Unit-III

Wave-optics

Lecture-18

Outline

- Interference of Light
- Coherent Source
- Condition of sustained interference
- Snell’s law

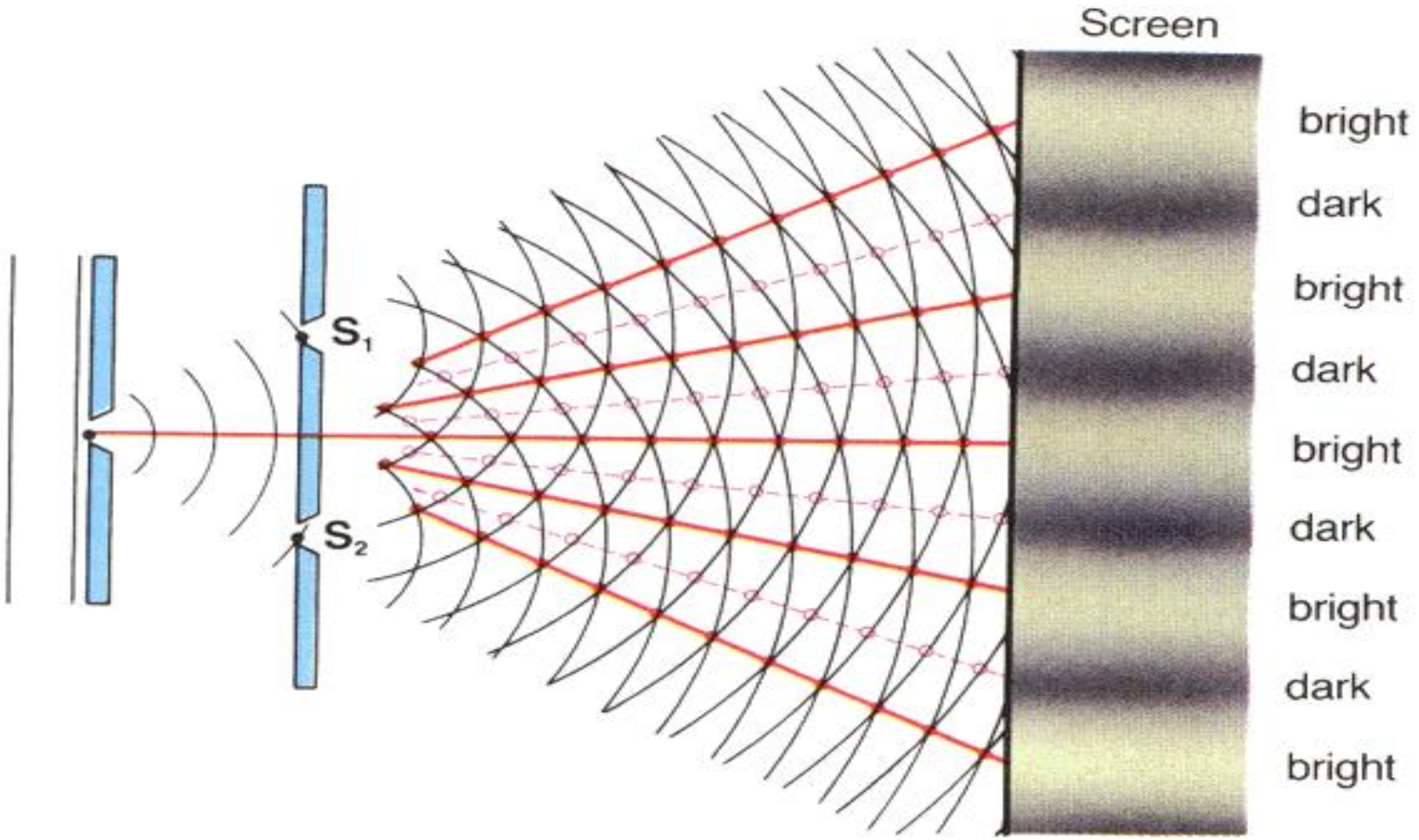
Interference of Light

- When two or more waves of the same frequency and having constant phase difference between them, travels simultaneously in the medium and cross each other, intensity of light changed.
- Interference is superposition of two or more waves of light.
- Interference is based on the principle of superposition of waves.
- There are two types of interference.

(i) Constructive Interference

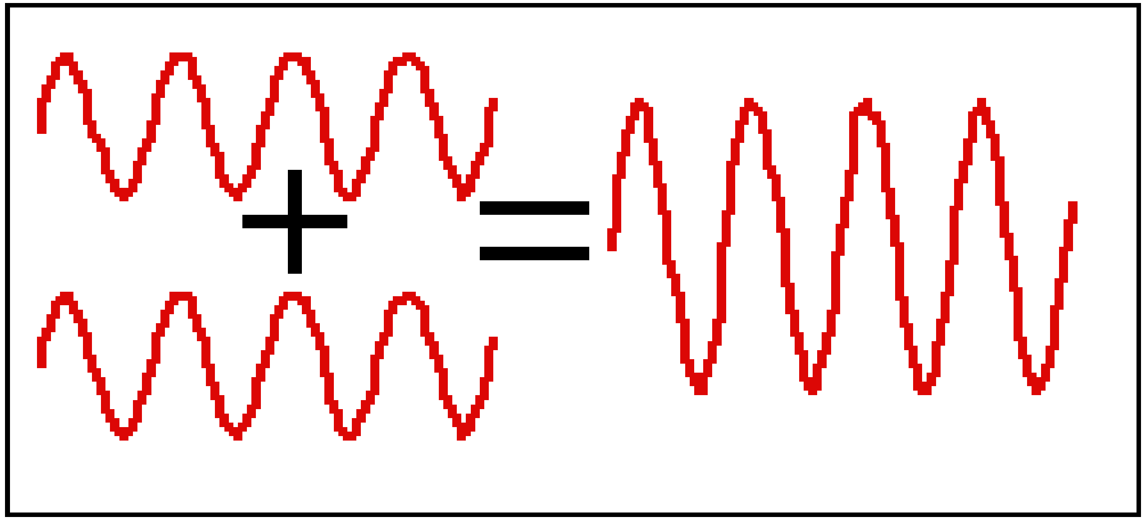
(ii) Destructive Interference

Interference Patterns



Constructive Interference

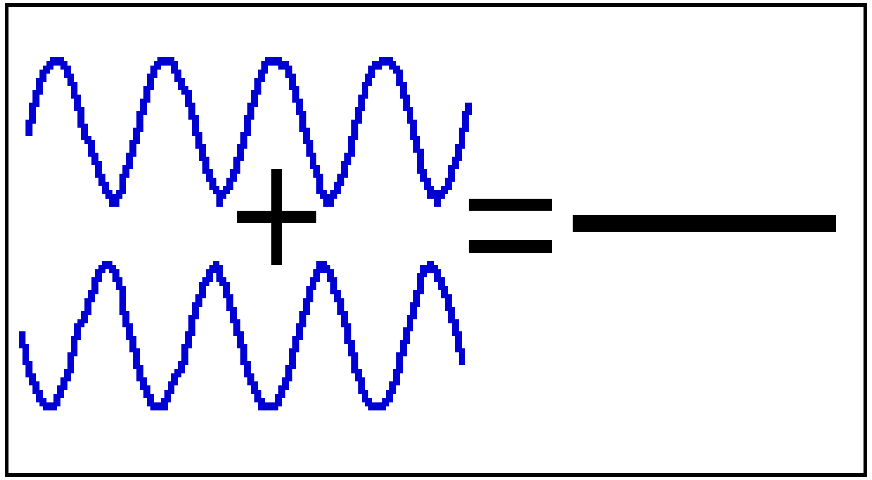
Due to superposition of two or more waves, intensity is maximum at some points. Interference at these points is called constructive interference.



Constructive Interference

Destructive Interference

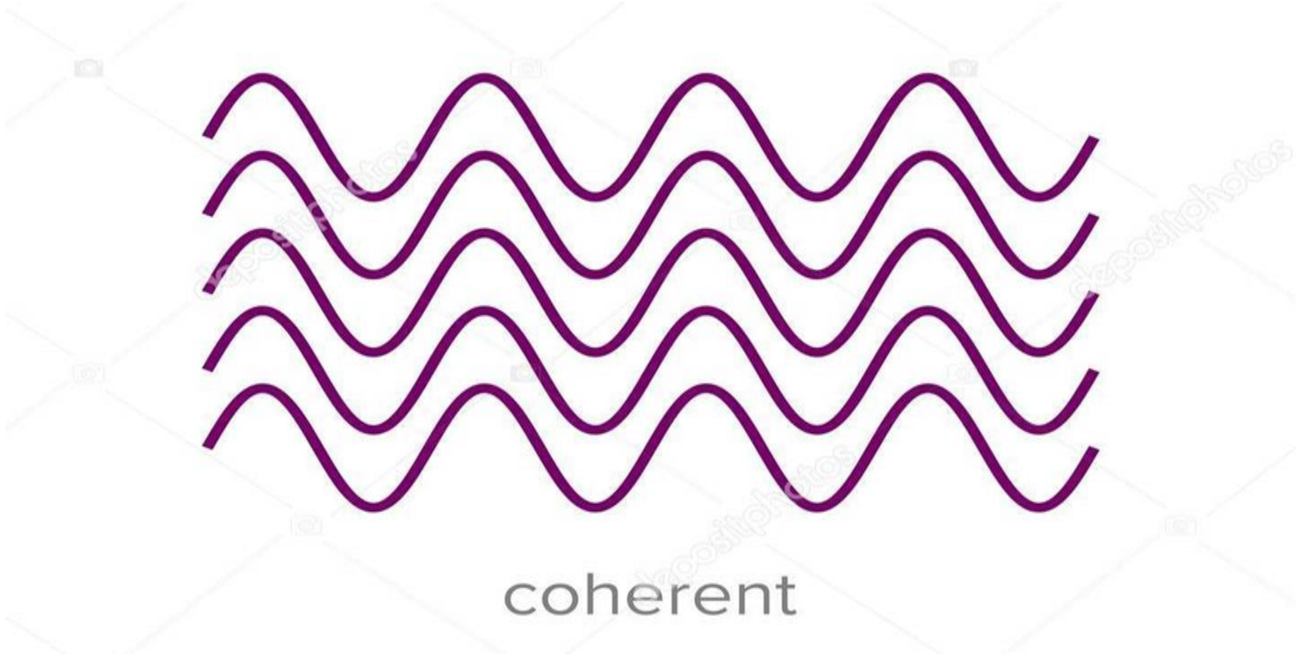
Due to superposition of two or more waves, intensity is minimum at some points. Interference at these points is called destructive interference.



Destructive Interference

Coherent Source

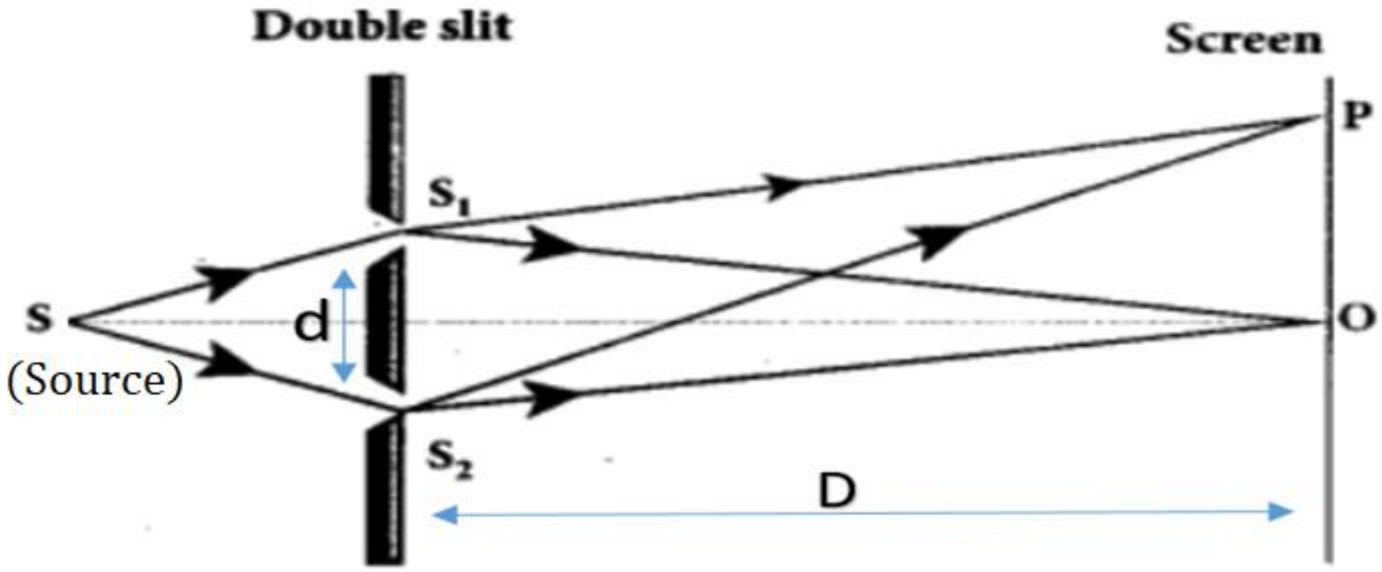
Two sources of light are said to be coherent if they emit light which have always a constant phase difference between them. It means that two sources must emit radiation of same wavelength.



