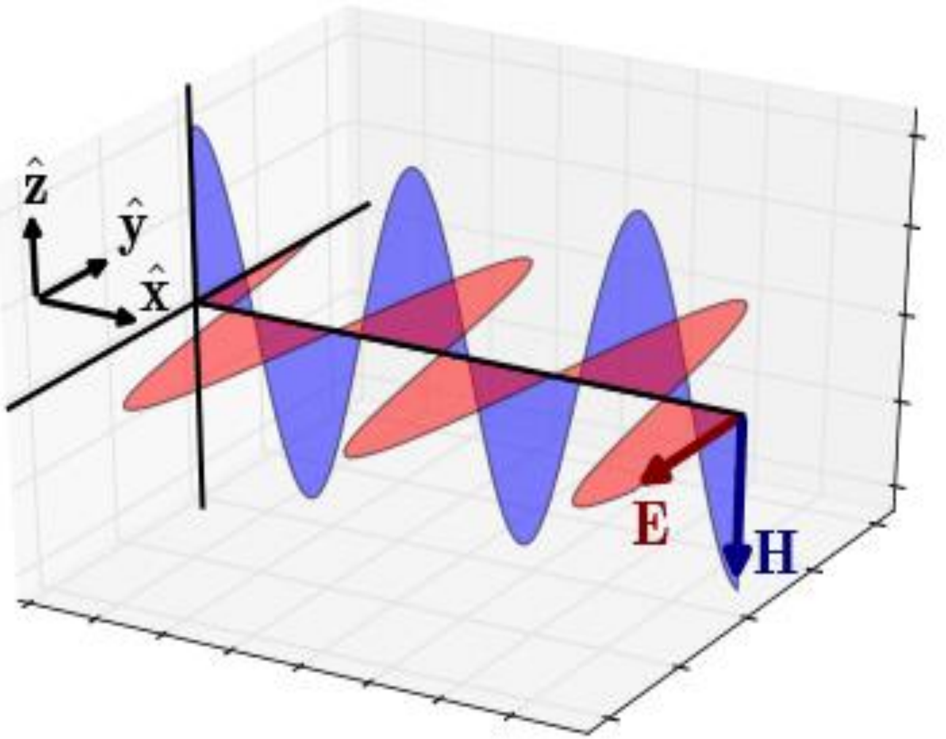
- For steady state

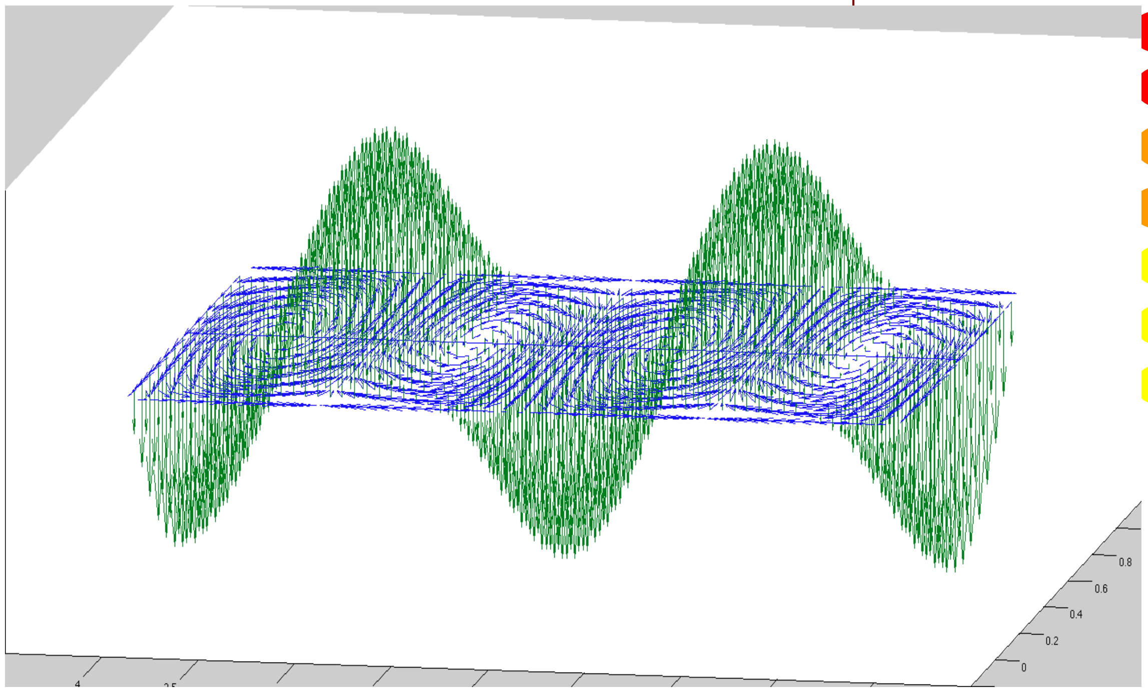
$$\frac{\partial \rho}{\partial t} = 0$$

$$\therefore \qquad \vec{\nabla}.\vec{\boldsymbol{J}} = 0$$

Lecture 11

ELECTROMAGNETIC FIELD THEORY





CONTENTS

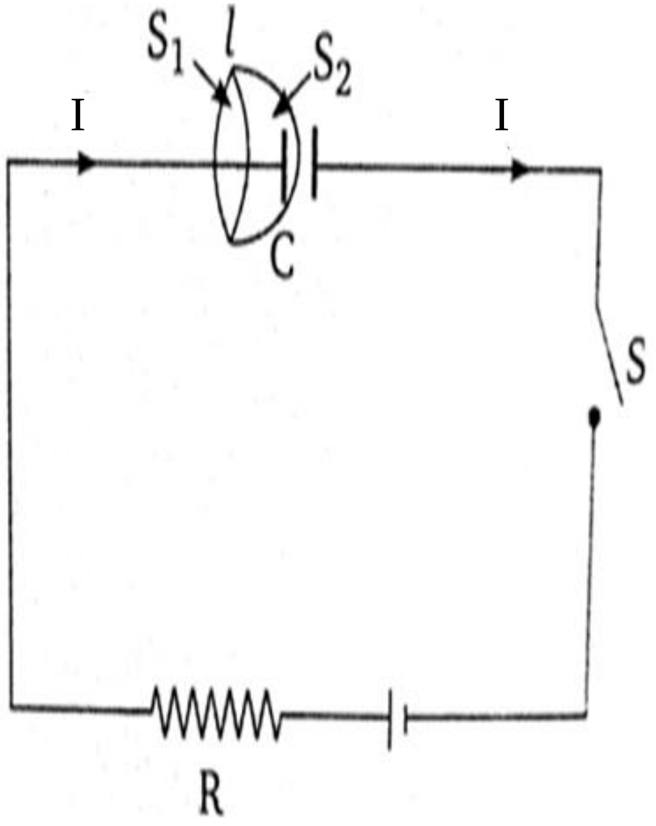
- Displacement current
- Modification of Ampere’s circuital law

Why Maxwell proposed that Ampere's law require modification?

[2018-19]

The idea of modification in Ampere's law arises in connection with capacitors with no medium between them.

Let us consider the circuit showing the process of charging of capacitor. Consider two surfaces S_1 (plane) and S_2 (hemispherical) bounded by common closed path *l*. Let an instant of time 't', the current is 'I'.



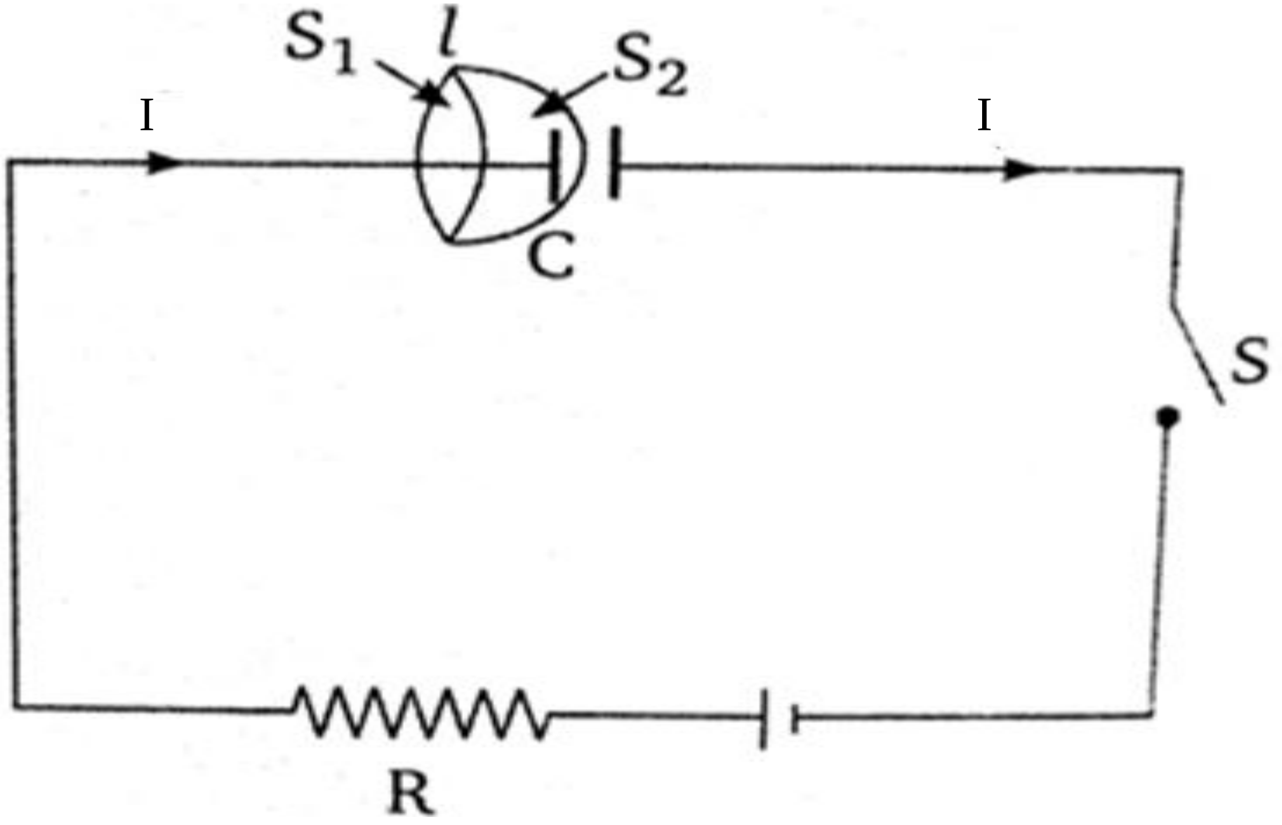
Applying Ampere's law for surface S_1, we get

$$\oint \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{l}} = \mu_0 I \ldots\ldots..(1)$$

Ampere's law for surface S_2 is given by,

$$\oint \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{l}} = 0\ldots\ldots\ldots(2)$$

Since, the dielectric current inside the capacitor is zero. We see that equation (1) & (2) are contradict to each other which is impossible. So, to remove this controversy, Maxwell introduced the modification in Ampere’s law.



What is displacement current? [2015-16, 2016-17, 2018-19, 2020-21]

or

Explain the concept of displacement current and show how it leads to the modification of Ampere's law.[2016-17, 2018-19, 2020-21]

Displacement current

The concept of displacement current was introduced to resolve the paradox of charging capacitor. Maxwell proposed that

Current —produces→ Magnetic field

Changing electric field —produces→ Magnetic field

It means that changing electric field is equivalent to a current which flows as long as the electric field is changing. This equivalent current in vacuum or dielectric produces same magnetic field as the conduction current in conductor and is known as Displacement current .