Write the main conditions for sustained interference. (2015-16)

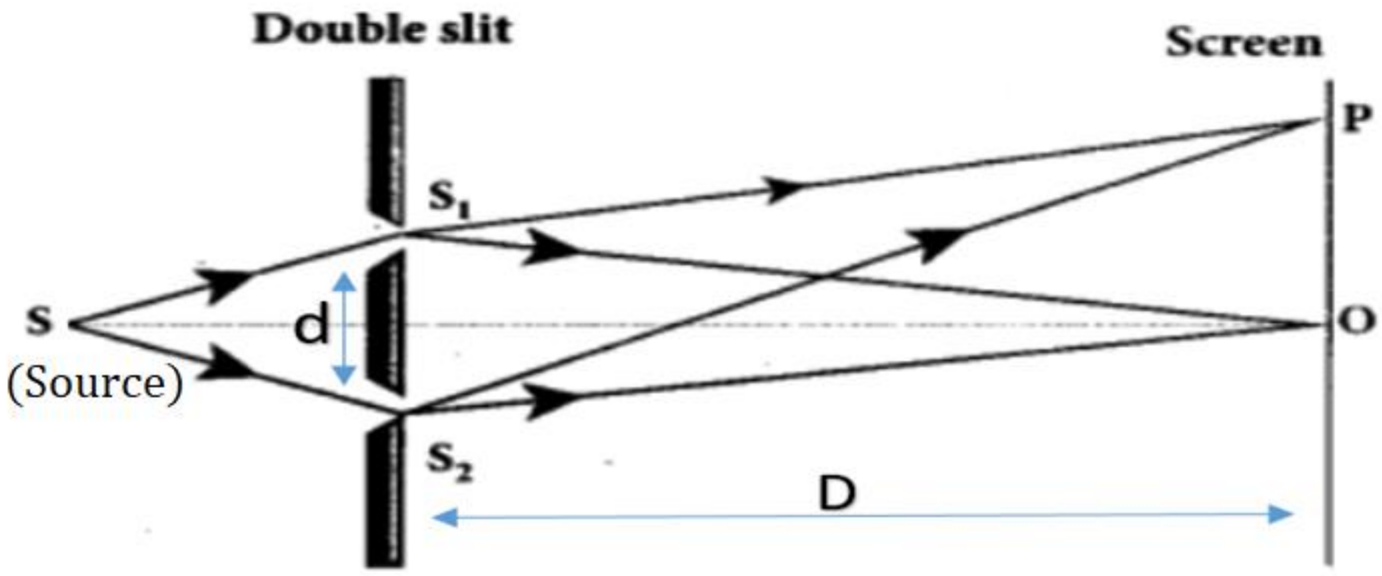
Essential Conditions of Sustained Interference

1. The two interfering waves should be coherent.
2. Light sources should be monochromatic.
3. The two coherent sources must be narrow.



Young's double slit experiment

- The separation between the coherent sources should be as small as possible.
- The distance of the screen from the two sources should be quite large.



Young's double slit experiment

What happens when Young’s double slit experiment immersed in water.

(2015-16)

If the Young’s double slit experiment immersed in water, fringe width will be narrower.

Fringe width,

$$\beta = \frac{D\lambda}{d}$$

where,
d = slit separation,
λ = wavelength of light,
D = distance between slit & screen,

Refractive index,

$$\mu = \frac{c}{v}$$

where, c-speed of light in vacuum, v-velocity of light in medium.

The wavelength of light is less in water than in air.

$$\lambda_{water} < \lambda_{air}$$

Two independent sources can not produces interference, why ?

- Two independent sources of light don’t have a constant phase relationship between them.
- The reason behind it is that every source undergoes haphazard changes of phase in every billionth of a second. Within a time interval of 10^{-8} sec, the phase will be randomly changed because of the fact that the excited atoms which are responsible for emitting vibrations, are replaced by other excited atoms. Hence two independent sources of light can not be coherent and can’t produce a sustained interference pattern.

Production of different coherent source

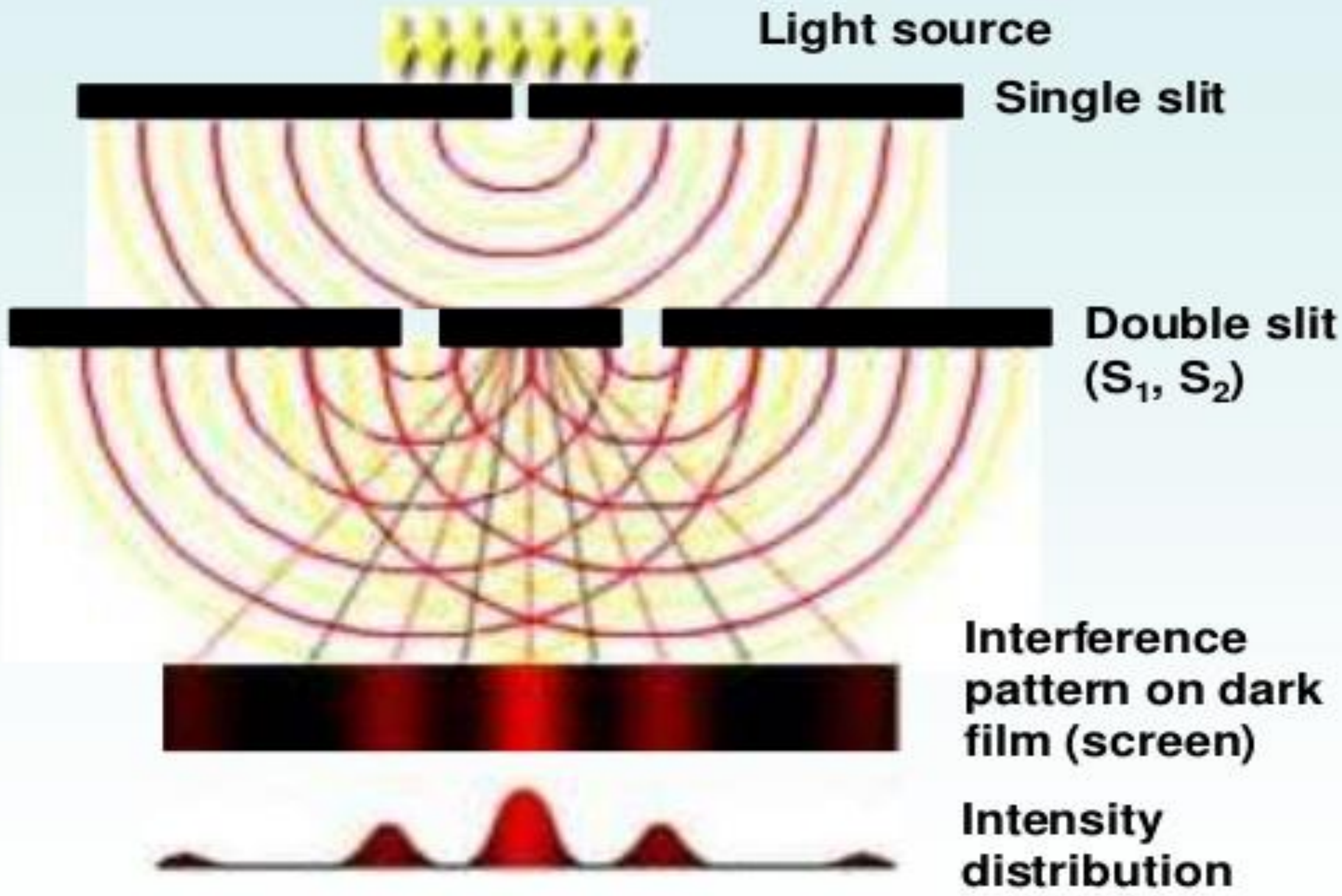
The coherent sources of light can be obtained by using following two methods.

Division of wavefront -

The coherent sources obtained by dividing the wave front, originating from the common source, by employing mirrors, biprism or lenses

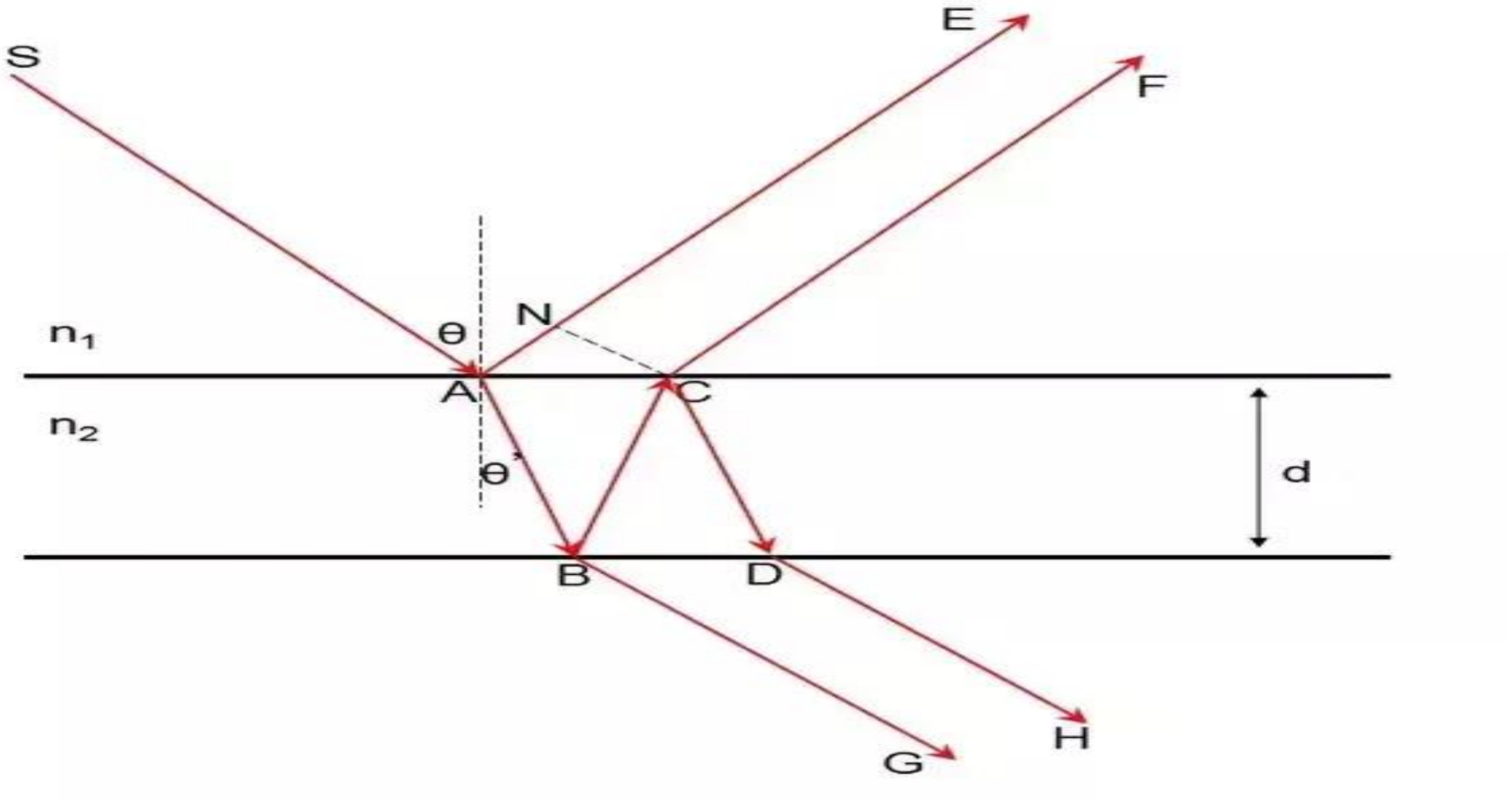
Example :- Fresnel biprism ,Lloyd mirror and laser.

Interference of Light by Division of Wavefront



Division of amplitude-

The amplitude of incident beam is divided into two or more parts either by partial reflection or refraction.



These beams travel different paths and are finally brought together to produce interference. This is referred as two beam interference resulting from the superposition of two waves**.**

Example: Thin film Interference, Newton ring, Michelson Interferometer etc.

Snell's law

Snell's law gives relation between angle of refraction and angle of incidence. Snell's law defined as the ratio of sine of angle of incidence to the sine of the angle of refraction, which is constant.

$$\frac{\sin i}{\sin r} = \mu = constant$$

where i = angle of incidence, r = angle of refraction

Unit-III

Wave-optics

Lecture-19

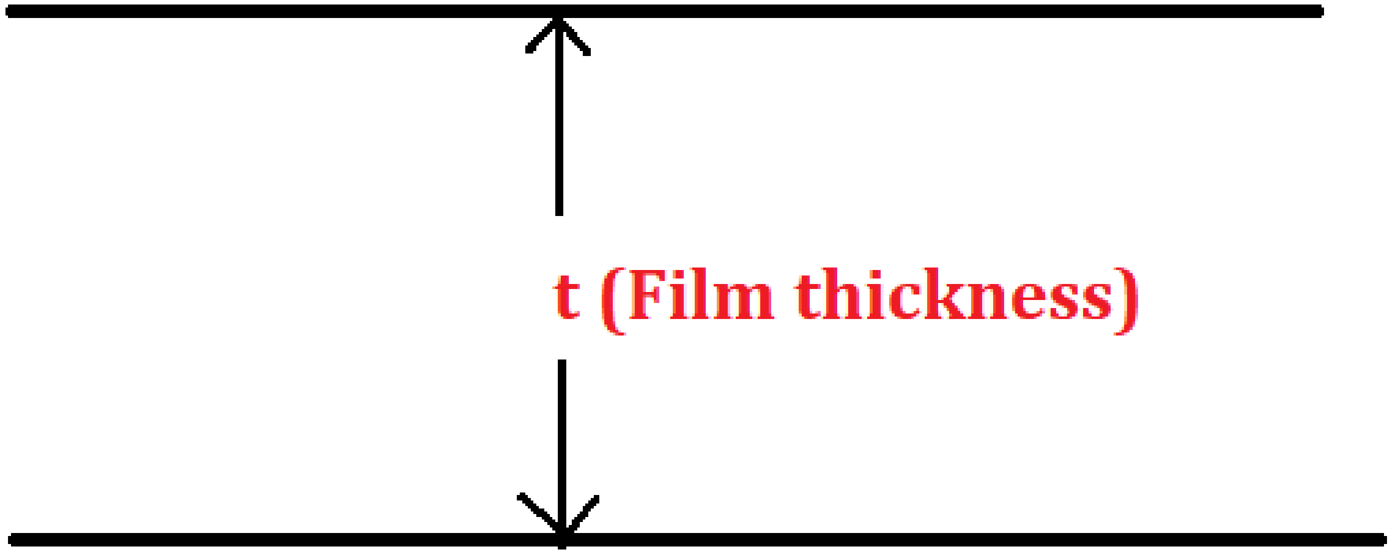
GROUP OF INSTITUTIONS

Outline

- Thin film
- Interference of light due to reflected light
- Interference of light due to transmitted light
- Interference due to wedge shaped thin film

Thin film

A thin film is layer of material ranging from 1nm to 100 nm in thickness. Film thickness is constant in whole film from one end to other.



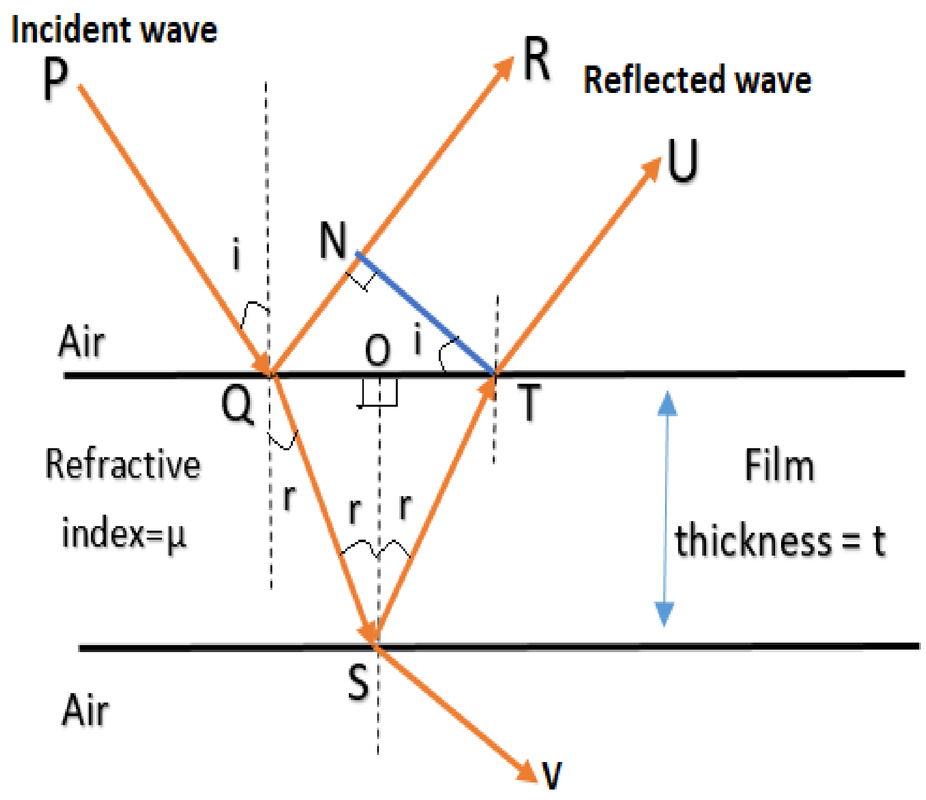
Discuss the phenomenon of interference in thin films due to reflected light.

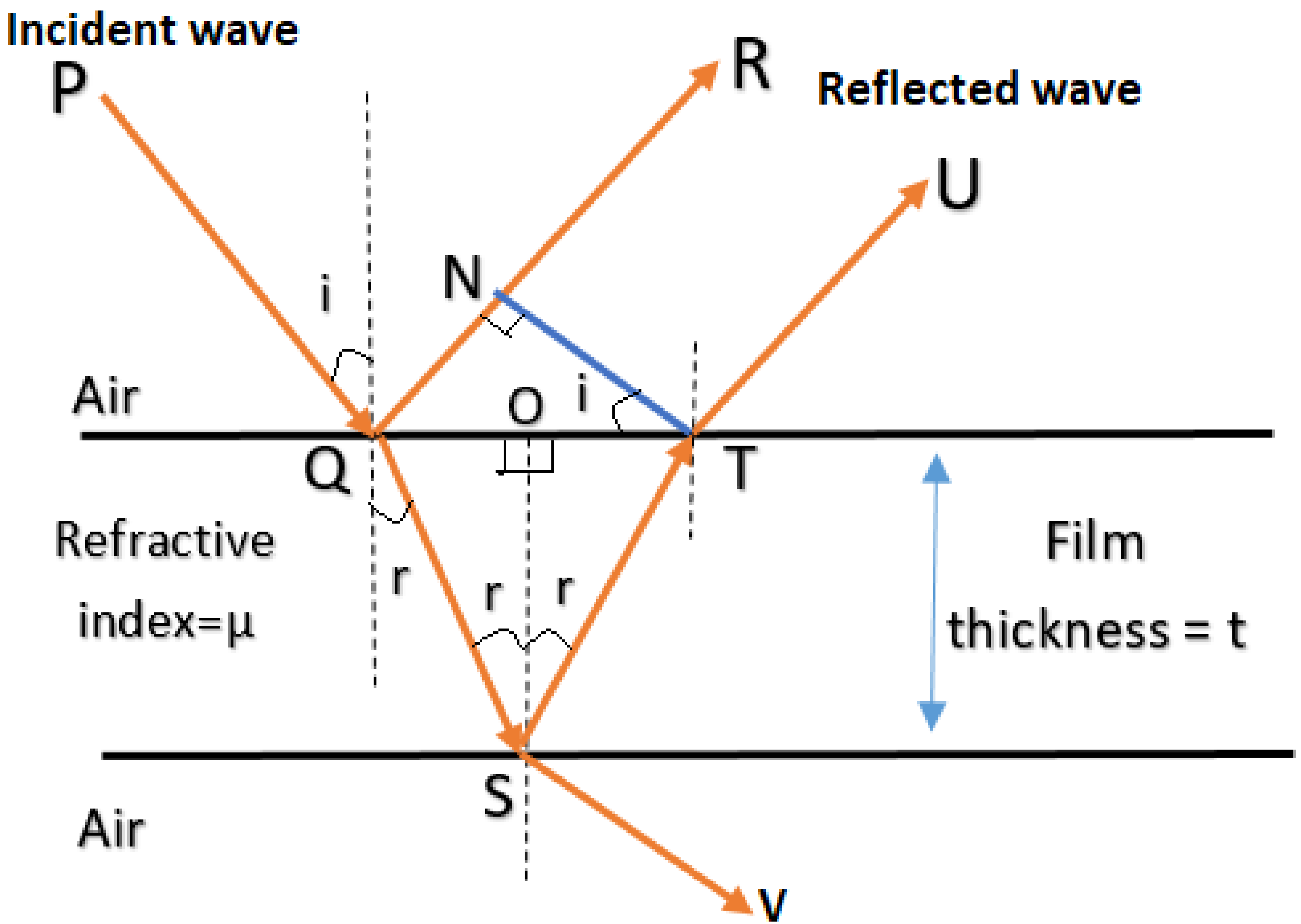
(2015-16)

Discuss the phenomenon of interference of light due to thin films and find the conditions of maxima and minima. Show that reflected and transmitted system are complementary in thin films. (2018-19)

Interference due to reflected light in thin film

- Consider a transparent film of thickness t & refractive index μ (>1). Let a ray of monochromatic light PQ be incident on the upper surface of the film.
- PQ ray is partly reflected along QR and partly refracted along QS. At point S, it is again partly reflected ST and partly refracted along SV and this process continues throughout the film.




Incident wave
P
R
Reflected wave
U
N
i
Air
O
i
Q
T
Refractive
index=μ
r
r
r
Film
thickness = t
S
Air
V

The optical path difference,

$$\Delta = \text{path (QS+ST) in medium} - \text{path QN in air}$$

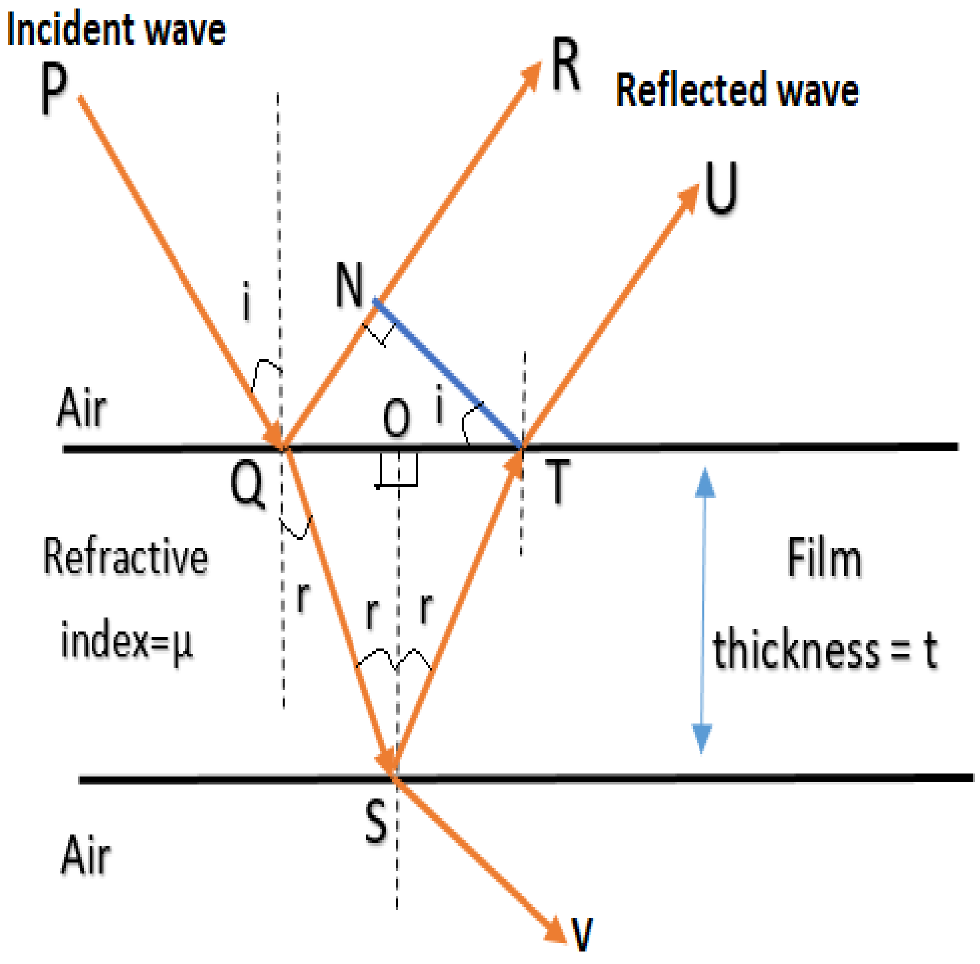
$$\Delta = \mu\,(QS+ST) - QN \qquad \ldots(1)$$

In right angled Δ QSO,

$$\cos r = \frac{SO}{QS} \quad or \quad QS = \frac{t}{\cos r} \qquad \ldots.(2)$$

Similarly in right angled Δ SOT,

$$\cos r = \frac{SO}{ST} \quad or \quad ST = \frac{t}{\cos r} \qquad \ldots\ldots.(3)$$