Therefore, Maxwell introduced a factor $\varepsilon_0 \frac{d\phi_E}{dt}$ to add in equation (1) instead of ‘I’ which is denoted as ‘I_d’ (displacement current). So,

$$\oint \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{l}} = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

Modification of Ampere's law

Ampere's law in differential form is given by

$$\vec{\nabla} \times \vec{\boldsymbol{H}} = \vec{\boldsymbol{J}} \ldots\ldots.(1)$$

The above equation stands only for steady state current but for time varying fields the current density should be modified. Thus taking divergence of both sides of above equation we get

$$\vec{\nabla}.(\vec{\nabla} \times \vec{\boldsymbol{H}}) = \vec{\nabla}.\vec{\boldsymbol{J}}$$

Since $\quad \vec{\nabla}.(\vec{\nabla} \times \vec{\boldsymbol{H}}) = 0$ (i.e. Divergence of curl (DC) is zero)

Therefore, $\quad \vec{\nabla}.\vec{\boldsymbol{J}} = 0 \ldots\ldots\ldots\ldots\ldots (2)$

But according to the equation of continuity,

$$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t} \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Maxwell realized the situation and suggested that the definition of total current density is incomplete and suggested to add something to J, such that equation 1 becomes

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}' \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Now taking divergence of both the sides we get

$$\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = \nabla.(\vec{J} + \vec{J}')$$

Or $$\vec{\nabla}.(\vec{J} + \vec{J}') = 0$$

$$\vec{\nabla}.\vec{J} = -\vec{\nabla}.\vec{J}'$$

Putting the value of $\vec{\nabla}.\vec{J}$ from equation (3) we get

Or $$\vec{\nabla}.\vec{J}' = \frac{\partial \rho}{\partial t}$$

From Maxwell's first equation,

$$\vec{\nabla}.\boldsymbol{D} = \rho$$

Therefore $$\vec{\nabla}.\vec{J}' = \frac{\partial}{\partial t}(\vec{\nabla}.\vec{D})$$

$$\vec{\nabla}.\vec{J}' = \vec{\nabla}.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

Hence $$\vec{J}' = \frac{\partial \vec{D}}{\partial t}$$

Substituting the value of J' in equation 4, we get

$$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}} = \vec{\boldsymbol{J}} + \frac{\partial \vec{\boldsymbol{D}}}{\partial t} \qquad \{\text{Modified Ampere's Law}\}$$

where $\frac{\partial \vec{\boldsymbol{D}}}{\partial t}$ is displacement current density which arises when the electric displacement vector $\vec{\boldsymbol{D}}$ is changing with time. It flows until the electric field is changing with time. In this way displacement current leads to the modification of Ampere's law.

What is the difference between conduction current and displacement current?

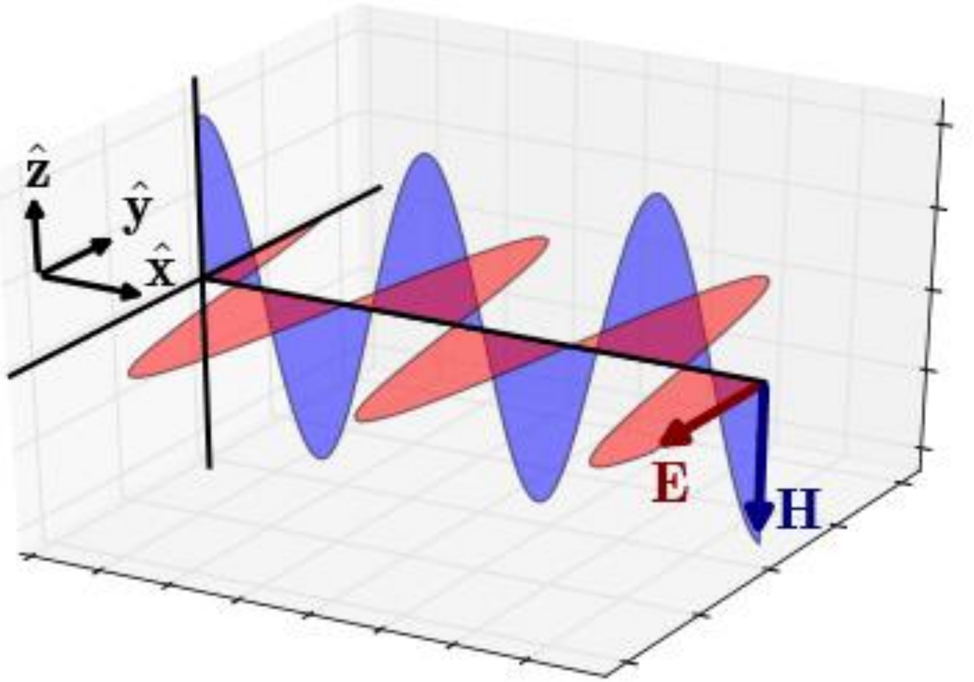
[2017-18]

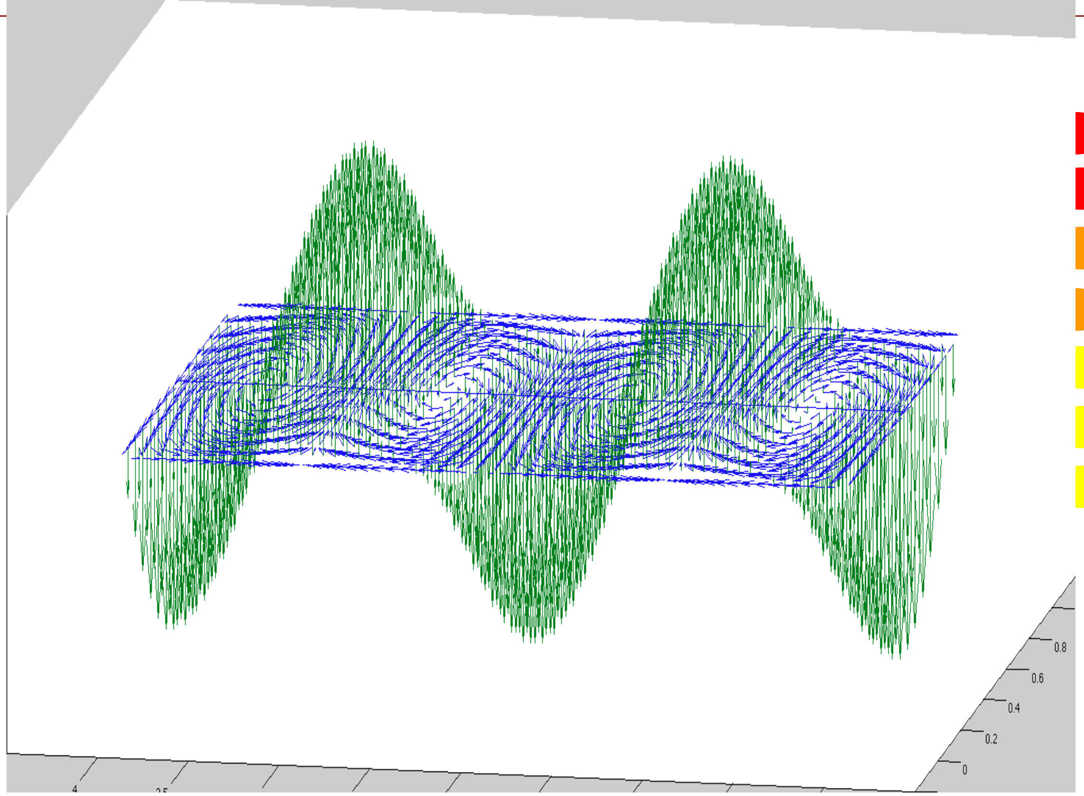
Difference between conduction and displacement current

S. No.	Conduction current	Displacement current
1.	The electric current carried by conductors due to flow of charge is called conduction current	The electric current due to changing electric field is called displacement current
2.	It exist even if flow of electron is at uniform rate.	It does not exist under steady state.
3.	I_c = V/R	$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

Lecture 12

ELECTROMAGNETIC FIELD THEORY





CONTENTS

- Maxwell's equations
  - **Differential form**
  - **Integral form**
- Physical Significance of Maxwell's equations

Derive Maxwell's equation in differential form. Give physical significance of each equation. [2017-18]

or

Write Maxwell's equation in integral and differential form and explain their physical significance with their proofs. [2018-19]

Maxwell's equations

Maxwell's equation in Differential form:

Maxwell's first equation

According to Gauss law in electrostatics

$$\oint \vec{\boldsymbol{E}}.\boldsymbol{d}\vec{\boldsymbol{S}} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int \rho dV$$

$$\oint \varepsilon_0 \vec{\boldsymbol{E}}.\boldsymbol{d}\vec{\boldsymbol{S}} = \int \rho dV$$

$$\oint \vec{\boldsymbol{D}}.\boldsymbol{d}\vec{\boldsymbol{S}} = \int \rho dV \qquad \left(Because\ \vec{\boldsymbol{D}} = \varepsilon_0 \vec{\boldsymbol{E}}\right)$$

where $\vec{\boldsymbol{D}}$ is called electric displacement vector.

Using Gauss divergence theorem, we get

$$\oint \overrightarrow{\boldsymbol{D}}.\boldsymbol{d}\overrightarrow{\boldsymbol{S}} = \int \left(\overrightarrow{\boldsymbol{\nabla}}.\overrightarrow{\boldsymbol{D}}\right)dV = \int \rho dV$$

Therefore,

$$\int \left(\overrightarrow{\boldsymbol{\nabla}}.\overrightarrow{\boldsymbol{D}} - \rho\right)dV = 0$$

Since the equation is true for any volume the integrand must vanish, thus

$$\left(\overrightarrow{\boldsymbol{\nabla}}.\overrightarrow{\boldsymbol{D}} - \rho\right) = 0$$

$$\overrightarrow{\boldsymbol{\nabla}}.\overrightarrow{\boldsymbol{D}} = \rho$$

This is Maxwell's first equation.

Maxwell's second equation

According to Gauss law in magnetostatics

$$\oint \vec{\boldsymbol{B}}.\,\boldsymbol{d}\vec{\boldsymbol{S}} = 0$$

Using Gauss divergence theorem, we get

$$\oint \vec{\boldsymbol{B}}.\,\boldsymbol{d}\vec{\boldsymbol{S}} = \int (\vec{\boldsymbol{\nabla}}.\,\vec{\boldsymbol{B}})dV$$

Therefore, $$\int (\vec{\boldsymbol{\nabla}}.\,\vec{\boldsymbol{B}})dV = 0$$

The integrand should vanish for the surface boundary as the volume is arbitrary

i.e. $$\vec{\boldsymbol{\nabla}}.\,\vec{\boldsymbol{B}} = 0$$

Maxwell's third equation

According to Faraday's law of electromagnetic induction

$$emf = -\frac{d\emptyset_B}{dt}$$

By the definition of emf we know

$$emf = \oint \vec{\boldsymbol{E}}\,.\,\boldsymbol{d}\vec{\boldsymbol{l}}$$

And by the definition of magnetic flux we know

$$\emptyset_B = \int \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{S}}$$

where S is the surface bounded by the circuit.

Thus, we have

$$\oint \vec{\boldsymbol{E}}\,.\,d\vec{\boldsymbol{l}} = -\frac{d}{dt}\int \vec{\boldsymbol{B}}.\,d\vec{\boldsymbol{S}}$$

Since the surface S does not change its shape or position with time, we can write above equation as

$$\oint \vec{\boldsymbol{E}}\,.\,d\vec{\boldsymbol{l}} = -\int \frac{\partial \vec{\boldsymbol{B}}}{\partial t}.\,d\vec{\boldsymbol{S}}$$

Using Stoke's theorem, we get

$$\oint \vec{\boldsymbol{E}}\,.\,d\vec{\boldsymbol{l}} = \int (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}}).\,d\vec{\boldsymbol{S}} = -\int \frac{\partial \vec{\boldsymbol{B}}}{\partial t}.\,d\vec{\boldsymbol{S}}$$

$$\int (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}} + \frac{\partial \vec{\boldsymbol{B}}}{\partial t}) . \boldsymbol{d}\vec{\boldsymbol{S}} = \mathbf{0}$$

This equation must hold for any arbitrary surface, thus the integrand should vanish

$$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}} + \frac{\partial \vec{\boldsymbol{B}}}{\partial t} = 0$$

Or

$$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}} = -\frac{\partial \vec{\boldsymbol{B}}}{\partial t}$$

This is Maxwell's third equation.