In right angled Δ QTN,

$$\sin i = \frac{QN}{QT}$$

$$QN = QT\sin i$$

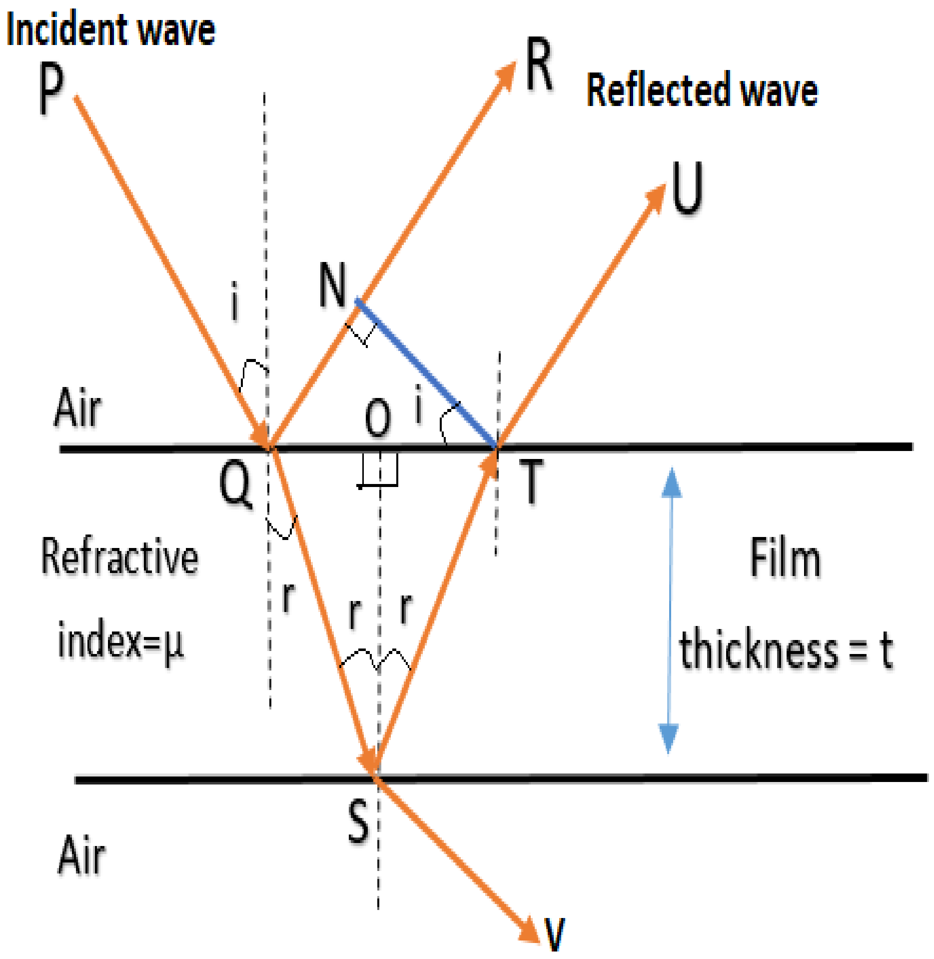
QN = $(QO + OT)\sin i$ ….(4)

Putting these values of equation 2, 3 & 4 into eq^{n} (1),

$$\Delta = \mu\,(QS+ST) - QN$$

$$\Delta = \mu\left(\frac{t}{\cos r}+\frac{t}{\cos r}\right) - (QO + OT)\sin i$$

$$\Delta = \frac{2\mu t}{\cos r} - (QO + OT)\sin i \;……..(5)$$



Now again in Δ QSO and Δ SOT,

$\tan r = \frac{QO}{OS}$ *or* $QO = OS \tan r$ ...(6)

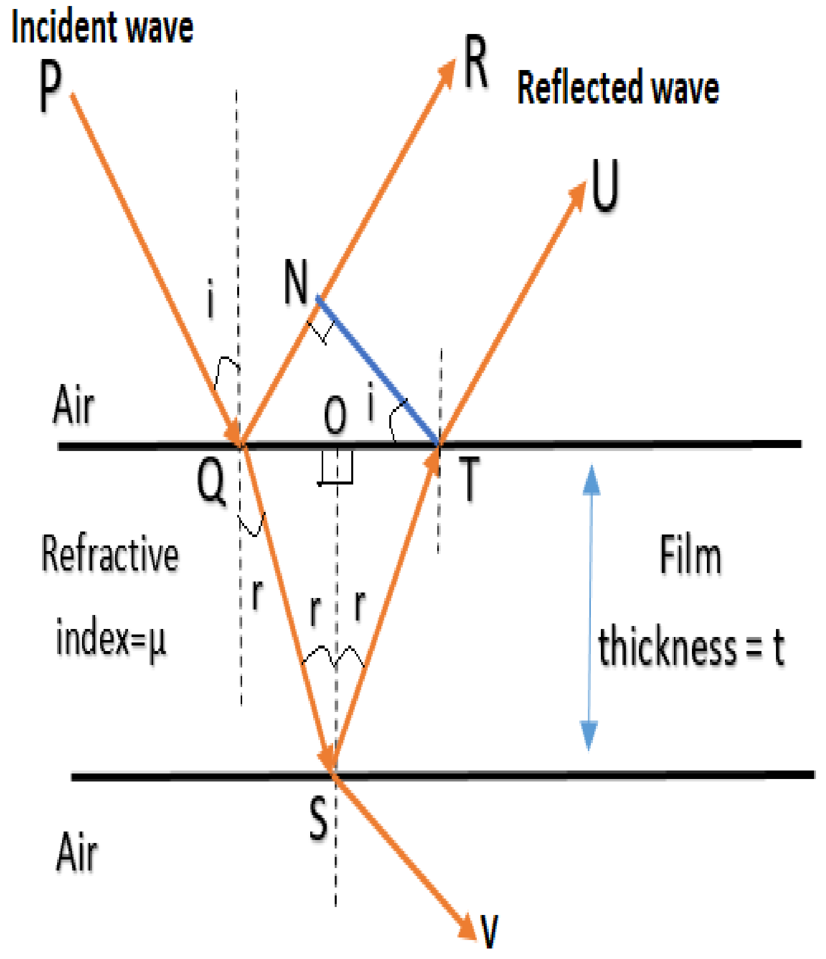
and $\tan r = \frac{OT}{OS}$ or $OT = OS \tan r$ ....(7)

Putting these values of equation 6 & 7 into eq^{n}(5),

$$\Delta = \frac{2\mu t}{\cos r} - (QO + OT) \sin i$$

$$\Delta = \frac{2\mu t}{\cos r} - (OS \tan r + OS \tan r) \sin i$$

$$\Delta = \frac{2\mu t}{\cos r} - (2t \tan r) \sin i \text{ ....(8)}$$



From Snell's law, $\dfrac{\sin i}{\sin r} = \mu$

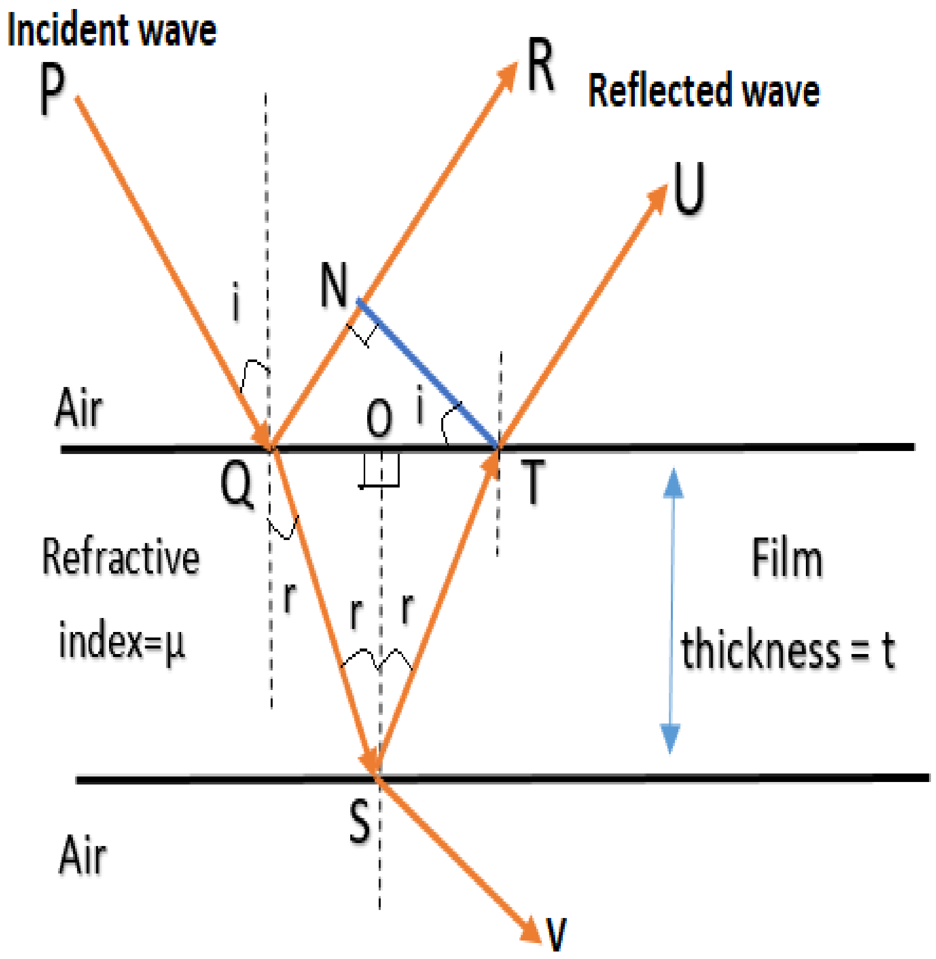
$$\sin i = \mu \sin r$$

$$\Delta = \frac{2\mu t}{\cos r} - (2\mu t \tan r)\sin r$$

$$\Delta = \frac{2\mu t}{\cos r} - \left(2\mu t \frac{\sin r}{\cos r}\right)\sin r$$

$$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\Delta = 2\mu t \cos r$$



According to Stoke's law, when the light is reflected from the surface of an optically denser medium, a phase change of π equivalent to a path difference of λ/2 occurs.

$$\therefore \qquad \Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

1. Condition for maxima :

If Δ = nλ, where n =1,2,3,…….constructive interference takes place and the film will appear bright.

$$\therefore \qquad 2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

$$or \quad 2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

2. Condition for minima

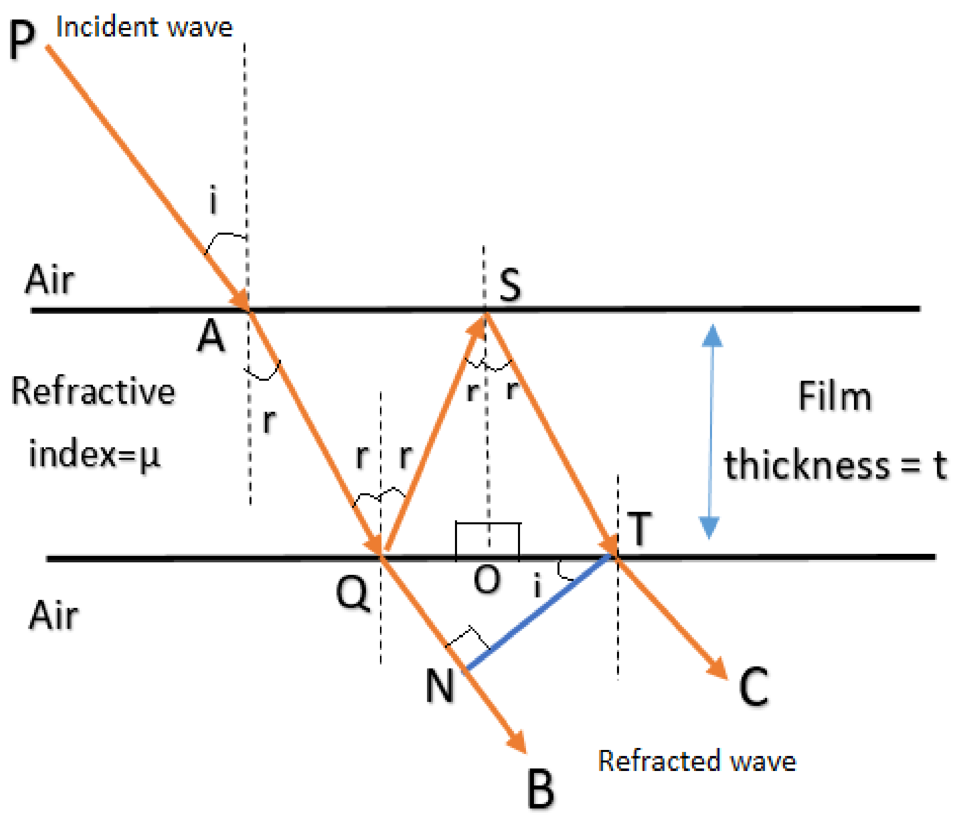
If $\Delta = (2n+1)\ \lambda/2$, where n = 0, 1, 2, 3…then destructive interference takes place and the film will appear dark.