Maxwell's fourth equation

According to Ampere's circuital law

$$\oint \vec{\boldsymbol{H}}.\boldsymbol{d}\vec{\boldsymbol{l}} = I[\oint \vec{\boldsymbol{B}}.\boldsymbol{d}\vec{\boldsymbol{l}} = \mu_0 I \ \ and\ B = \mu_0 H]$$

We also know $I = \int \vec{\boldsymbol{J}}.\boldsymbol{d}\vec{\boldsymbol{S}}$

So, $\oint \vec{\boldsymbol{H}}.\boldsymbol{d}\vec{\boldsymbol{l}} = \int \vec{\boldsymbol{J}}.\boldsymbol{d}\vec{\boldsymbol{S}}$

Using Stoke's theorem, we get

$$\oint \vec{\boldsymbol{H}}.\boldsymbol{d}\vec{\boldsymbol{l}} = \int (\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}}).\boldsymbol{d}\vec{\boldsymbol{S}} = \int \vec{\boldsymbol{J}}.\boldsymbol{d}\vec{\boldsymbol{S}}$$

$$\int (\vec{\nabla} \times \vec{H} - \vec{J}).\, d\vec{S} = 0$$

For any arbitrary surface, this equation must hold. Thus, the integrand should vanish

$$\vec{\nabla} \times \vec{H} - \vec{J} = 0$$

$\vec{\nabla} \times \vec{H} = \vec{J}$...................(1)

This equation derived on the basis of ampere's law stands only for steady state current but for time varying fields the current density should be modified. Thus taking divergence of both sides of above equation we get,

$$\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = \vec{\nabla}.\vec{J}$$

$\vec{\nabla}.\vec{J} = 0$................. (2) [since, $\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = 0$]

But by equation of continuity, we have

$\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t}$........................(3)

Maxwell realized the situation and suggested that the definition of total current density is incomplete and suggested to add something to J, such that equation 1 becomes

$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}'$.......................... (4)

Now taking divergence of both the sides we get

$$\vec{\nabla}.\left(\vec{\nabla} \times \vec{H}\right) = \nabla.(\vec{J} + \vec{J}')$$

Or $$\vec{\nabla}.(\vec{J} + \vec{J}') = 0$$

Or $$\vec{\nabla}.\vec{J} = -\vec{\nabla}.\vec{J}'$$

Putting the value of $\vec{\nabla}.\vec{J}$ from equation (3) we get

Or $$\vec{\nabla}.\vec{J}' = \frac{\partial \rho}{\partial t}$$

But we know from Maxwell's first equation,

$$\vec{\nabla}.\vec{D} = \rho$$

Therefore $$\vec{\nabla}.\vec{J}' = \frac{\partial}{\partial t}(\vec{\nabla}.\vec{D})$$

$$\vec{\nabla}.\vec{J}' = \vec{\nabla}.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

Hence

$$\vec{\boldsymbol{J}}' = \frac{\partial \overrightarrow{\boldsymbol{D}}}{\partial t}$$

Substituting the value of J' in equation 4, we get

$$\vec{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{H}} = \vec{\boldsymbol{J}} + \frac{\partial \overrightarrow{\boldsymbol{D}}}{\partial t}$$

This is Maxwell's fourth equation. The second term arises when the electric displacement vector $\overrightarrow{\boldsymbol{D}}$ is changing with time and is called displacement current density, $\vec{\boldsymbol{J}}$ is called conduction current density.

Maxwell's equations

Maxwell's equation in Integral form

(1) **First Equation: -**

We know Maxwell's first equation in differential form

$$\vec{\nabla}.\vec{D} = \rho$$

Integrating over an entire volume we get

$$\int (\vec{\nabla}.\vec{D})dV = \int \rho\, dV$$

Applying Gauss divergence theorem on LHS we get

$$\oint \vec{D}.d\vec{S} = \int \rho\, dV$$

$$\oint (\varepsilon_0 \vec{\boldsymbol{E}}).\, \boldsymbol{d}\vec{\boldsymbol{S}} = \int \rho \, dV$$

$$\varepsilon_0 \oint \vec{\boldsymbol{E}}.\, \boldsymbol{d}\vec{\boldsymbol{S}} = \int \rho \, dV$$

$$\oint \vec{\boldsymbol{E}}.\, \boldsymbol{d}\vec{\boldsymbol{S}} = \frac{1}{\varepsilon_0} \int \rho \, dV = \frac{q}{\varepsilon_0}$$

$$\oint \vec{\boldsymbol{E}}.\, \boldsymbol{d}\vec{\boldsymbol{S}} = \frac{q}{\varepsilon_0}$$

(2) Second Equation:

We know Maxwell's second equation in differential form

$$\vec{\nabla}.\vec{B} = 0$$

Integrating over an entire volume we get

$$\int \left(\vec{\nabla}.\vec{B}\right) dV = 0$$

Applying Gauss divergence theorem on LHS we get

$$\oint \vec{B}\,.\,d\vec{S} = 0$$

(3) Third Equation:

We know Maxwell’s third equation in differential form

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Integrating over an open surface area we get

$$\int (\vec{\nabla} \times \vec{E}).\, d\vec{S} = -\int \frac{\partial \vec{B}}{\partial t}.\, d\vec{S}$$

Applying Stoke's theorem on LHS, $\int (\vec{\nabla} \times \vec{E}).\, d\vec{S} = \oint \vec{E} .\, d\vec{l}$

we get,

$$\oint \vec{E} .\, d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}.\, d\vec{S}$$

$$\oint \vec{E} .\, d\vec{l} = -\frac{d}{dt}\int \vec{B}.\, d\vec{S}$$

(4) Fourth Equation:

We know

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Integrating over an open surface area we get

$$\int (\vec{\nabla} \times \vec{H}).\, d\vec{S} = \int \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right).\, d\vec{S}$$

$$\int (\vec{\nabla} \times \vec{H}).\, d\vec{S} = \int \vec{J}.\, d\vec{S} + \int \frac{\partial \vec{D}}{\partial t}.\, d\vec{S}$$

$$\int (\vec{\nabla} \times \vec{H}).\, d\vec{S} = \int \vec{J}.\, d\vec{S} + \int \frac{\partial \vec{D}}{\partial t}.\, d\vec{S}$$

Applying Stoke's theorem on LHS we get

$$\oint \vec{H}.\, d\vec{l} = I + \int \frac{\partial \vec{D}}{\partial t}.\, d\vec{S}$$

It is modified Ampere`s law. It states that any current carrying conductor as well as time varying electric field produces a magnetic field.

Physical Significance of Maxwell's equation

First Equation:

It is Gauss law in electrostatics. It states that the surface integral of electric field over any closed surface area is equal to $\frac{1}{\varepsilon_0}$ times of net charge enclosed by that surface.

Second Equation:

Ques: Show that magnetic monopoles does not exist?[2020-21]

It is Gauss law in magneto statics. It states that there is no existence of magnetic monopoles or the net magnetic flux through any closed surface area is zero.
It also signifies that magnetic field lines are closed curves.

Physical Significance of Maxwell’s equation

Third Equation:

It is Faraday`s law in electromagnetic induction. It states that induced emf around any closed path is equal to the negative rate of change of magnetic flux bounded by the surface w.r.t. time. i.e. any changing magnetic field produces a electric field.

Fourth Equation:

It is modified Ampere`s law. It states that any current carrying conductor as well as time varying electric field produces a magnetic field.

Derive Coulomb`s law from Maxwell`s first equation. [2018-19]

Maxwell`s first equation , $\vec{\nabla}.\vec{D} = \rho$

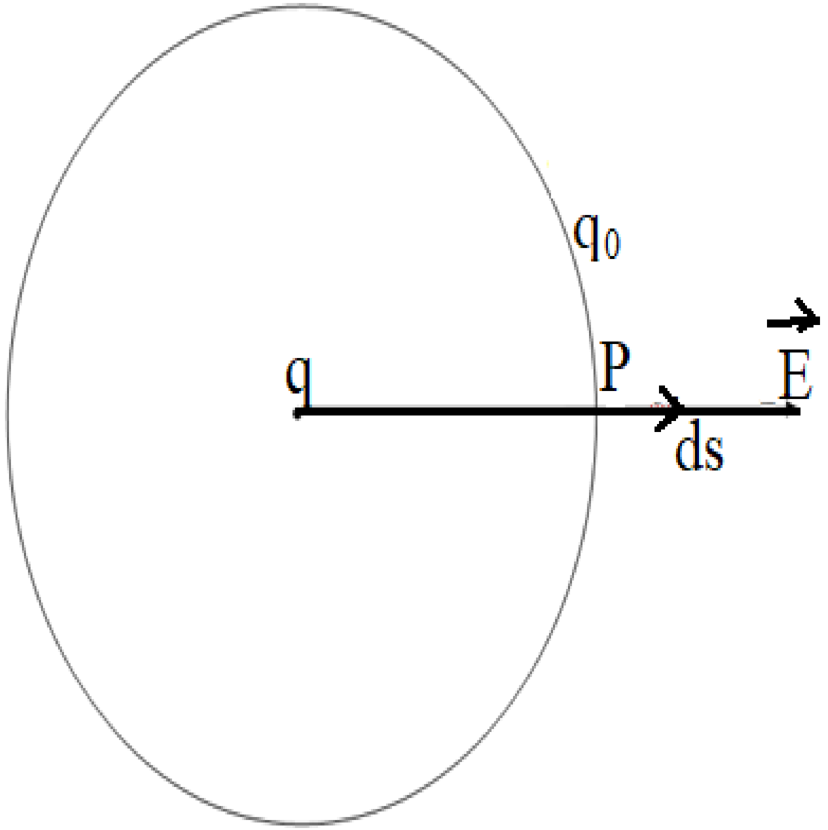
Integrating over an entire volume we get

$$\int (\vec{\nabla}.\vec{D}) dV = \int \rho \, dV$$

Applying Gauss divergence theorem on LHS we get

$$\oint \vec{D}.d\vec{S} = \int \rho \, dV$$

$$\oint (\varepsilon_0 \vec{E}).d\vec{S} = \int \rho \, dV$$



$$\varepsilon_0 \oint \vec{\boldsymbol{E}}.\boldsymbol{d}\vec{\boldsymbol{S}} = q$$

$$\oint \mathrm{E}\boldsymbol{dS} = \frac{q}{\varepsilon_0}$$ (‘q’ is point charge)

For a positive point charge electric field will be radial outward and so it will along$\boldsymbol{d}\vec{\boldsymbol{S}}$

$$\mathrm{E} \oint \boldsymbol{dS} = \frac{q}{\varepsilon_0}$$

$$\mathrm{E}.\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\mathrm{E} = \frac{q}{4\pi\varepsilon_0 r^2}$$

If test charge q_0 is placed at point P, then electrostatic force experienced by q_0 is given by

$$F = q_0 E$$

$$F = \frac{q_0 q}{4\pi\varepsilon_0 r^2}$$

This is Coulomb`s law

Derive equation of continuity from Maxwell`s fourth equation.

Maxwell`s fourth equation is , $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Taking divergence on both sides we get

$$\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = \vec{\nabla}.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$\vec{\nabla}.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0;$ [since $\vec{\nabla}.(\vec{\nabla} \times \vec{H}) = 0$]

$$\vec{\nabla}.\vec{J} + \vec{\nabla}.\left(\frac{\partial \vec{D}}{\partial t}\right) = 0$$

$$\vec{\nabla}.\vec{J} + \frac{\partial(\vec{\nabla}.\vec{D})}{\partial t} = 0$$

From Maxwell`s first equation we know,

$$\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{D}} = \rho$$

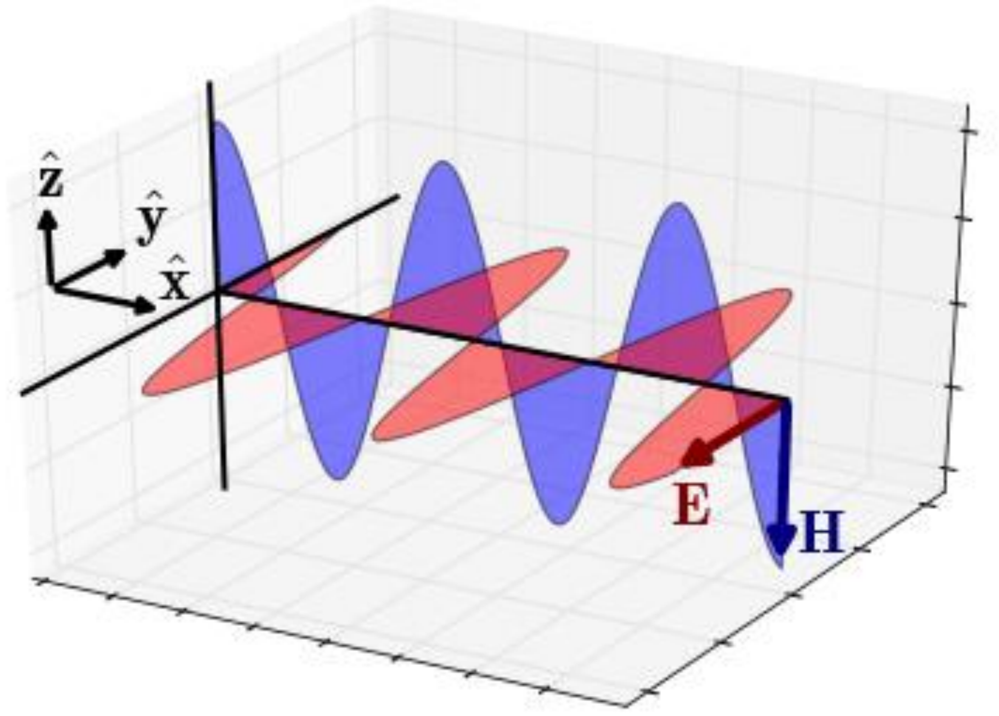
Putting the value of $\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{D}}$,