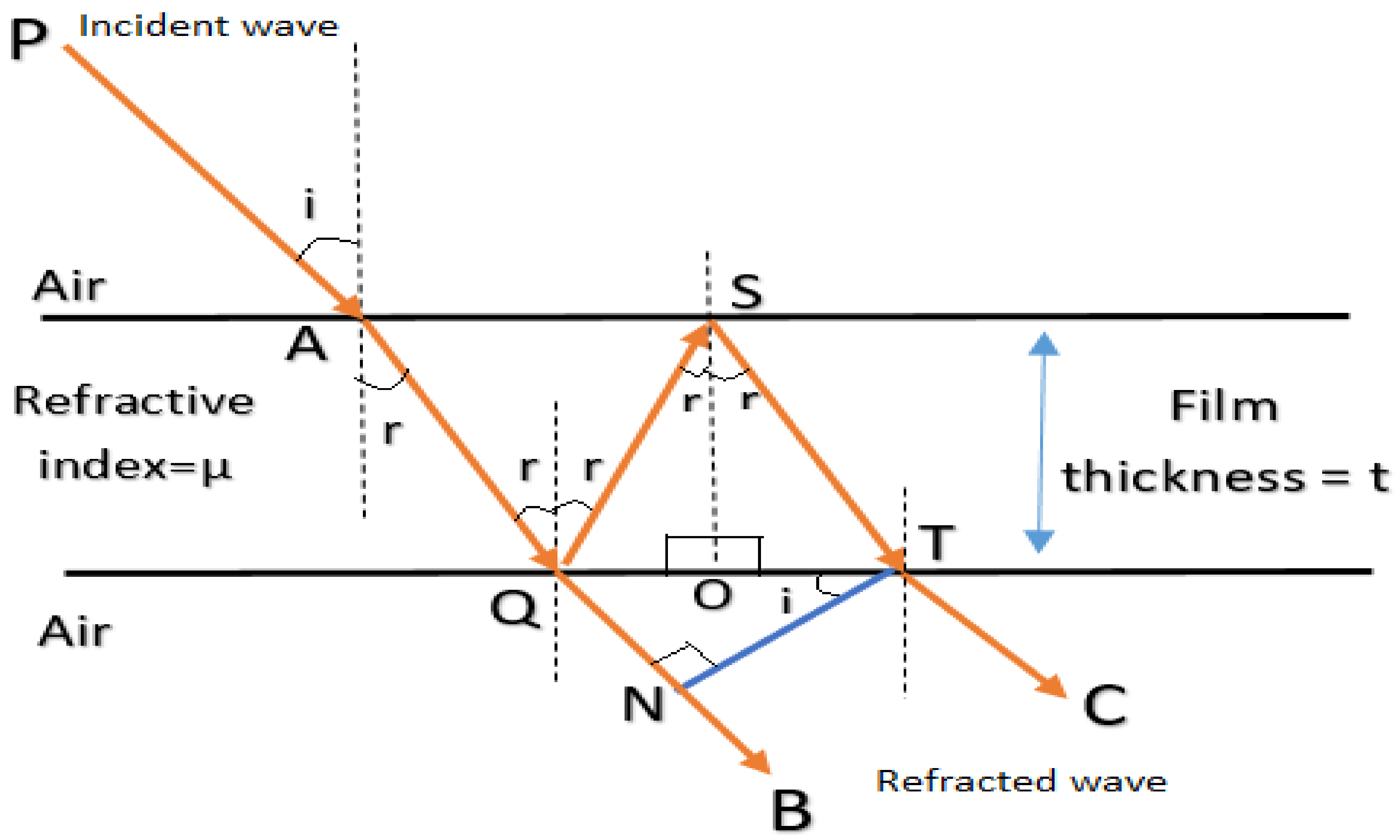
$$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t\ \cos r = n\lambda$$

Interference due to transmitted light in thin film

- Consider a transparent film of thickness 't' & refractive index 'μ'. Let a ray of monochromatic light be incident on the upper surface of the film.
- The ray PA ray is refracted along AQ at an angle r. The refracted part AQ is partly reflected along QS and partly refracted along QB.




P
Incident wave
i
Air
A
S
r
r
r
Refractive
index=μ
r
r
Film
thickness = t
T
Q
O
i
Air
N
C
B
Refracted wave

- The reflected part QS is again reflected from point S on the upper surface of the film along ST and finally emerges out through TC.

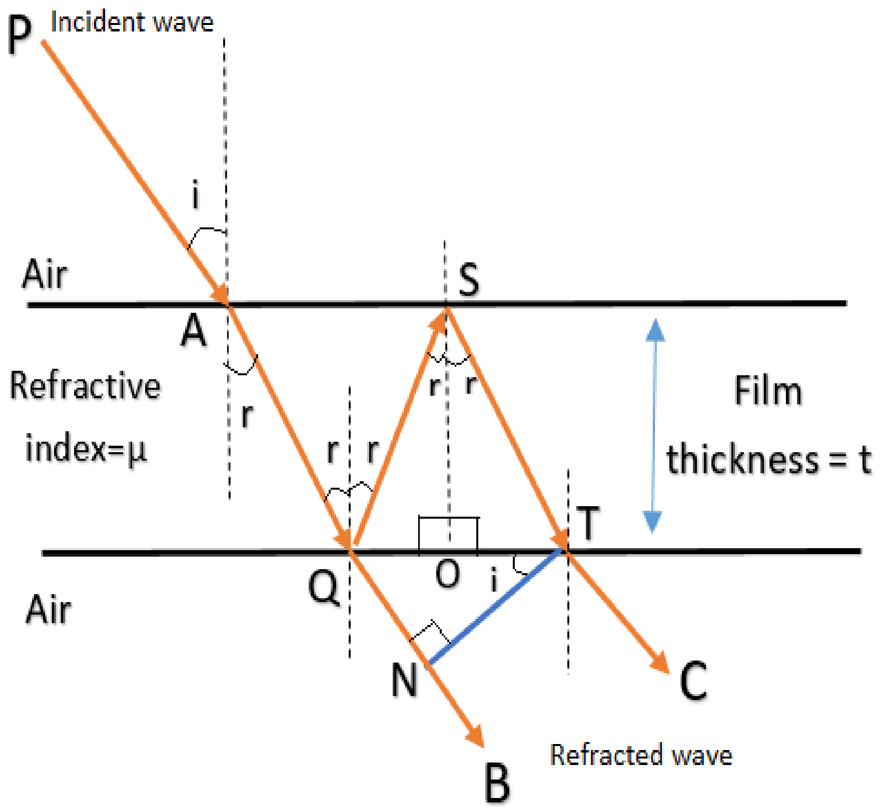
The optical path difference,

$$\Delta = \text{path } (QS+ST) - \text{path } QN$$

$$\Delta = \mu\,(QS+ST) - QN \qquad \ldots\ldots.(1)$$

In right angled Δ QSO,

$$\cos r = \frac{SO}{QS} \quad or \quad QS = \frac{t}{\cos r} \qquad \ldots..(2)$$



Similarly in right angled Δ SOT,

$$\cos r = \frac{SO}{ST} \quad or \;\; ST = \frac{t}{\cos r} \qquad \text{.....(3)}$$

Putting these values of equation (2) & (3) into eq^n (1),

$$\Delta = \mu\,(QS + ST) - QN$$

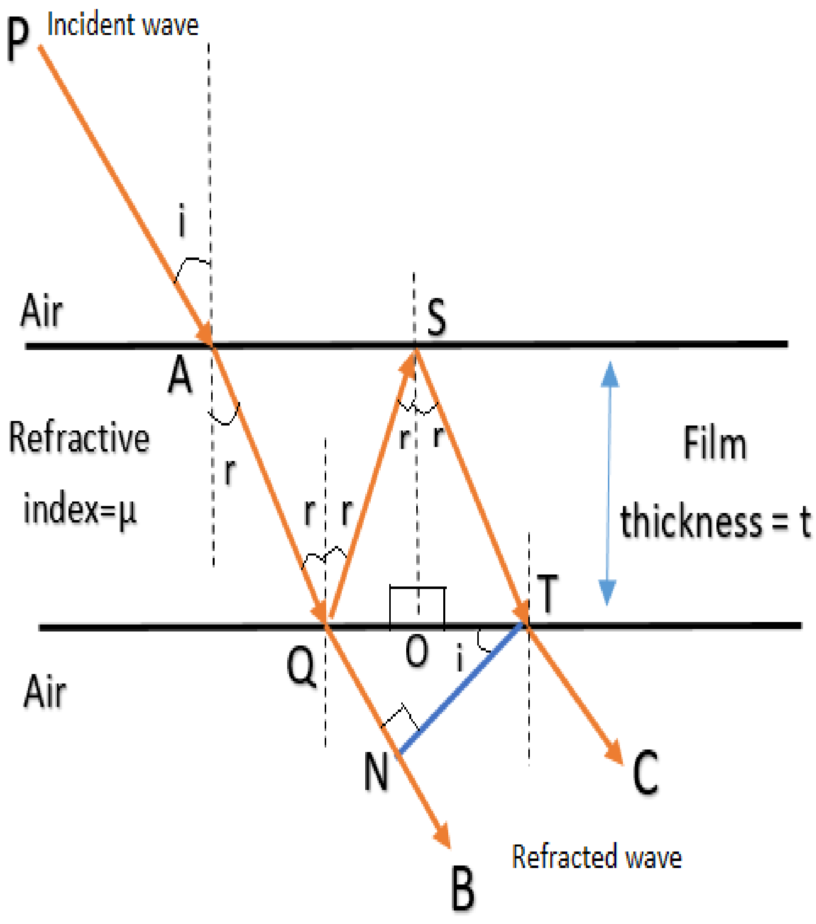
$$\Delta = \mu\,\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - QN \qquad \text{......(4)}$$

In right angled Δ QTN,

$$\sin i = \frac{QN}{QT}$$

$$QN = QT\sin i$$

$$= (QO + OT)\sin i \text{.. (5)}$$



Putting the values of equation (5) in eq^n (4), we get

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - (QO + OT) \sin i \qquad .....(6)$$

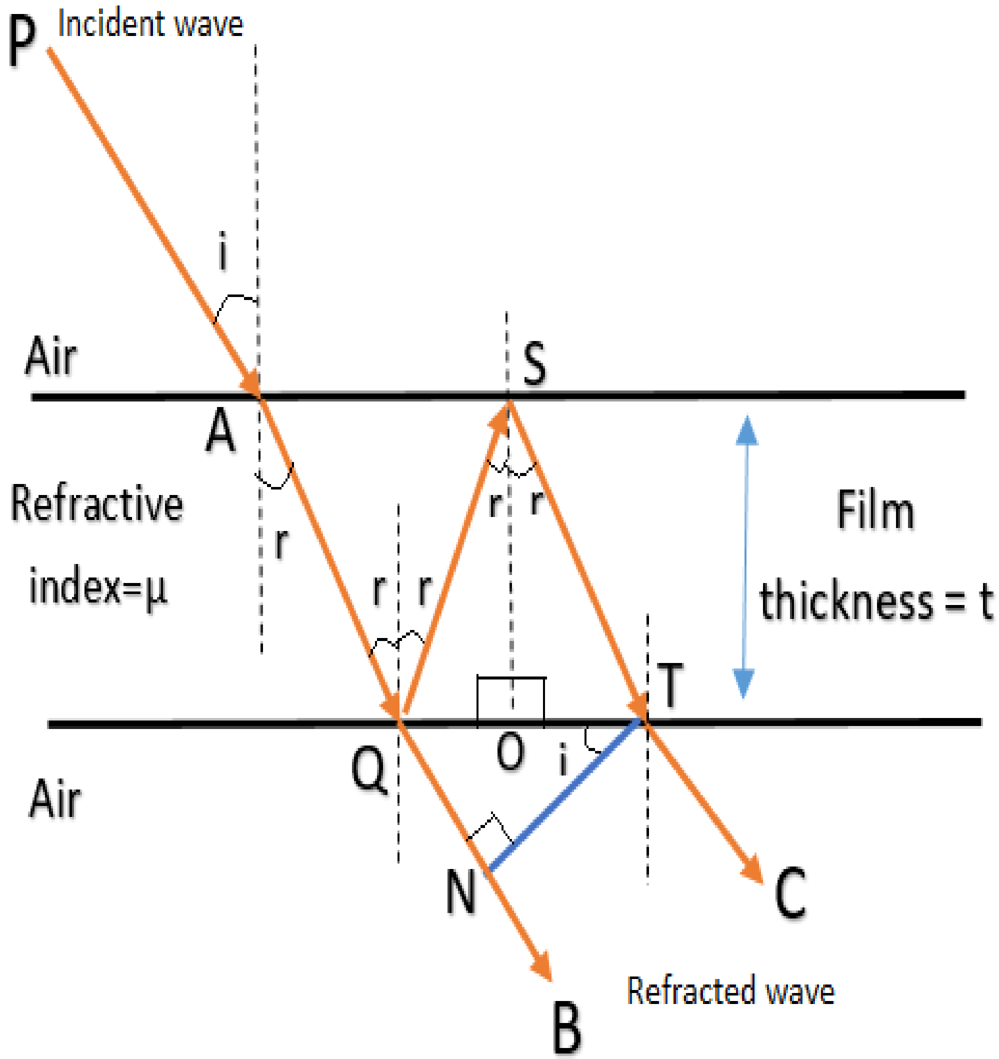
Now again in Δ QSO and Δ SOT,

$$\tan r = \frac{QO}{OS} \quad and \quad \tan r = \frac{OT}{OS}$$

Putting these values in eq^n(6), we get

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - (\mathrm{OS} \tan \mathrm{r} + \mathrm{OS} \tan \mathrm{r}) \sin i$$

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - 2(\mathrm{OS} \tan \mathrm{r}) \sin i \qquad ....(7)$$



From Snell's law, $\frac{\sin i}{\sin r} = \mu$, sin i = μ sin r

Putting these value in equation (7),

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - (2t \tan r)\, \mu \sin r \qquad \text{....(7)}$$

$$\Delta = \mu \left(\frac{2t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$$

$$\Delta = \frac{2\mu t \cos^2 r}{\cos r}$$

$$\Delta = 2\mu t \cos r$$

1.Condition for maxima :

If Δ = nλ, where n = 0,1,2,3,…….constructive interference takes place and the film will appear bright

$$\therefore \quad 2\mu t \cos r = n\lambda$$

2.Condition for minima:

If Δ = (2n-1) λ/2, where n =1, 2, 3 … destructive interference takes place and the film will appear dark.

$$\therefore \quad 2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

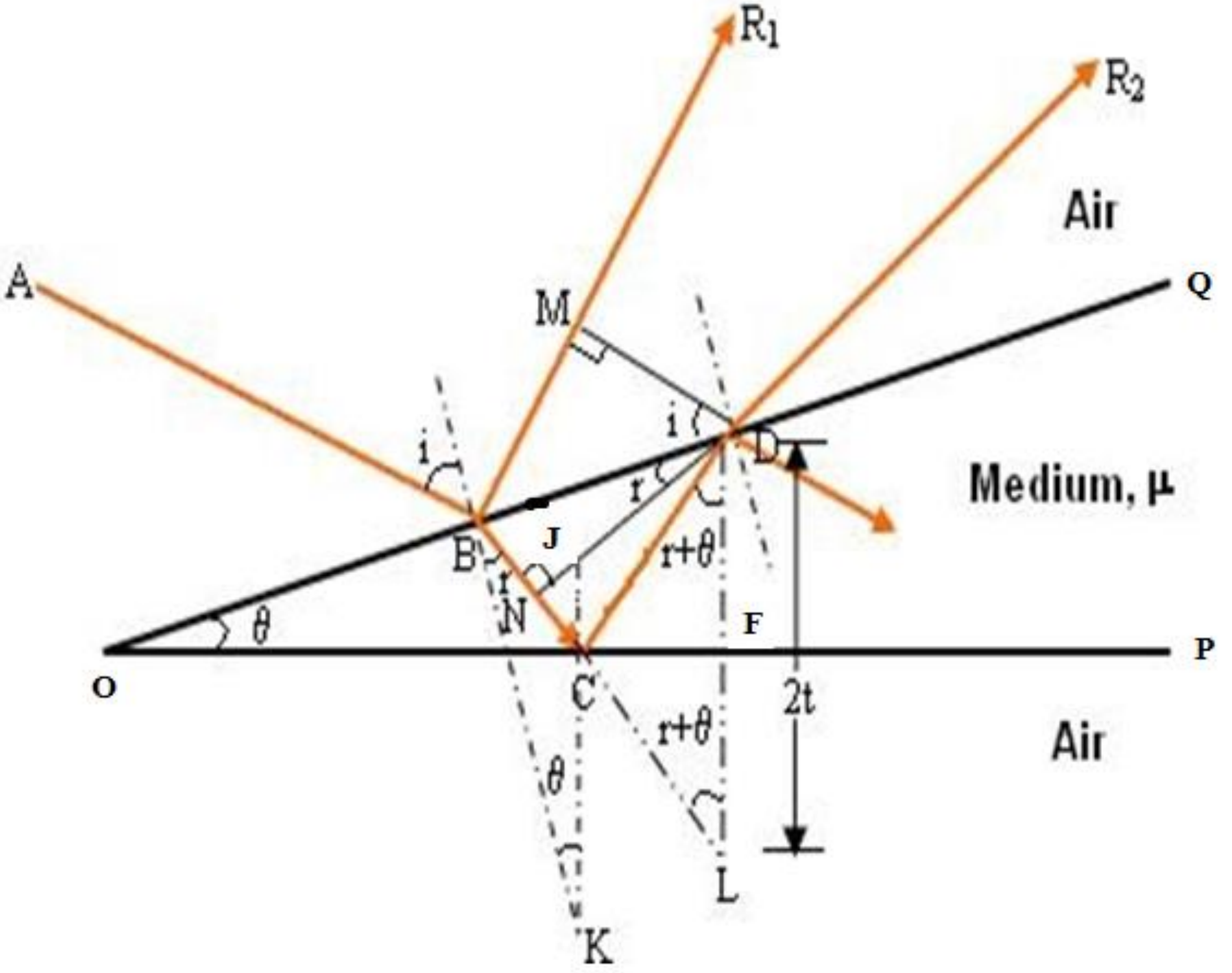
Therefore, the point of film which appears bright in reflected light appears dark in transmitted light.

Hence, **the interference pattern of reflected and transmitted monochromatic light are complementary to each other.**

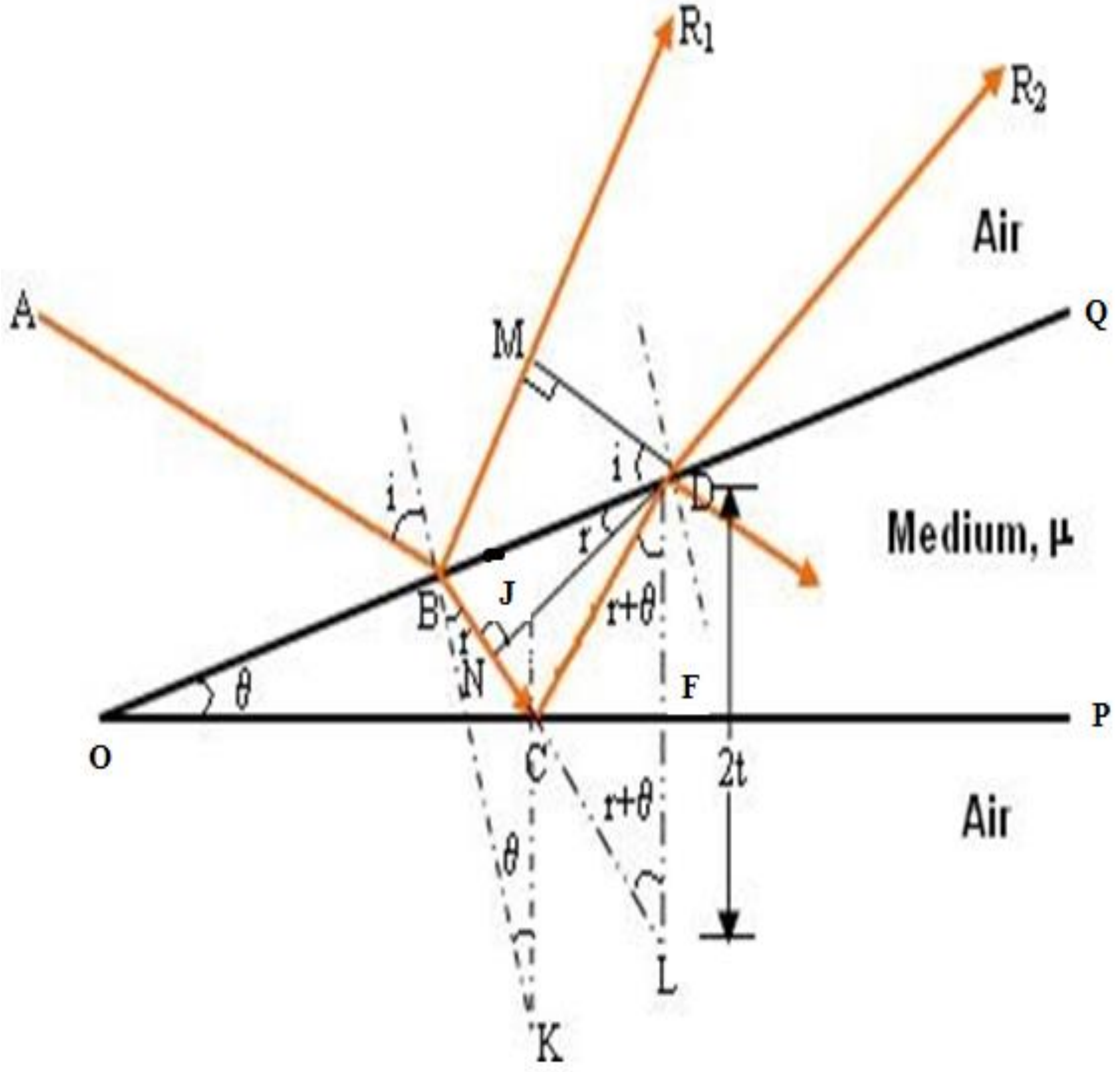
Discuss the formation of interference fringes due to wedge shaped thin films seen by normally reflected monochromatic light and derive an expression for fringe width in wedge shaped films. (2015-16, 2017-18)

Interference in Wedge shaped film

- A wedge shaped thin film is one whose plane surfaces are slightly inclined to each other at small angle θ and encloses a film of transparent material of refractive index μ.
- The thickness of the film increases from one side to another. At the point of contact thickness is zero.



- Consider a film of non-uniform thickness, bounded by two surfaces OP and OQ inclined at an angle θ. The thickness of the film gradually increases from O to P. The point O at which the thickness is zero is known as the edge of the wedge.
- Let a beam AB of monochromatic light of wavelength λ be incident at an angle 'i' on the upper surface of the film. It is reflected along BM and is transmitted along BC.

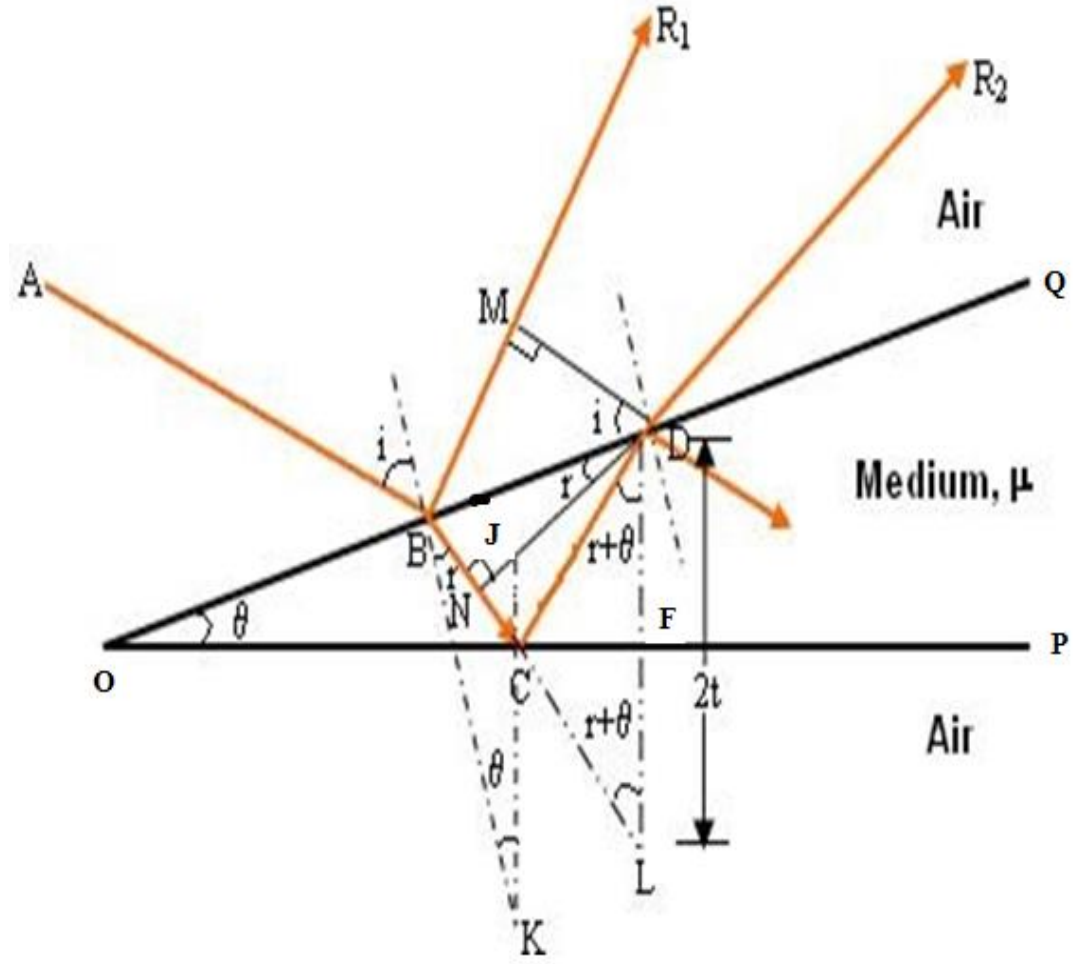


- At C also the beam suffers partial reflection and refraction and finally we have the ray DR_2 in the reflected system.
- The optical path difference between the two reflected rays R_1 and R_2 will be

$$\triangle = \mu(BC + CD)\ in\ film - BM\ in\ air$$
$$\triangle = \mu(BN + NC + CD) - BM \quad ..... (1)$$