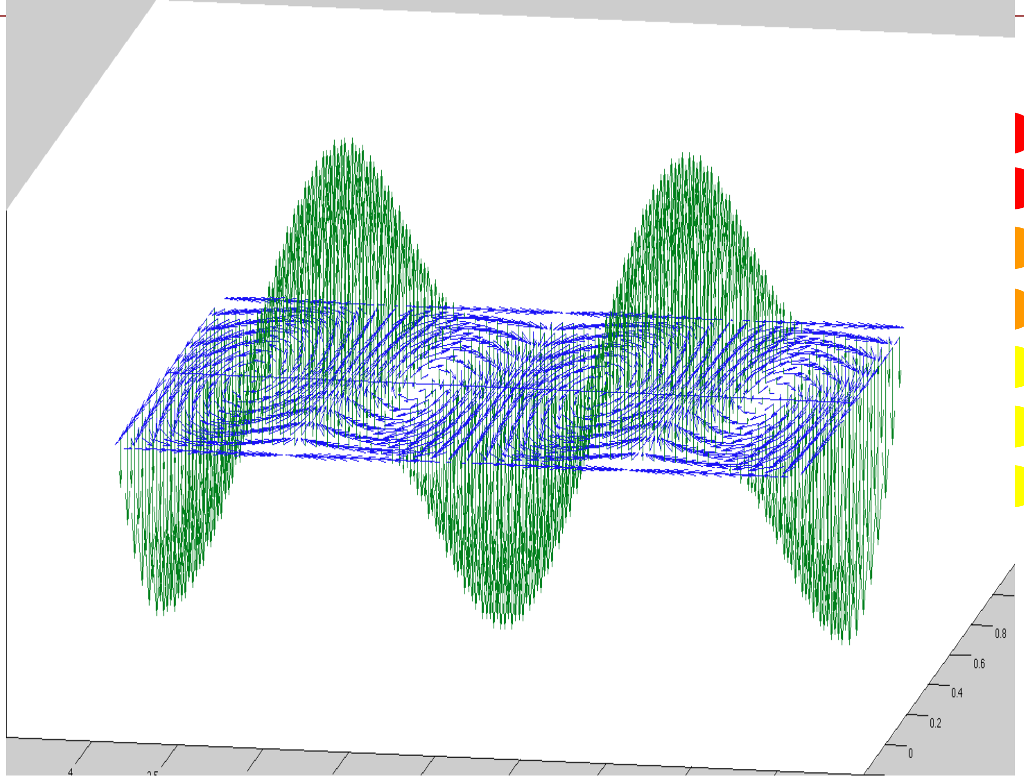
we get, $\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{J}} + \frac{\partial \rho}{\partial t} = 0$

This is the equation of continuity.

Lecture 13

ELECTROMAGNETIC FIELD THEORY





CONTENTS

State and deduce Poynting theorem for the flow of energy in an electromagnetic field. Also, discuss its physical significance.

[**2016-17, 2019-20, 2020-21**]

or

Discuss the work-energy theorem for the flow of energy in an electromagnetic field.

[**2018-19**]

Poynting theorem

Poynting theorem describes the flow of energy or power in an electromagnetic field during the propagation of uniform plane wave. Maxwell's third and fourth equations are given

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (1)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{........} \qquad (2)$$

Taking dot product of equation (1) with $\vec{\boldsymbol{H}}$ and equation (2) with $\vec{\boldsymbol{E}}$, we get

$$\vec{\boldsymbol{H}}.\left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}}\right) = -\vec{\boldsymbol{H}}.\frac{\partial \vec{\boldsymbol{B}}}{\partial \mathrm{t}} \qquad .... (3)$$

$$\vec{\boldsymbol{E}}.\left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}}\right) = \vec{\boldsymbol{E}}.(\vec{\boldsymbol{J}} + \frac{\partial \vec{\boldsymbol{D}}}{\partial t}) \qquad .... (4)$$

Subtracting equation (4) from equation (3) we get

$$\vec{\boldsymbol{H}}.\left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}}\right) - \vec{\boldsymbol{E}}.\left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}}\right) = -\vec{\boldsymbol{H}}.\frac{\partial \vec{\boldsymbol{B}}}{\partial \mathrm{t}} - \vec{\boldsymbol{E}}.\vec{\boldsymbol{J}} - \vec{\boldsymbol{E}}.\frac{\partial \vec{\boldsymbol{D}}}{\partial t} \qquad ... (5)$$

Using vector identity, $\quad \vec{\boldsymbol{\nabla}}.\left(\vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}\right) = \vec{\boldsymbol{H}}.\left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}}\right) - \vec{\boldsymbol{E}}.\left(\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{H}}\right) ... (6)$

From equations (5) and (6), we get

$$\vec{\nabla}.\left(\vec{E}\times\vec{H}\right) = -\vec{H}.\frac{\partial\vec{B}}{\partial t} - \vec{E}.\vec{J} - \vec{E}.\frac{\partial\vec{D}}{\partial t} \qquad \ldots.(7)$$

on rearranging, $\vec{\nabla}.\left(\vec{E}\times\vec{H}\right) = -\vec{E}.\vec{J} - \vec{E}.\frac{\partial\vec{D}}{\partial t} - \vec{H}.\frac{\partial\vec{B}}{\partial t}$

or $\quad -\vec{\nabla}.\left(\vec{E}\times\vec{H}\right) = \vec{E}.\vec{J} + \vec{E}.\frac{\partial\vec{D}}{\partial t} + \vec{H}.\frac{\partial\vec{B}}{\partial t} \qquad \ldots(8)$

Now, since $\vec{D}= \varepsilon\vec{E}$ and $\vec{B}= \mu\vec{H}$

$$\vec{E}.\frac{\partial\vec{D}}{\partial t} = \varepsilon\vec{E}.\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\varepsilon\frac{\partial(\vec{E}.\vec{E})}{\partial t} = \frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon\left(E^2\right)\right]$$

Similarly,

$$\vec{H}.\frac{\partial \vec{B}}{\partial t} = \mu\vec{H}.\frac{\partial \vec{H}}{\partial t} = \frac{1}{2}\mu\frac{\partial(\vec{\boldsymbol{H}}.\vec{\boldsymbol{H}})}{\partial t} = \frac{\partial}{\partial t}[\frac{1}{2}\mu(H^2)]$$

Making these substitutions in equation (8), we get

$$-\vec{\nabla}.\left(\vec{E}\times\vec{H}\right) = \vec{E}.\vec{J} + \frac{\partial}{\partial t}[\frac{1}{2}\varepsilon(E^2)] + \frac{\partial}{\partial t}[\frac{1}{2}\mu(H^2)]$$

Taking the volume integral of the above equation we get

$$-\int \vec{\nabla}.\left(\vec{E}\times\vec{H}\right)dV = \int(\vec{E}.\overrightarrow{J})dV + \int\frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon(E^2)\right]dV + \int\frac{\partial}{\partial t}\left[\frac{1}{2}\mu(H^2)\right]dV$$

Applying Gauss divergence theorem on LHS, we get

$$-\int(\vec{E}\times\overrightarrow{H}).\overrightarrow{dS} = \int(\vec{E}.\vec{J})dV + \frac{d}{dt}\int\frac{1}{2}[\varepsilon(E^2)+\mu(H^2)]dV$$

Or $$\underbrace{\int(\vec{E}.\vec{J})dV}_{\textbf{I term}} = -\underbrace{\frac{d}{dt}\int\frac{1}{2}[\varepsilon(E^2)+\mu(H^2)]dV}_{\textbf{II term}} - \underbrace{\int(\vec{E}\times\overrightarrow{H}).\overrightarrow{dS}}_{\textbf{III term}} \;\ldots\ldots\ldots.(9)$$

Above equation represent work-energy theorem and is called Poynting theorem. In above equation,

(i) The first term represents the total power dissipated in a volume 'V'.

(ii)The second term represents the sum of energy stored in electric field $\{\frac{1}{2}\int[\varepsilon(E^2)\}$ and in magnetic field$\{\frac{1}{2}\int\mu(H^2)]dV\}$.

(iii) The third term represents the rate at which the energy is carried out of volume ‘V’, across its boundary surface by electromagnetic wave.

- **Physical significance :** Above (i), (ii), and (iii) statement represent the physical significance of Poynting theorem or we can say that physical significance of Poynting theorem is ‘**conservation of energy**’, as it is clear from equation (9) that the work done on the charge by an EM force is equal to decrease in energy stored in field, less than the energy which flowed out through surface.

What is Poynting vector? [2016-17, 2018-19]

Poynting vector

The amount of field energy passing through unit area of the surface per unit time is called Poynting vector. It is represented by

$$\vec{S} = \vec{E} \times \vec{H}$$

Since, $\vec{E}$ and $\vec{H}$ are perpendicular to each other ,thus the magnitude of $\vec{S}$ is given by

$\vec{S} = EH$ [$\vec{S} = \vec{E} \times \vec{H}$= EH sinθ]

The direction of $\vec{S}$ is along the direction of wave propagation.

What is the unit and dimension of Poynting vector?

Unit of Poynting vector is $\text{Joule}/(\text{m}^2.\text{sec})$ or Watt/m^2.

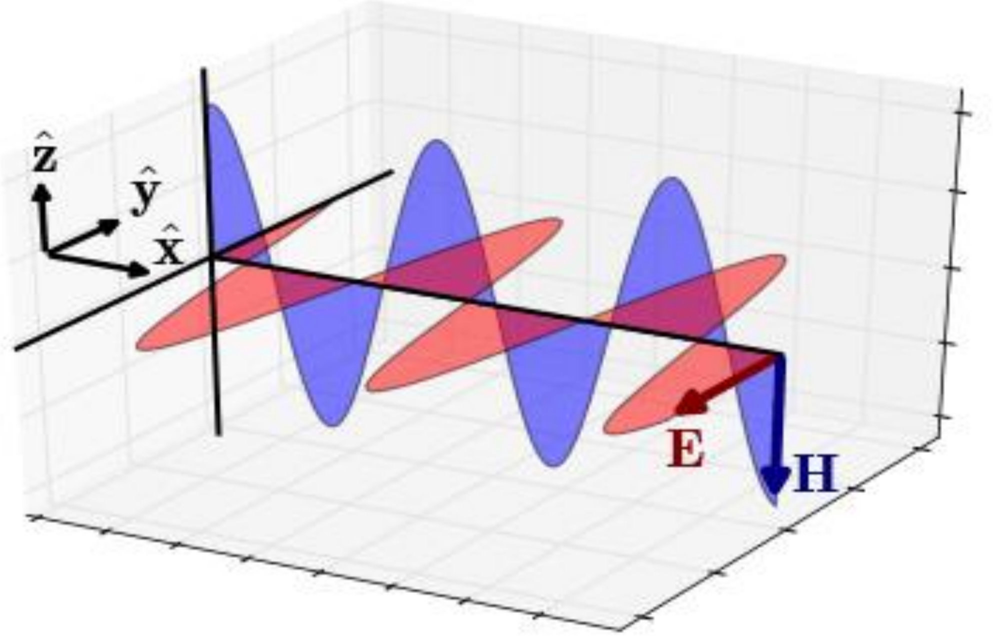
Dimensions of Poynting vector are

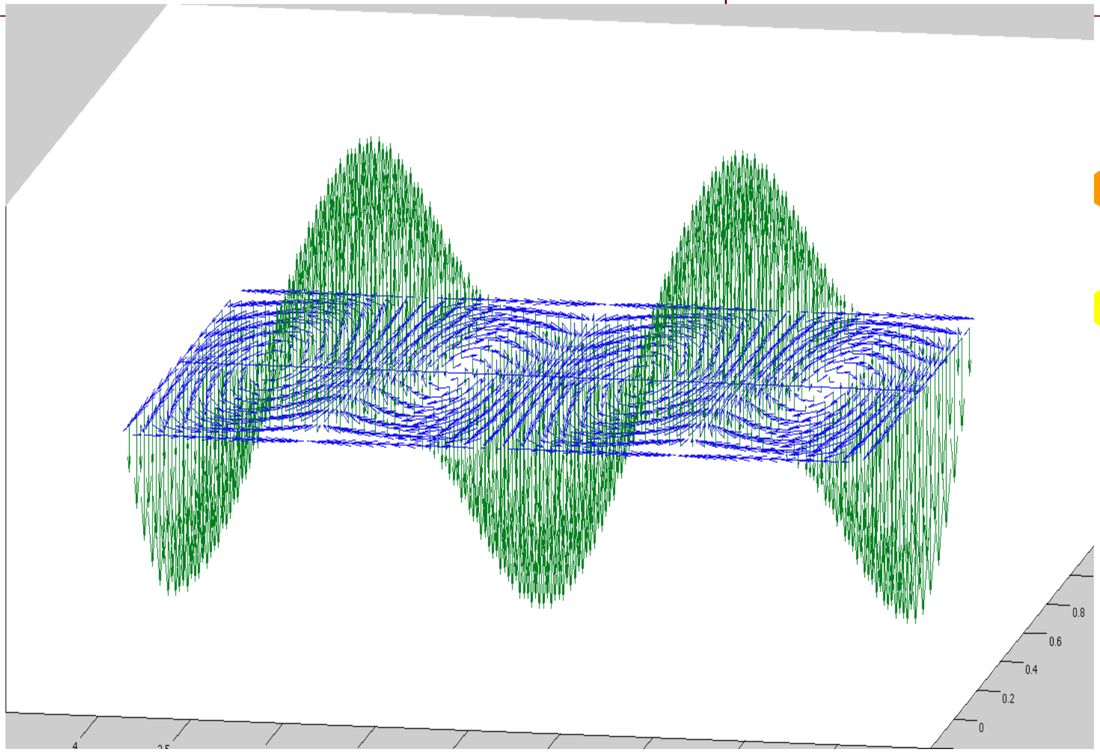
$$\frac{\text{Energy}}{\text{Area. Time}} = \frac{ML^2T^{-2}}{L^2T}$$

$$= MT^{-3}$$

Lecture 14

ELECTROMAGNETIC FIELD THEORY





CONTENTS

- Electromagnetic wave in free space
- Electromagnetic wave in non conducting medium
- Transverse nature of electromagnetic waves

Deduce four Maxwell’s equation in free space.[2019-20, 2020-21]

Electromagnetic wave in free space

Maxwell's equations are

$$\vec{\nabla}.\vec{D} = \rho \qquad \dots\dots.. \quad (1)$$

$$\vec{\nabla}.\vec{B} = 0 \qquad \dots\dots\dots \quad (2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \dots\dots\dots\dots(3)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \dots\dots. \quad (4)$$

Vacuum means that there is no free charge in the region so there is no current produced, i.e., no conduction current. So, we have

$\rho = 0, \overrightarrow{\boldsymbol{J}} = 0, \overrightarrow{\boldsymbol{D}} = \varepsilon_0 \overrightarrow{\boldsymbol{E}}, \overrightarrow{\boldsymbol{B}} = \mu_0 \overrightarrow{\boldsymbol{H}}$

Making these substitutions in above equations. Then, Maxwell`s equations in free space is given as

$$\vec{\nabla}.\overrightarrow{\boldsymbol{E}} = 0 \qquad \dots(5)$$

$$\vec{\nabla}.\overrightarrow{\boldsymbol{H}} = 0 \qquad \dots(6)$$

$$\vec{\nabla} \times \overrightarrow{\boldsymbol{E}} = -\mu_0 \frac{\partial \overrightarrow{\boldsymbol{H}}}{\partial t} \qquad \dots.(7)$$

$$\vec{\nabla} \times \overrightarrow{\boldsymbol{H}} = \varepsilon_0 \frac{\partial \overrightarrow{\boldsymbol{E}}}{\partial t} \qquad \dots(8)$$

Derive the electromagnetic wave equations in free space. Prove that the electromagneticwavespropagatewithspeedof light in free space.

[**2017-18**]

or

Derive the equation for the propagation of plane electromagnetic wave in free space. Show that the velocity of plane electromagnetic wave in free space is given by c =1/$\sqrt{\mu_0 \epsilon_0}$. [**2015-16, 2018-19**]

Maxwell`s equations in free space is given as

$$\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{E}} = 0 \qquad \ldots(1)$$

$$\vec{\boldsymbol{\nabla}}.\vec{\boldsymbol{H}} = 0 \qquad \ldots(2)$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{E}} = -\mu_0 \frac{\partial \vec{\boldsymbol{H}}}{\partial t} \qquad \ldots.(3)$$

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \ldots(4)$$

Taking curl of equation (3), we get

$$\vec{\nabla}\times(\vec{\nabla}\times\overrightarrow{E}) = -\vec{\nabla}\times\left(\mu_0 \frac{\partial \overrightarrow{H}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\overrightarrow{E}) = -\mu_0 \frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H}) \;\ldots\ldots(5)$$

Putting the value of $\overrightarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{H}}$ in equation (5) from equation (4) we get

$$\vec{\nabla}\times(\vec{\nabla}\times\overrightarrow{E}) = -\mu_0 \frac{\partial}{\partial t}\left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\overrightarrow{E}) = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} \text{ ....}\mathbf{(6)}$$

Using vector identity (vector triple product),

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = \vec{\nabla}\left(\vec{\nabla}.\vec{E}\right) - \nabla^2\vec{E}$$

From equation (1) we have $\vec{\nabla}.\vec{E} = 0,$

Therefore $\vec{\nabla}\times\left(\vec{\nabla}\times\vec{E}\right) = -\nabla^2\vec{E}$

Putting this in equation (6), we get

$$-\nabla^2\vec{E} = -\mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \text{............}\mathbf{(7)}$$

Similarly, $\nabla^2 \vec{H} = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$ ....(8)

By general wave equation we have

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \qquad \text{.. ... (9)}$$

where ψ is the wave function which propagates with velocity *v*. Thus, comparing equations (7) & (8) with the above equation (9), we observe that field vectors E & H propagate as waves in free space and the velocity of propagation is

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times \ 8.85 \times 10^{-12}}}$$

$$= \ 2.99 \times 10^{8} m/s = c \text{ (Speed of light)}$$

Thus, EM waves travel in free space with speed of light.So, equations (7) and (8) may be written as

$$\nabla^2 \vec{\boldsymbol{E}} - \frac{1}{c^2}\frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2} = 0 \ \ \& \ \nabla^2 \vec{\boldsymbol{H}} - \frac{1}{c^2}\frac{\partial^2 \vec{\boldsymbol{H}}}{\partial t^2} = 0$$

Write the Maxwell’s equation in non conducting medium. Also, derive electromagnetic wave equation in non conducting medium.

Electromagnetic wave in non conducting medium

Maxwell's equations are

$$\vec{\nabla}.\vec{D} = \rho \qquad \dots\dots..(1)$$

$$\vec{\nabla}.\vec{B} = 0 \qquad \dots\dots\dots(2)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \dots\dots\dots\dots(3)$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \dots\dots.(4)$$

Non conducting medium means non-charged, current free dielectric are non-conducting media. So, we have

$$\rho = 0, \vec{J} = 0, \vec{D} = \varepsilon\vec{E}, \vec{B} = \mu\vec{H}$$

Making these substitutions in above equations, we get Maxwell`s equations in non conducting medium

$$\vec{\nabla}.\vec{\boldsymbol{E}} = 0 \qquad \dots(5)$$

$$\vec{\nabla}.\vec{\boldsymbol{H}} = 0 \qquad \dots(6)$$

$$\vec{\nabla} \times \vec{\boldsymbol{E}} = -\mu\frac{\partial\vec{\boldsymbol{H}}}{\partial t} \qquad \dots(7)$$

$$\vec{\nabla} \times \vec{\boldsymbol{H}} = \varepsilon\frac{\partial\vec{\boldsymbol{E}}}{\partial t} \qquad \dots(8)$$

Taking curl of equation (7), we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times(\mu\frac{\partial\vec{H}}{\partial t})$$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H}) \;\ldots\ldots(9)$$

Putting the value of $\vec{\nabla}\times\overrightarrow{\boldsymbol{H}}$ in equation (9) from equation (8) we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\frac{\partial}{\partial t}\left(\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\mu\varepsilon\frac{\partial^2 E}{\partial t^2} \;\ldots\ldots..(10)$$

Using vector identity (vector triple product),

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}.\vec{E}) - \nabla^2\vec{E}$$

From equation (1) we have $\vec{\nabla}.\vec{E} = 0,$

Therefore $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\nabla^2\vec{E}$

Putting this in equation (6), we get

$$\nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2 E}{\partial t^2}$$

Similarly, $$\nabla^2\vec{H} = \mu\varepsilon\frac{\partial^2 H}{\partial t^2}$$

Prove that electromagnetic waves are transverse in nature.

[2017-18, 2018-19]

or

Show that electric and magnetic vectors are normal to the direction of propagation of electromagnetic wave. [2020-21]

or

Show that vector E, H and direction of propagation form a set of orthogonal vectors.

[2016-17]

Transverse nature of electromagnetic waves

The equation for electric and magnetic field in free space are given by

$$\nabla^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}\,;\ \nabla^2\vec{H} = \mu_0\varepsilon_0\frac{\partial^2 H}{\partial t^2}$$

The solution of above equations may be written as:

$$\vec{\boldsymbol{E}}(r,t) = \boldsymbol{E_0}e^{i(\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}}-\omega t)}$$

$$\vec{\boldsymbol{H}}(r,t) = \boldsymbol{H_0}e^{i(\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}}-\omega t)}$$

where E_0 *and* H_0 *are* amplitudes of electric & magnetic fields and k ispropagation vector defined as

$$\vec{\boldsymbol{k}} = k\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi\nu}{c}\hat{n} = \frac{\omega}{c}\hat{n}$$

Here $\hat{n}$ is a unit vector in the direction of wave propagation.

Since there are no angular coordinates in $\overrightarrow{\boldsymbol{E}}$ and $\overrightarrow{\boldsymbol{H}}$, therefore

$$\vec{\nabla} = \frac{\partial}{\partial r}$$

Now $$\frac{\partial \vec{\boldsymbol{E}}}{\partial r} = \frac{\partial}{\partial r}\boldsymbol{E_0} e^{i\left(\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}}-\omega t\right)} (\text{since } \frac{\partial\,(\boldsymbol{e^{ix}})}{\partial \boldsymbol{x}} = \boldsymbol{ie^{ix}})$$

$$= ik\boldsymbol{E_0} e^{i\left(\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}}-\omega t\right)}$$

$$= ik\vec{\boldsymbol{E}}$$

$$\Rightarrow \vec{\nabla} = \frac{\partial}{\partial r} = ik \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now $\frac{\partial \vec{\boldsymbol{E}}}{\partial t} = \frac{\partial}{\partial t} \boldsymbol{E_0} e^{i(\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}} - \omega t)}$

$$= (-i\omega)\boldsymbol{E_0} e^{i(\vec{\boldsymbol{k}}.\vec{\boldsymbol{r}} - \omega t)}$$

$$= -i\omega\vec{\boldsymbol{E}}$$

$$\Rightarrow \quad \frac{\partial}{\partial t} = -i\omega \ldots\ldots\ldots\ldots\ldots(2)$$

Making the substitutions for ∇ in equation $\vec{\nabla}.\vec{\boldsymbol{E}} = \boldsymbol{0}$, we get

$$i\vec{\boldsymbol{k}}.\vec{\boldsymbol{E}} = \boldsymbol{0}$$

$$\vec{\boldsymbol{k}}.\vec{\boldsymbol{E}} = 0$$

i.e. $\qquad \vec{\boldsymbol{k}} \perp \vec{\boldsymbol{E}}$

Similarly making the substitutions for ∇ in equation $\vec{\nabla}.\vec{E} = 0$ we get

$$i\overrightarrow{\boldsymbol{k}}.\overrightarrow{\boldsymbol{H}} = \mathbf{0}$$

$$\overrightarrow{\boldsymbol{k}}.\overrightarrow{\boldsymbol{H}} = 0$$

i.e. $\overrightarrow{\boldsymbol{k}} \perp \overrightarrow{\boldsymbol{H}}$

Therefore, $\overrightarrow{\boldsymbol{k}} \perp \overrightarrow{\boldsymbol{H}} \perp \overrightarrow{\boldsymbol{E}}$

Now making the substitutions for ∇ and $\frac{\partial}{\partial t}$ from eq (1) and (2) in equation

$\overrightarrow{\boldsymbol{\nabla}} \times \overrightarrow{\boldsymbol{E}} = -\mu_0 \frac{\partial \overrightarrow{\boldsymbol{H}}}{\partial t}$, we get

$$i(\vec{\boldsymbol{k}} \times \vec{\boldsymbol{E}}) = \mu_0 i\omega \vec{\boldsymbol{H}}$$

$$k\,(\hat{\boldsymbol{n}} \times \vec{\boldsymbol{E}}) = \mu_o\, \omega\, \vec{\boldsymbol{H}}$$