In ΔBMD and ΔBND ;

$$\sin i = \frac{BM}{BD}; \quad \sin r = \frac{BN}{BD}$$



According to Snell's Law,

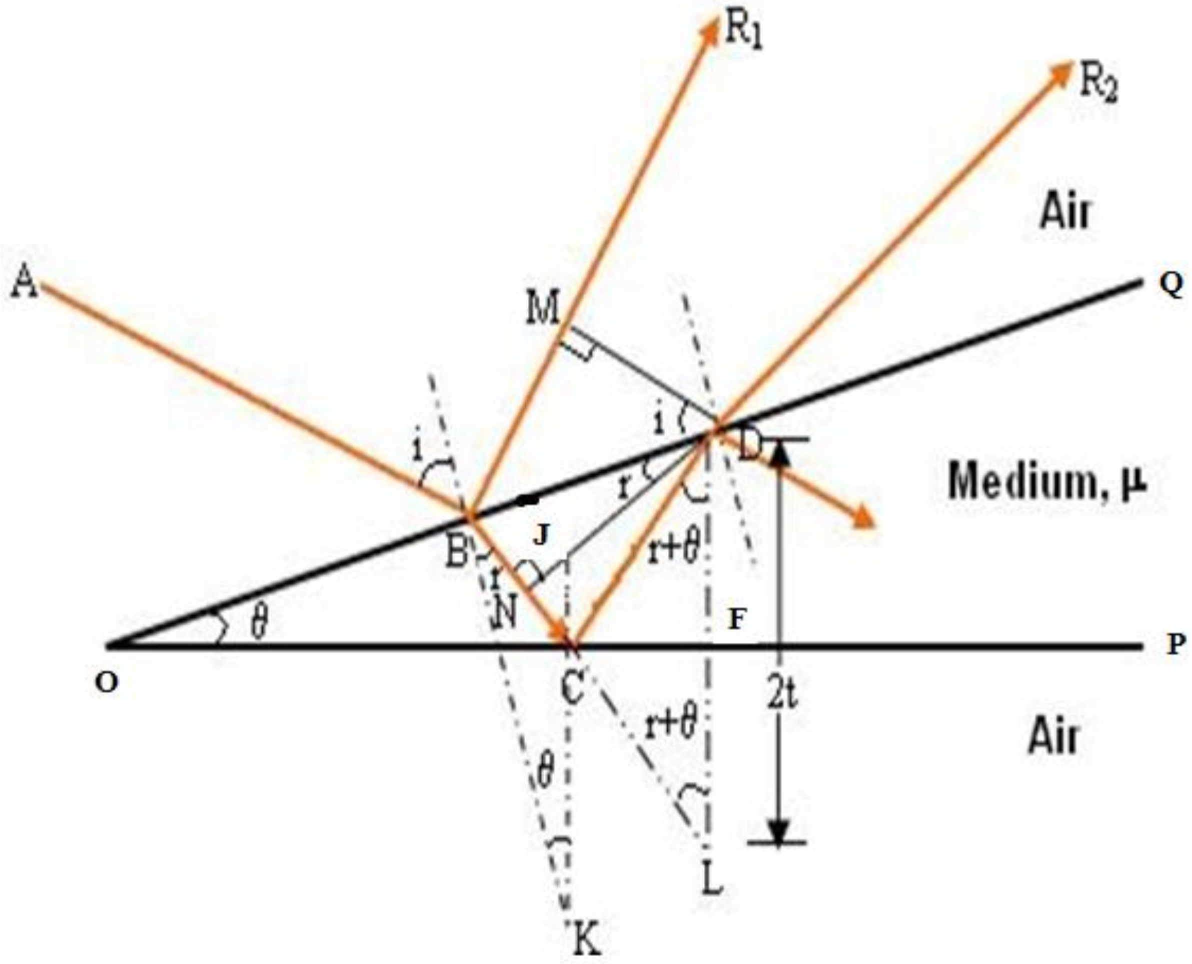
$$\mu = \frac{\sin i}{\sin r} = \frac{BM/BD}{BN/BD} = \frac{BM}{BN}$$

$$BM = \mu BN$$

Substitute the value BM in equation (1) become

$$\triangle = \mu(BN + NC + CD) - \mu\, BN$$

$$\triangle = \mu(NC + CD) \text{ .... } (2)$$



Now draw perpendicular DF from D on OP and produce BC. They meet at L.

Now triangle CDF and CFL are congruent,

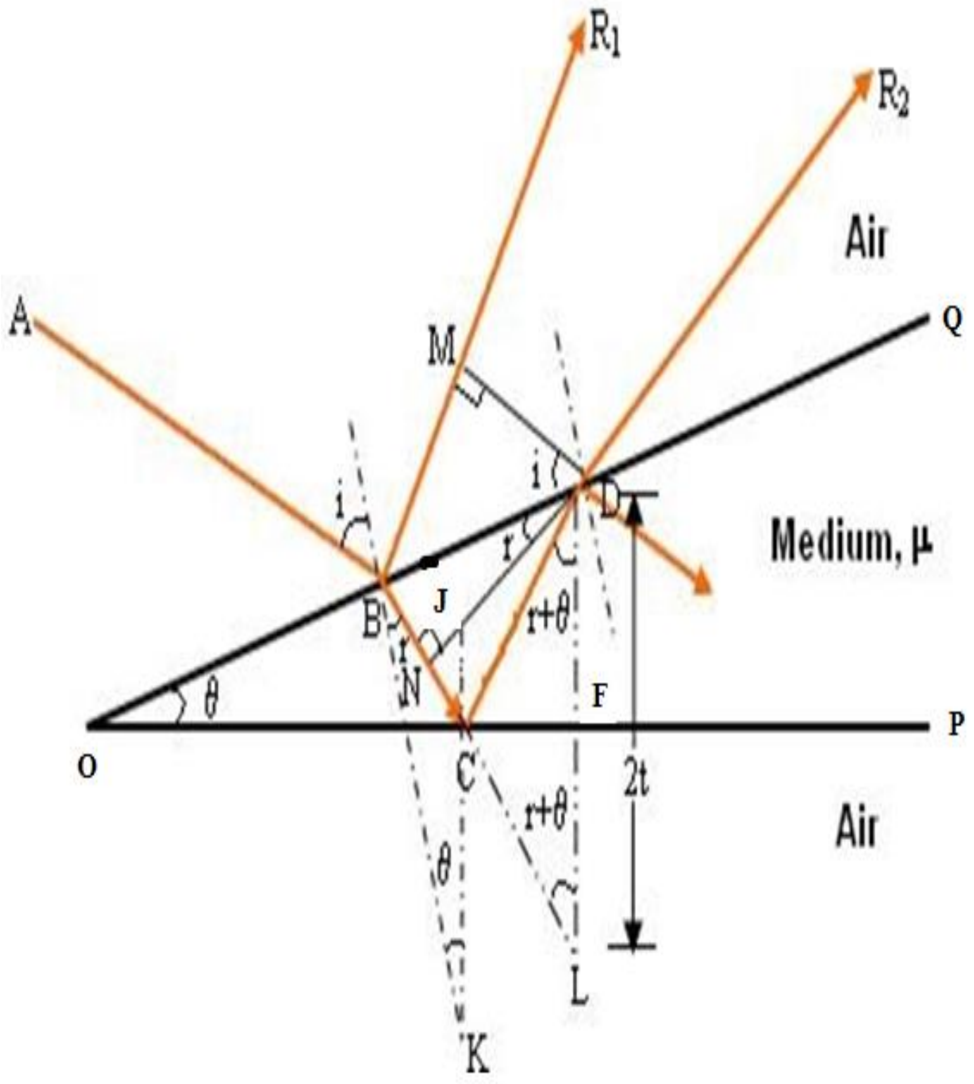
Since$\angle CDF = \angle CLF = r + \theta$

$\angle DFC = \angle CFL = 90^0$

And CF is common

DF = FL = t

CD = CL



Substituting value of CD in equation (2) we get

$$\triangle = \mu(NC + CL)$$

$\triangle = \mu NL$ .... (3)

- The angle of incidence BCJ at C is, therefore (r + θ). CJ and DL are normal to the surface OP therefore CJ and DL are parallel as ACL cuts the parallel line CJ and DL, we must have

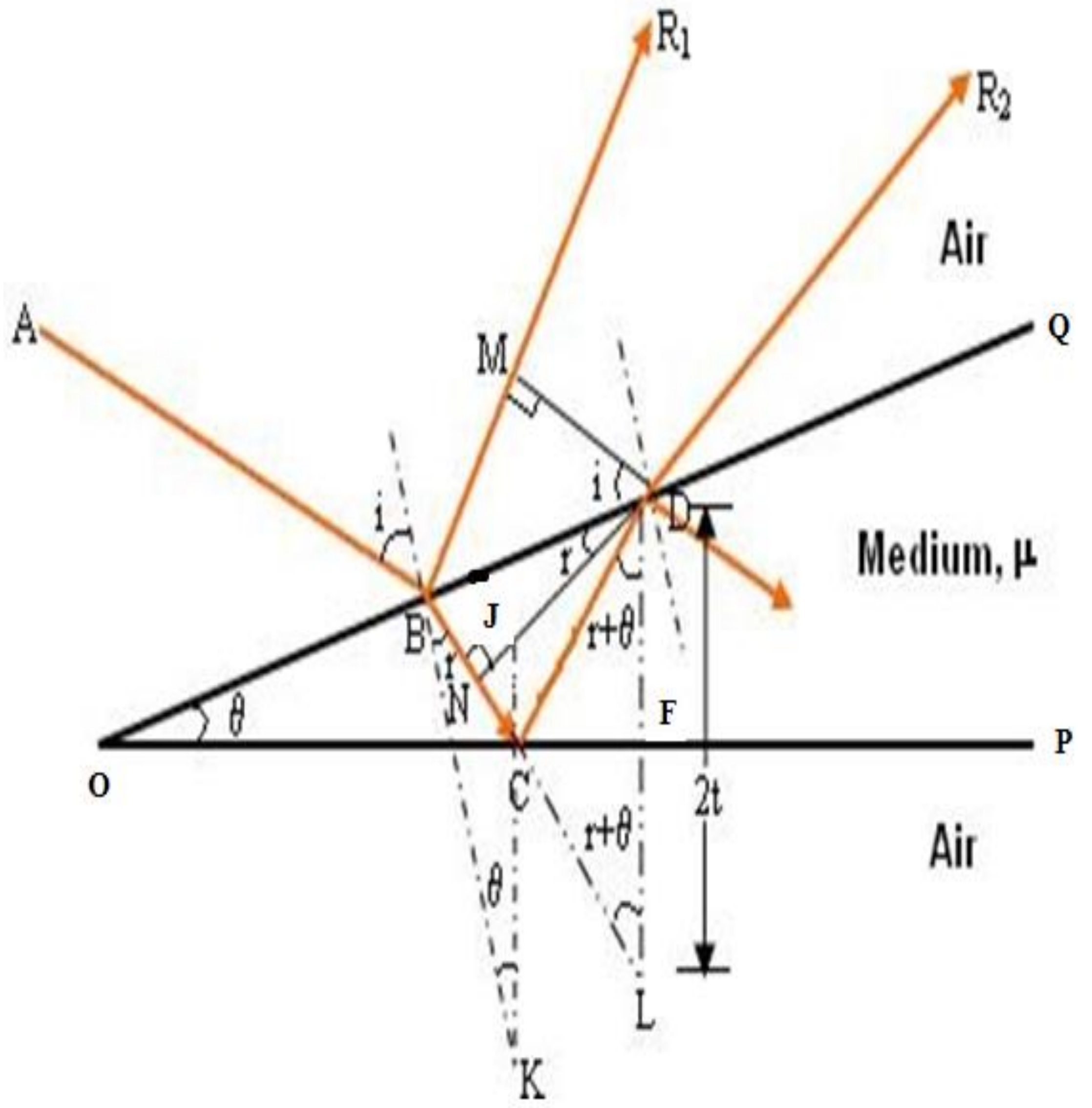
$$\angle ACJ = \angle CLD = r + \theta$$

As in ΔNDL $\cos(r + \theta) = \frac{NL}{2t}$

$$\mathrm{NL} = 2\,\mathrm{t}\cos(r + \theta)$$

Equation (3) gives

$$\Delta = 2\mu t \cos(r + \theta)$$



As the wave train along BR_1 is the reflected wave train from a denser medium, therefore there occurs a phase change of π or path difference λ/2.Therefore the effective path difference is,

$$\triangle = 2\mu t\cos\ (r+\theta) + \frac{\lambda}{2} \ldots\ldots\ldots\ldots (4)$$

Condition for maxima (Bright Fringe):

$$2\mu t\cos\ (r+\theta) + \frac{\lambda}{2} = n\lambda$$

or $$2\mu t\cos\ (r+\theta)\ = (2n-1)\frac{\lambda}{2}$$

where n = 1, 2, 3...........

Condition for minima (Dark Fringe)

$$2\mu t \cos\ (r+\theta) + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\mu t \cos\ (r+\theta)\ = n\lambda$$

where n = 0,1,2..

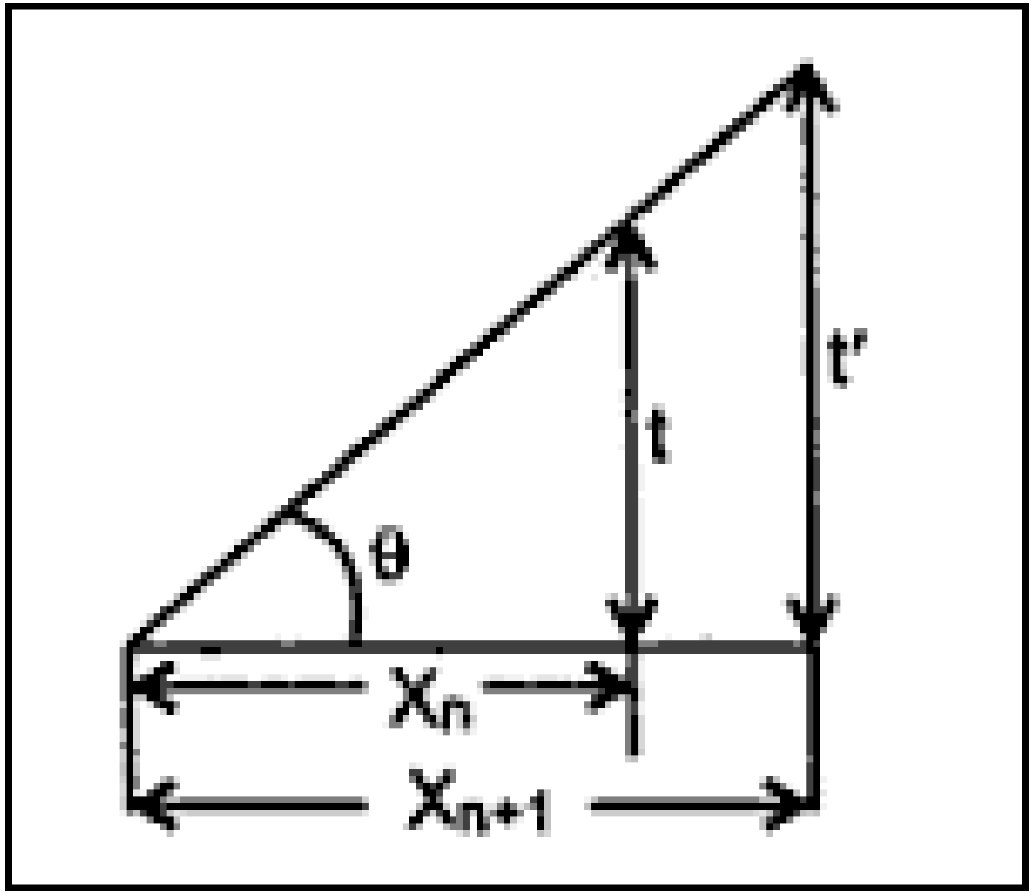
Fringe Width

For bright fringe, Let x_n be the distance of n^{th} bright fringe from the edge of the film, then

$$\tan\theta = \frac{t}{x_n} \text{or} \qquad x_n \tan\theta = t$$

Putting this value of t in effective path difference equation,

$$2\mu t \cos\,(r+\theta) = (2n-1)\frac{\lambda}{2}$$



$$2\mu x_n \tan\theta \cos\ (r+\theta)\ = (2n-1)\frac{\lambda}{2} \ldots\ldots.(1)$$