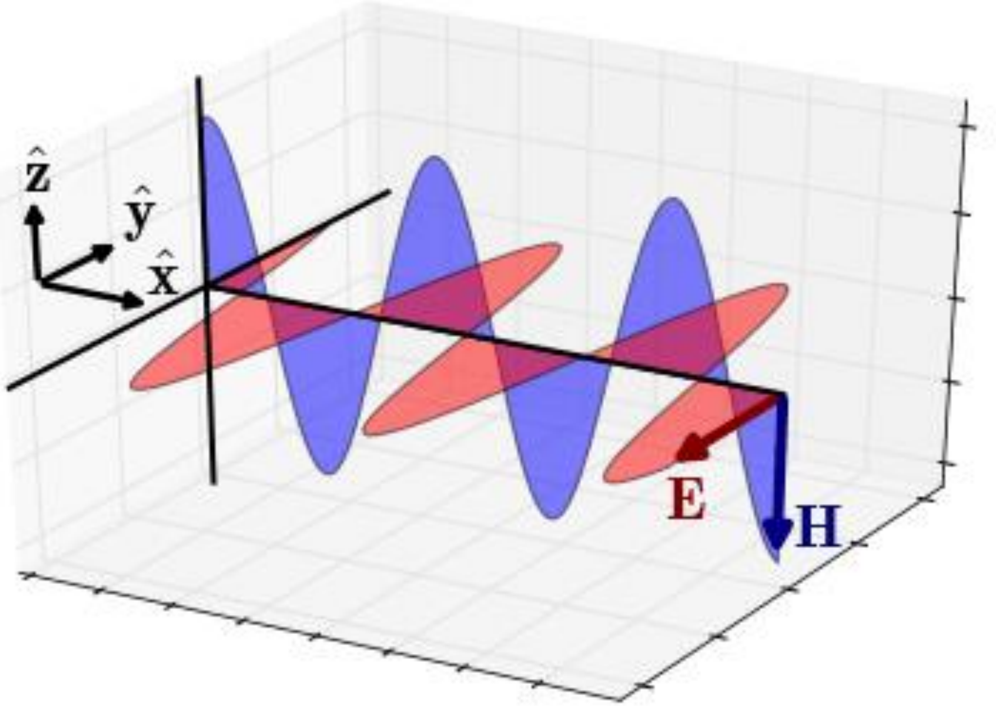
$$\frac{k}{\mu_o\, \omega}(\hat{\boldsymbol{n}} \times \vec{\boldsymbol{E}}) = \vec{\boldsymbol{H}}$$

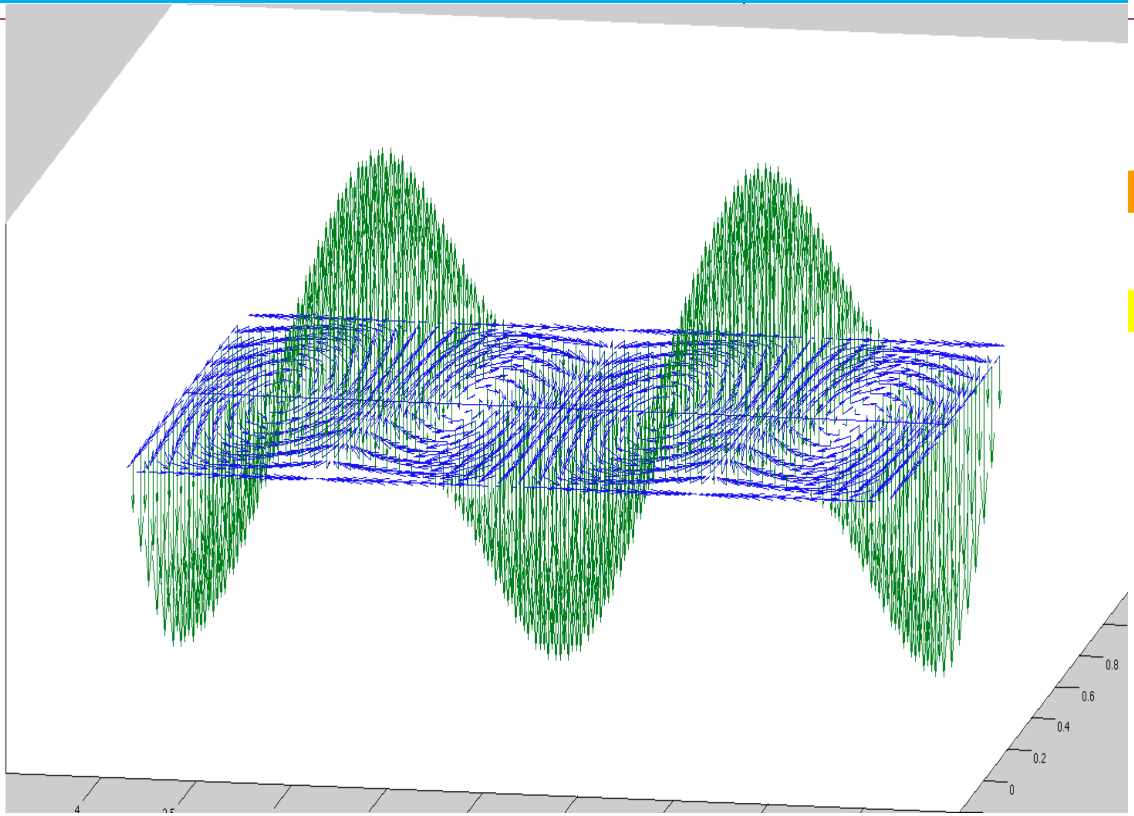
$$\frac{1}{\mu_o\, c}(\hat{\boldsymbol{n}} \times \vec{\boldsymbol{E}}) = \vec{\boldsymbol{H}}\left[\text{since } \frac{\omega}{k} = c\right]$$

From above equation, it is clear that field vector $\vec{H}$ is perpendicular to both $\vec{k}$ and $\vec{E}$. Thus, we see that electric field vectors E, magnetic field vectors H and the direction of propagation vector $\vec{\boldsymbol{k}}$ are mutually perpendicular to each other. This implies that electromagnetic waves are transverse in nature.

Lecture 15

ELECTROMAGNETIC FIELD THEORY





CONTENTS

What do you mean by impedance of a wave? [2019-20]

or

What do you understand by characteristics impedance?

Relation between electric and magnetic field

The ratio of magnitude of E to the magnitude of H is symbolized as Z_o and has the dimensions of electric resistance.

We know $$(\vec{\boldsymbol{k}} \times \vec{\boldsymbol{E}}) = \mu_0 \omega \vec{\boldsymbol{H}}$$

$$kE\ sin90^0 = \mu_0 \omega H$$

$$kE = \mu_0 \omega H$$

$$E = \mu_0 \frac{\omega}{k} H$$

$$E = \mu_0 c\boldsymbol{H} \qquad \left(v = \frac{\omega}{k} = c\right)$$

$$E = \mu_0 \frac{1}{\sqrt{\varepsilon_0\mu_0}} H \qquad \left(c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

$$E = \sqrt{\frac{\mu_0}{\varepsilon_0}} H$$

Therefore, $$\boldsymbol{Z_o} = \left|\frac{\vec{\boldsymbol{E}}}{\vec{\boldsymbol{H}}}\right| = \frac{\boldsymbol{E_o}}{\boldsymbol{H_o}} = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

$$where,\ \mu_o = 4\pi \times 10^{-7}, \qquad \varepsilon_o = 8.85 \times 10^{-12}$$

The ratio Z_o is a universal constant and is called characteristic impedance or wave impedance of free space.

$$\boldsymbol{Z_o} = 376.72\ ohms$$

Obtain the expression for total electromagnetic energy density.

Energy density of plane EM wave in free space

The electric energy per unit volume or electric field energy density ‘u_E’ is given by

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \ldots\ldots\ldots\ldots(1)$$

Similarly, the magnetic field energy density U_B is given by

$$u_B = \frac{1}{2}\mu_o H^2 \ldots\ldots(2)$$

In electromagnetic field, sum of energy densities due to both $\vec{E}$ and $\vec{B}$ is

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_o H^2,$$

$$\frac{E}{H} = \sqrt{\frac{\mu_o}{\varepsilon_o}} \quad \text{or} \quad H = E\sqrt{\frac{\varepsilon_o}{\mu_o}} \qquad \text{.........(3)}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 \frac{\boldsymbol{\varepsilon_o}}{\boldsymbol{\mu_o}} E^2$$

$$u = \varepsilon_0 E^2 \text{.......(4)}$$

or $$u = \mu_o H^2 \qquad \text{........(5)}$$

Show that magnetic field and electric field of electromagnetic wave carries equal energies in free space.

We know that electric energy density, $u_E = \frac{1}{2}\varepsilon_0 E^2$ ....(1)

And magnetic energy density, $u_B = \frac{1}{2}\mu_o H^2$............(2)

Divide (1) from (2), $$\frac{u_E}{u_B} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{1}{2}\mu_0 H^2} = \frac{\varepsilon_0 E^2}{\mu_0 H^2}$$

$\frac{u_E}{u_B} = \frac{\varepsilon_0 E^2}{\varepsilon_0 E^2} = 1$; [Since, $E = \sqrt{\frac{\mu_0}{\varepsilon_0}} H$]

$$\Rightarrow \qquad u_E = u_B$$

i.e., EM wave consists of equal electric and magnetic field energy.

Derive a relation between Poynting vector and total energy density.

Since, $\vec{\boldsymbol{S}} = \vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}$

If $\vec{\boldsymbol{E}}$ is along $\hat{x}$ – axis, $\vec{\boldsymbol{H}}$ is along $\hat{y}$ – axis. Then, $\vec{\boldsymbol{S}}$ is along $\hat{z}$ - axis

Therefore, $\vec{\boldsymbol{S}} = EH(\hat{x} \times \hat{y}) = EH\hat{z}$

$$\vec{\boldsymbol{S}} = \sqrt{\frac{\varepsilon_0 \; x \; \varepsilon_0}{\mu_0 \varepsilon_0}} E^2 \hat{z} (\text{Since, } \; E = \sqrt{\frac{\mu_0}{\varepsilon_0}} H)$$

$\vec{\boldsymbol{S}} = c\varepsilon_0 \, E^2 \hat{z}$

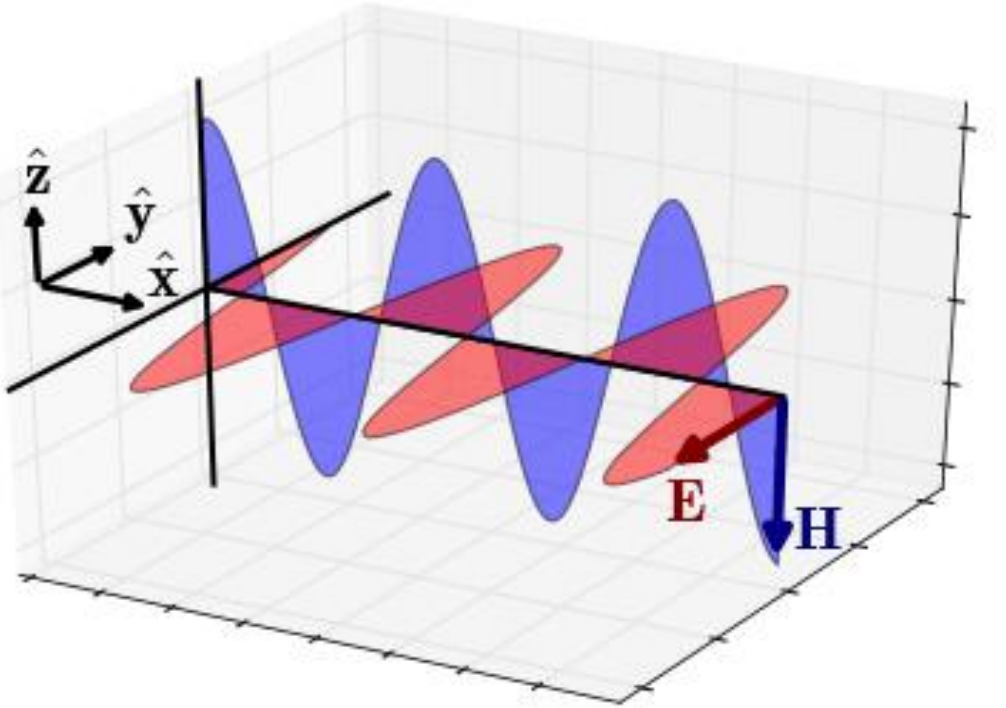
Since, $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

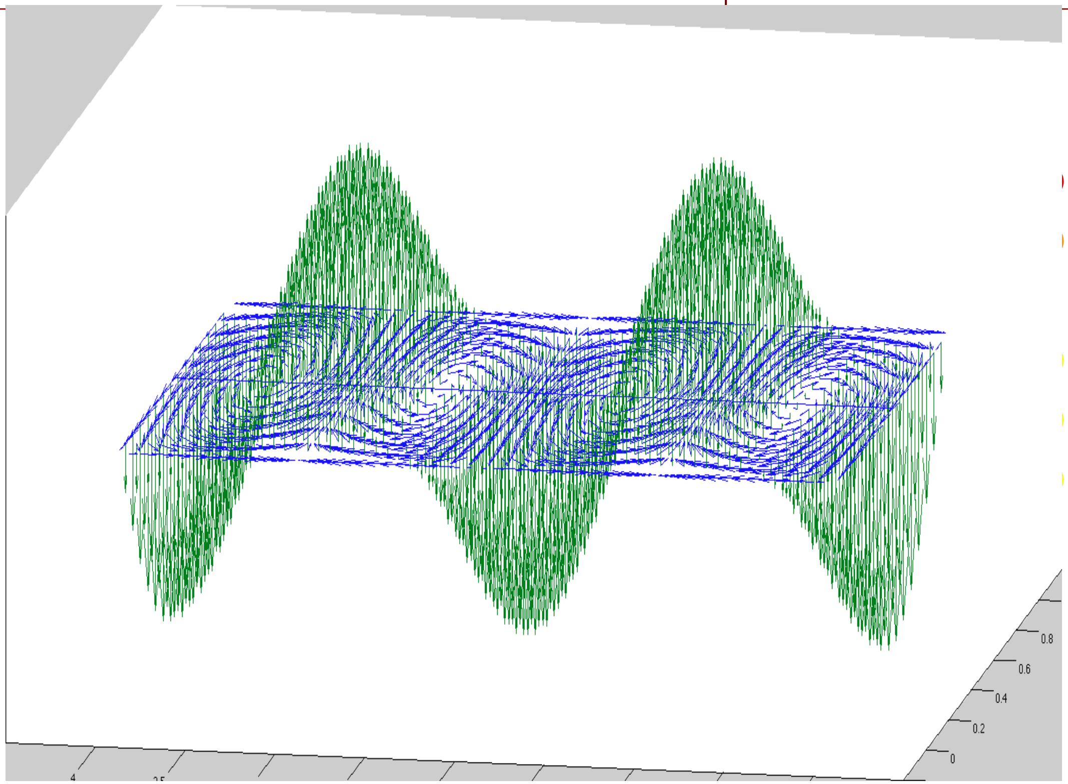
$$\vec{S} = \varepsilon_0 E^2 c\hat{z} = \mathrm{u}c\hat{z}; \qquad\qquad [\mathrm{u} = \varepsilon_0 E^2]$$

Thus, energy density associated with an electromagnetic wave in free space travels with speed equal to velocity of light with which the field vectors propagate.

Lecture 16

ELECTROMAGNETIC FIELD THEORY





CONTENTS

- Radiation pressure (Resultant pressure)
- Skin depth
- Numericals

What is radiation pressure?

Radiation pressure (Resultant pressure)

- When electromagnetic radiation strikes to a surface area; its momentum gets changed. It means some momentum get transferred to the surface. If electromagnetic radiation is striking continuously to a surface, then the momentum of the surface will be continuously changed.
- According to Newton`s second law the rate of change of momentum of the surface will be equal to the force exerted on the surface, i.e.

$$\vec{F} = \frac{d\vec{p}}{dt}$$

- The force per unit area on the surface is equal to the pressure. Therefore, if electromagnetic radiation strikes to a surface, then it exerts a pressure on that surface, which is known as radiation pressure.

Derive a suitable expression for momentum and radiated pressure of an EM wave.

or

Derive a suitable expression for energy density and radiated pressure of an EM wave.

Maxwell predicted that electromagnetic waves transport linear momentum in the direction of propagation. The momentum of the particle of mass m moving with velocity v is given by

$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$ ………….(1)

According to Einstein’s mass-energy relation,

$$U = mc^2 \quad \text{or } m = \frac{U}{c^2} \dots \dots (2)$$

Using equation (2) in (1), then momentum density associated with EM wave is

$$\vec{\boldsymbol{p}} = \frac{U}{c^2}\vec{\boldsymbol{v}}$$

If electromagnetic wave propagating along x direction, then $\vec{v} = c\hat{\imath}$

so, $$\vec{p} = \frac{U}{c}\hat{\imath} \Rightarrow p = \frac{U}{c}$$

Let a plane EM wave incident normally on a perfectly absorbing surface having area A. Also let this surface area absorbs energy U in time t.

Then momentum transferred to the surface is

$$p = \frac{U}{c} \;\ldots\ldots..(3)$$

We know $S = \frac{U}{At}$

where S is magnitude of poynting vector.

Therefore,

$$p = \frac{SAt}{c}$$

$$p = uAt \qquad (S = uc)$$

Where u is energy density.

By Newton`s second law , $$F = \frac{dp}{dt} = uA \qquad \dots\dots\dots(4)$$

Now, force exerted per unit area by EM wave is called radiation pressure, i.e., $P = \frac{F}{A}$

$$P = \frac{1}{A}\frac{dp}{dt} \quad \ldots\ldots\ldots(5)$$

Equation (5) gives required relation between radiation pressure and momentum.

Now, Using equation (4) and (5)

$$P = u$$

i.e. Radiation pressure is equal to energy density.

What do you mean by depth of penetration? [2016-17, 2018-19]

Skin depth (Depth of penetration)

When an EM wave propagate in a medium its amplitude decreases with the distance inside the medium from the surface. This phenomenon is known as attenuation.

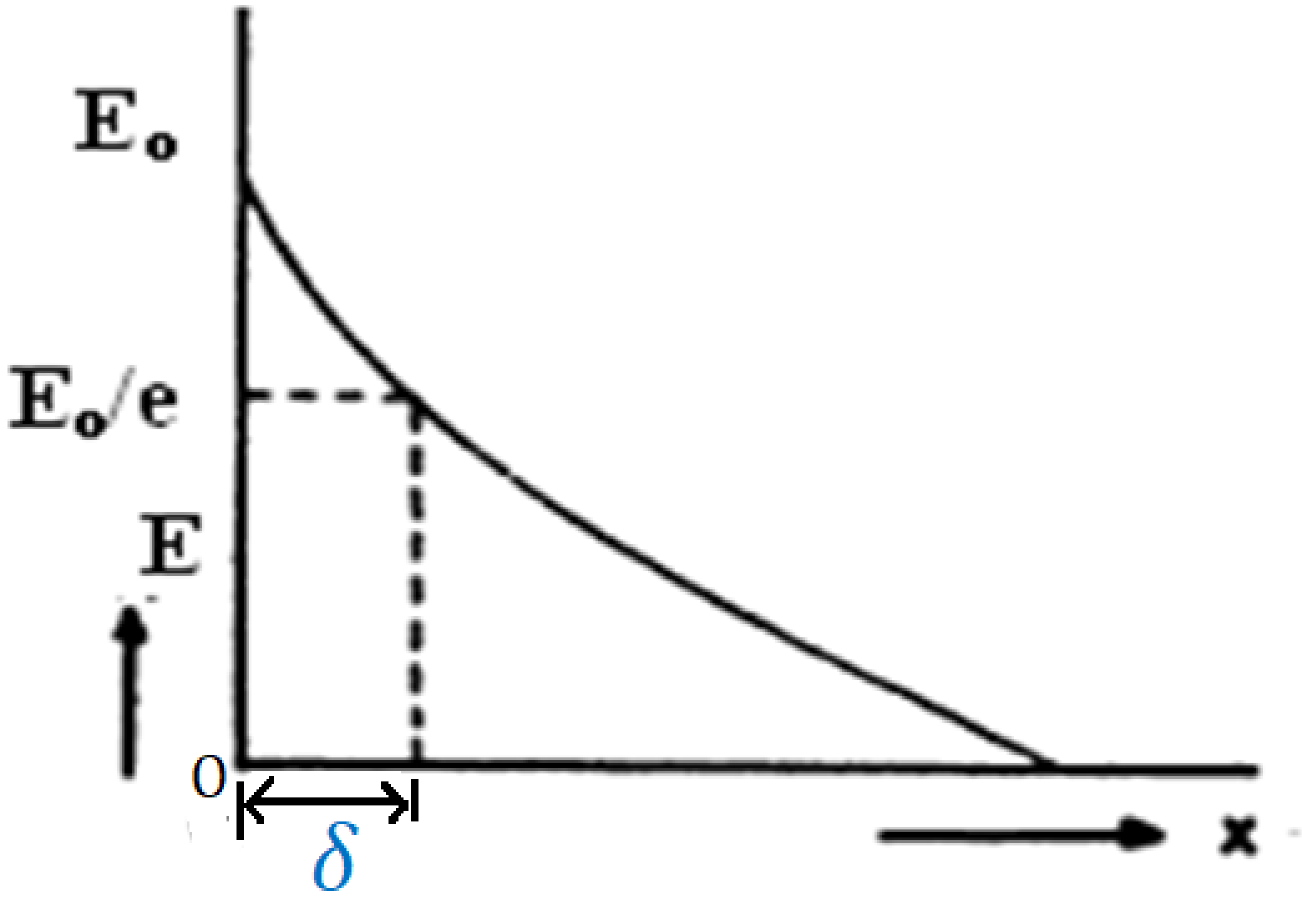
The amplitude of an EM wave at a depth x is given by

$$E = E_0 e^{-\alpha x}$$

Where- E_0 is amplitude of the wave at the surface of the medium,

α is attenuation constant

The depth of penetration is defined as the depth in which the strength of electric field associated with the electromagnetic wave reduces to $\frac{1}{e}$ times to its initial value.



What is the relation between skin depth and attenuation constant?

Since, $E = E_0 e^{-\alpha x}$

At skin depth $x = \delta$, $E = \frac{E_0}{e}$

$$\frac{E_0}{e} = E_0 e^{-\alpha\delta}$$

$$e^{-1} = e^{-\alpha\delta}$$

$$\delta = \frac{1}{\alpha}$$

Thus,

$$skin\ depth = \frac{1}{attenuation\ constant}$$

Derive an expression of skin depth for good conductor and insulator. Also, show that skin depth for insulator does not depends on frequency of EM wave.

Attenuation constant 'α' is given as

$$\alpha = \omega\left[\frac{\mu\varepsilon}{2}\left\{\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{\frac{1}{2}}-1\right\}\right]^{\frac{1}{2}} \ldots\ldots\ldots\ldots\ldots(1)$$

For good conductors:

$$\frac{\sigma}{\omega\varepsilon} \gg 1$$

Therefore neglecting 1 with respect to $\frac{\sigma}{\omega\varepsilon}$ we get

$$\alpha = \omega\left\{\frac{\mu\varepsilon}{2}\left(\frac{\sigma}{\omega\varepsilon} - 1\right)\right\}^{\frac{1}{2}}$$

$$\alpha = \omega\left(\frac{\mu\varepsilon}{2}\frac{\sigma}{\omega\varepsilon}\right)^{\frac{1}{2}}$$

$$\alpha = \left(\frac{\mu\sigma\omega}{2}\right)^{\frac{1}{2}}$$

$$\alpha = \sqrt{\frac{\mu\sigma\omega}{2}}$$

Skin depth, $\delta = \frac{1}{\alpha}$

$$\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$$

Since $\omega = 2\pi f$, where f is a linear frequency

$$\delta = \sqrt{\frac{1}{\mu\sigma\pi f}}$$

- It is clear from the above expression that skin depth for good conductor depends upon the frequency of EM waves.

For poor conductors or insulators:

$$\frac{\sigma}{\omega\varepsilon} \ll 1$$

Therefore

$$\left(1+\frac{\sigma^2}{\omega^2\varepsilon^2}\right)^{\frac{1}{2}} \approx 1+\frac{\sigma^2}{2\omega^2\varepsilon^2}$$

$$\alpha = \omega\left\{\frac{\mu\varepsilon}{2}\left(1+\frac{\sigma^2}{2\omega^2\varepsilon^2}-1\right)\right\}^{\frac{1}{2}}$$

$$\alpha = \omega\left(\frac{\mu\varepsilon}{2}\frac{\sigma^2}{2\omega^2\varepsilon^2}\right)^{\frac{1}{2}}$$

$$\alpha = \frac{\sigma}{2}\left(\frac{\mu}{\varepsilon}\right)^{\frac{1}{2}}$$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

- From this expression it is clear that skin depth for insulator does not depend upon frequency of EM waves.

For a conducting medium, $\sigma = 5.8\times10^{6}\,S/m$ and $\varepsilon_r = 1$. Find conduction and displacement current densities if magnitude of electric field intensity E is given by $E = 150\sin(10^{10}t)\,V/m$. [2018-19]

Conduction current density is given by $J_c = \sigma E$

$$\Rightarrow \qquad J_c = 5.8{\times}10^6 \times 150\, sin(10^{10}t)\ A/m^2$$

$$= 8.7 \times 10^8 sin(10^{10}t)\,\frac{A}{m^2}$$

And, $$J_d = \frac{\partial D}{\partial t} = \varepsilon\frac{\partial E}{\partial t}$$

$$\Longrightarrow \qquad J_d = \varepsilon_r\varepsilon_0\frac{\partial}{\partial t}\{150 sin(10^{10}t)\}$$

$$J_d = 1 \times 8.854 \times 10^{-12} \times 150\ \times 10^{10}\ \times cos(10^{10}t)$$

$$J_d = \qquad 13.28\ \ cos(10^{10}t) A/m^2$$

Calculate the magnitude of Poynting vector at the surface of the sun. Given the power radiated by sun is 5.4×10^{28} Watts and radius of the sun is 7×10^{8} m.

[2018-19]

We know Poynting vector is given by $S = \frac{P}{4\pi r^2}$

$$\Rightarrow \qquad S = \frac{3.8 \times 10^{26}\ W}{4 \times 3.14 \times (7 \times 10^{8}\ m)^2}$$

$$\Longrightarrow \qquad S = \frac{5.4 \times 10^{28}\ Joule}{4 \times 3.14 \times (7 \times 10^{8})^2 m^2 . s}$$

$$\Rightarrow \qquad S = 8.77 \times 10^{9}\ Joule/(m^2 . s)$$

If the magnitude of $\overrightarrow{H}$ in a plane wave is 1 Amp/meter, find the magnitude of $\overrightarrow{E}$ for plane wave in free space. [2015-16]

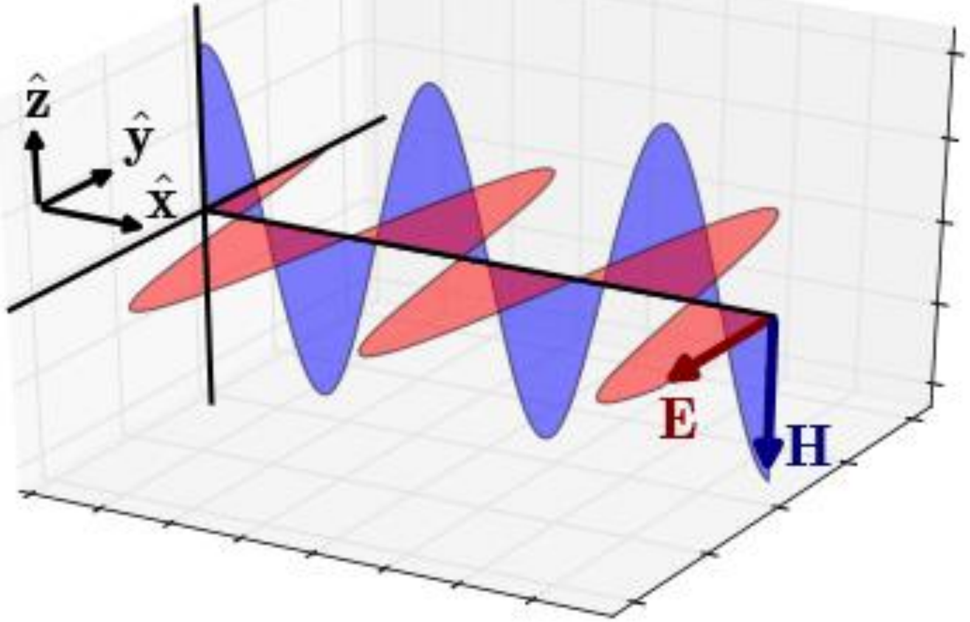
We know that, $\frac{H_o}{E_o} = \sqrt{\frac{\varepsilon_o}{\mu_o}}$

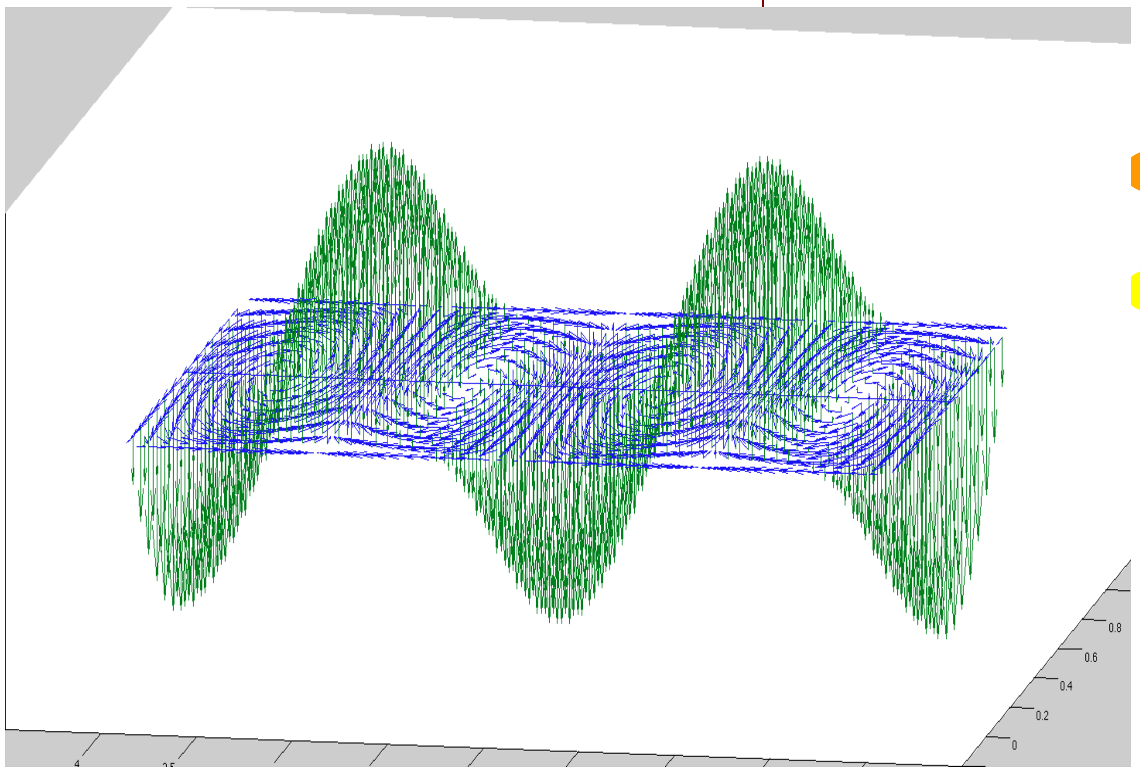
or $E_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} H_o$

$$E_o = 1 \times \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}}$$
$$= 376.72\ V/m$$

Lecture 17

ELECTROMAGNETIC FIELD THEORY





CONTENTS

- Numericals

Earth receives solar energy from the Sun which is $\frac{2cal}{cm^2.Min}$. what are the amplitude of electric and magnetic fields of radiation? **(M. IMP)**

or

The sunlight strikes upper atmosphere of earth with energy $1400\ \text{Watt/m}^2$. Calculate peak values of electric and magnetic fields at points.

$$S = 2cal/(cm^2.Min)$$
$$= \frac{2 \times 4.2\ Joule}{(10^{-2}m)^2 \times 60\ s}$$
$$= 1400\ Joule/(m^2.s)$$

We know Poynting vector is given by,

$S = EXH$= EH sin90 = EH

Therefore $$EH = 1400 \frac{Joule}{m^2.s} \ldots \ldots \ldots \ldots \ldots \ldots \ldots . (1)$$

We also know by characteristic impedance of vacuum is given by

$$\frac{E}{H} = 376.77\ \Omega \quad \ldots\ldots\ldots\ldots..(2)$$

Multiplying equations (1) and equation (2) we get

$$E^2 = 527478\ (N/C)^2$$

$$\Rightarrow\ E = 726.27\ N/C$$

Putting the value of E in equation (1) we get ,

$$H = \frac{1400}{726.27}\ N/(A.m)$$

$$\mathrm{H} = 1.93\ N/(A.m)$$

$$E_0 = E\sqrt{2}$$

$$E_0 = 726.27\sqrt{2}\ N/C$$

$$\Rightarrow \qquad E_0 = 1027.1\ N/C$$

$$H_0 = H\sqrt{2}$$

$$\Rightarrow \qquad H_0 = 1.93\sqrt{2}\ N/(A.m)$$

$$H_0 = 2.73\ N/(A.m)$$

A 100 Watt sodium lamp radiating its power. Calculate electric and magnetic fields strength at a distance of 5 m from the lamp.

[2015-16,2018-19]

We know Poynting vector is given by, $S = \frac{P}{4\pi r^2}$

$$\Rightarrow \qquad S = \frac{100\ W}{4 \times 3.14 \times (5\ m)^2}$$

$$\Rightarrow \qquad S = \frac{100\ Joule}{4 \times 3.14 \times 25\ m^2.s}$$

$$\Rightarrow \qquad S = 3.185 \times 10^{-1} Joule/(m^2.s)$$

We know $S = \vec{E} X \vec{H}$= EH sin90 = EH

Therefore $EH = 3.185 \times 10^{-1}\, J/(m^2.s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$

We also know by characteristic impedance of vacuum is given by

$$\frac{E}{H} = 376.6\,\Omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Multiplying equations (1) and equation (2) we get

$$E^2 = 119.95(V/m)^2$$

$$E = 10.95\, V/m$$

Putting the value of E in equation (2), we get

$$H = \frac{10.95}{376.6}\,\frac{amp - turn}{m}$$

$$H = 0.0291\,\frac{amp - turn}{m}$$

The sunlight strikes the upper atmosphere of earth with energy flux 1.38 kWm^{-2}. What will be the peak values of electric and magnetic fields at the points.

[2019-20]

As we know that ‘S’ is given by energy per unit time per unit area.

$$S = 1.38\ \mathrm{kW}m^{-2}$$

$$S = 1.38 \times 10^{3}\ \mathrm{Joule}/m^{2}s$$

We know Poynting vector is given by, $S = EXH$= EH sin90 = EH

Therefore $$EH = 1.38 \times 10^{3}\ \mathrm{Joule}/m^{2}s \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (1)$$

We also know by characteristic impedance of vacuum is given by

$$\frac{E}{H} = 376.6\,\Omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Multiplying equations (1) and equation (2) we get

$$E^2 = 519708(V/m)^2$$

$$\Rightarrow\ E = 720.91\frac{V}{m}$$

Now, $E_0 = E\sqrt{2} = 720.91 \times 1.414\ V/m$

$$\Rightarrow \quad E_0 = 1.02\ kV/m$$

Putting the value of E in equation (1), we get ,

$$H = \frac{\mathbf{1}.38 \times 10^3}{720.91}\ amp - turn/m$$

H = 1.91 $amp - turn/m$

Now,
$$H_0 = H\sqrt{2}$$

$$\Rightarrow \qquad H_0 = 1.91\sqrt{2}\ amp - turn/m$$

$$\Rightarrow \qquad H_0 = 2.7 amp - turn/m$$

For silver, $\mu = \mu_0$ and $\sigma = 3 \times 10^7\ mhos/m$. Calculate the skin depth at $10^8\ Hz$ frequency. Given, $\mu_0 = 4\pi \times 10^{-7} N/A^2$. **[2016-17]**

Sol: Skin depth is given by , $\delta = \sqrt{\frac{2}{\mu\sigma\omega}}$

$$\delta = \sqrt{\frac{2}{\mu\sigma 2\pi f}}$$

$$\delta = \sqrt{\frac{1}{\mu\sigma\pi f}}$$

$$\delta = \sqrt{\frac{1}{4\pi \times 10^{-7} N/A^2 \times 3 \times 10^7 mho/m \times \pi \times 10^8 \mathrm{Hz}}}$$

$$\delta = 9.18 \times 10^{-6}\ m$$

Using Maxwell's equation, curl $\vec{B} = \mu_0(\vec{J} + \frac{\partial \vec{D}}{\partial t})$, prove that div $\vec{D} = \rho$

According to the given problem, curl $\vec{B} = \mu_0(\vec{J} + \frac{\partial \vec{D}}{\partial t})$

Taking divergence of both sides, we get

$$\vec{\nabla}.(\vec{\nabla} \times \vec{B}) = \mu_o \vec{\nabla}.(\vec{J} + \frac{\partial \vec{D}}{\partial t})$$

$$\vec{\nabla}.(\vec{\nabla} \times \vec{B}) = 0$$

$$\vec{\nabla}.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0$$

$$\vec{\nabla}.\vec{J} + \frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{D}\right) = 0$$

From equation of continuity, $\vec{\nabla}.\vec{J} = -\frac{\partial \rho}{\partial t}$

Hence, $\frac{\partial}{\partial t}\left(\vec{\nabla}.\vec{D}\right) = \frac{\partial \rho}{\partial t}$

$$\vec{\nabla}.\vec{D} = \rho$$

Thank you