Similarly, if x $_{n+1}$ is the distance of (n+1)th bright fringe, then

$$2\mu x_{n+1} \tan\theta \cos(r+\theta) = [2(n+1)-1]\frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \ldots\ldots..(2)$$

Subtracting eq^n (1) from eq^n(2),

For normal incidence, i = r = 0 and cos (r + θ) = cos θ

$$\omega = \frac{\lambda}{2\mu \tan\theta\ cos\,\theta} = \frac{\lambda}{2\mu \sin\theta}$$

For very small value of θ, $\sin\theta \approx \theta$;

$$\omega = \frac{\lambda}{2\mu\theta}$$

For air film $\mu = 1$

$$\omega = \frac{\lambda}{2\theta}$$

Unit-III

Wave-optics

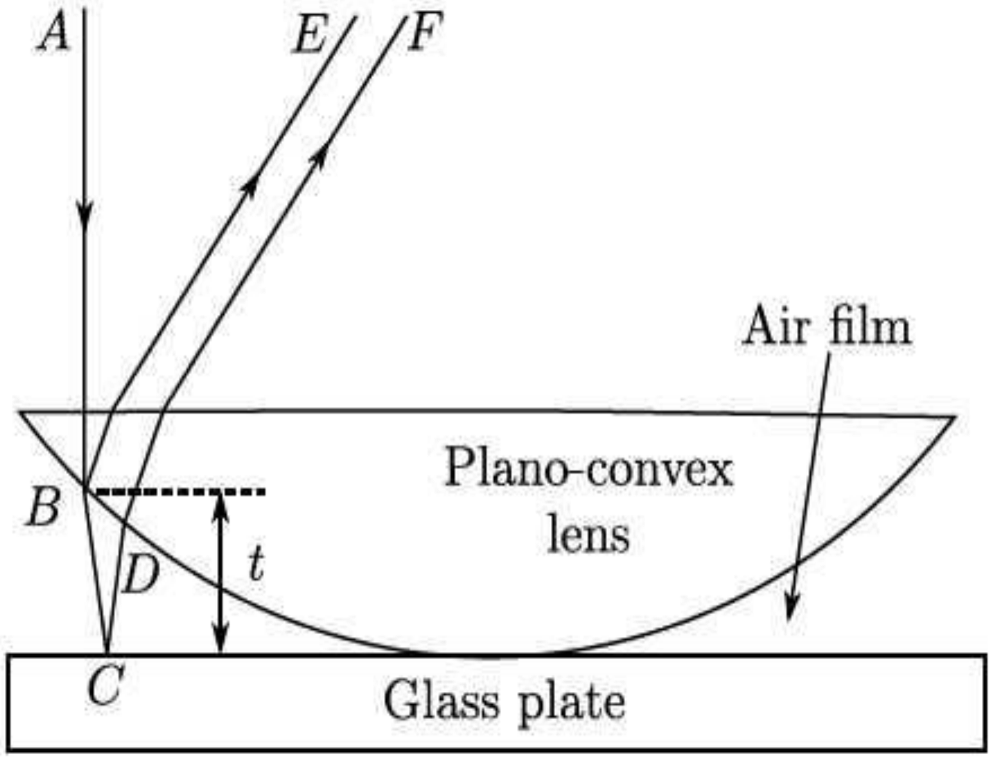
Lecture-20

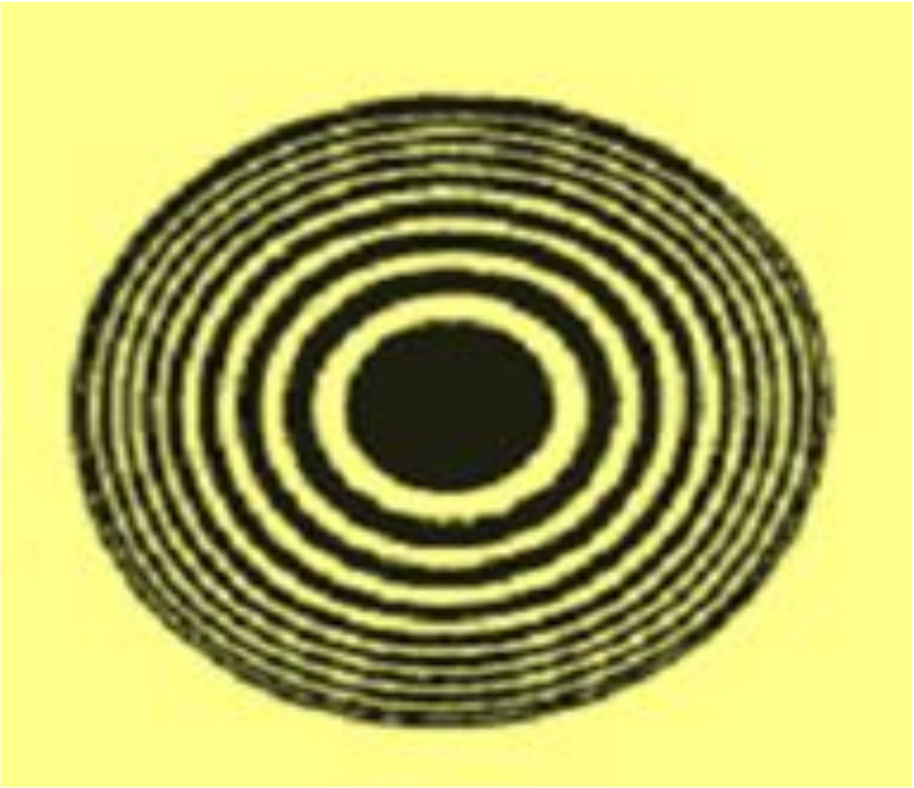
Outline

- Necessity of an extended source
- Newton’s rings

What is Newton’s rings? **(2015-16, 2018-19)**

Newton’s rings are the locus of constant thickness of air film formed between glass plate and Plano-convex lens. It is based on interference phenomena. As a result, alternate dark and bright fringes are obtained.





Why the centre of Newton's rings is dark in reflected system?

(2015-16)

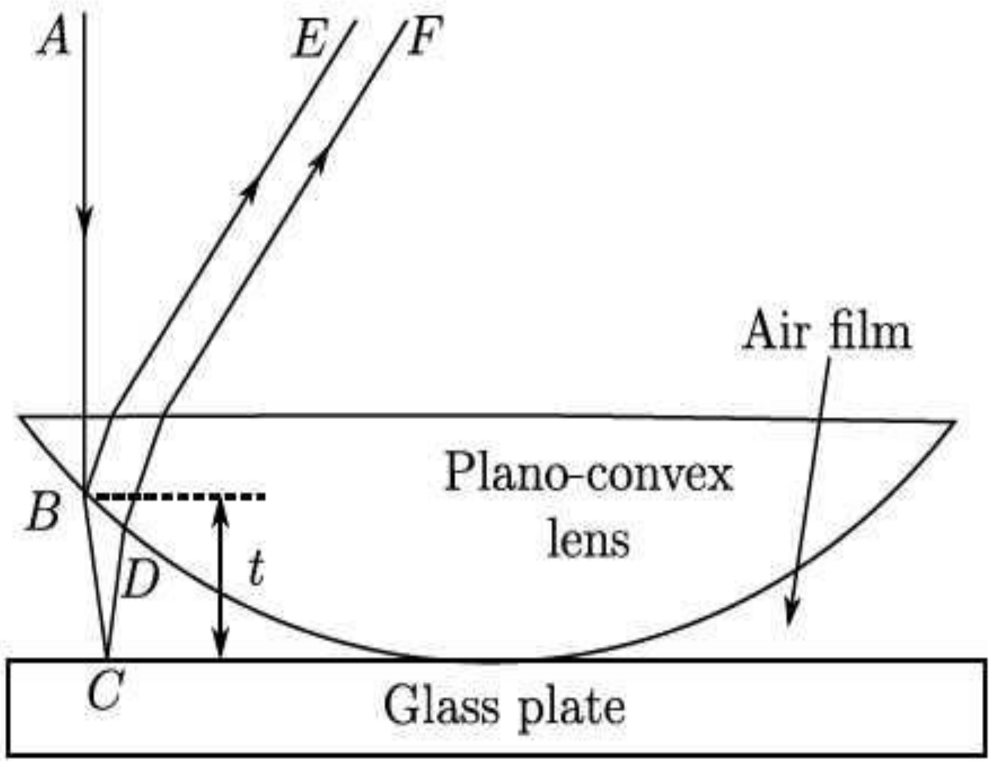
As the rings are observed in reflected light, the effective path difference is given by $2\mu t \cos r + \lambda/2$, where μ is the refractive index of the film, t is the thickness of the film at point of incidence. For normal incidence $r = 0$, therefore path difference is $2\mu t + \lambda/2$.
At $t = 0$, the effective path difference is $\lambda/2$. This is the condition of minimum intensity.

Hence the centre of Newton's ring is dark.

What do you understand by Newton's rings? Explain the experimental arrangement. How can you determine the wavelength of light with this experiment ? (2016-17)

Newton's Rings

- When a Plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, an air film of gradually increasing thickness from the point of contact is formed between the upper surface of the plate and the lower surface of the lens.



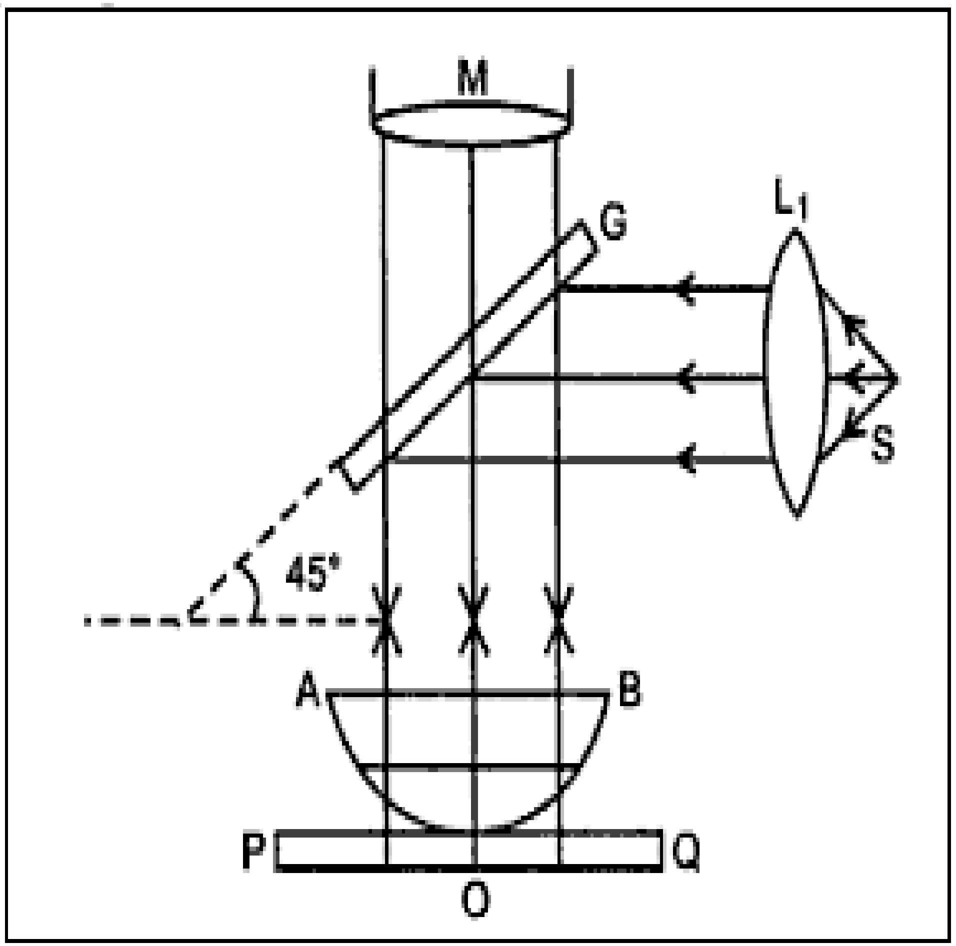
- If monochromatic light is allowed to fall normally on this film, then alternate bright and dark concentric rings with their centre dark are formed. These rings are known as Newton’s rings.

- Newton’s rings are formed because of the interference between the waves reflected from the top & bottom surfaces of an air film formed between the lens and plate.

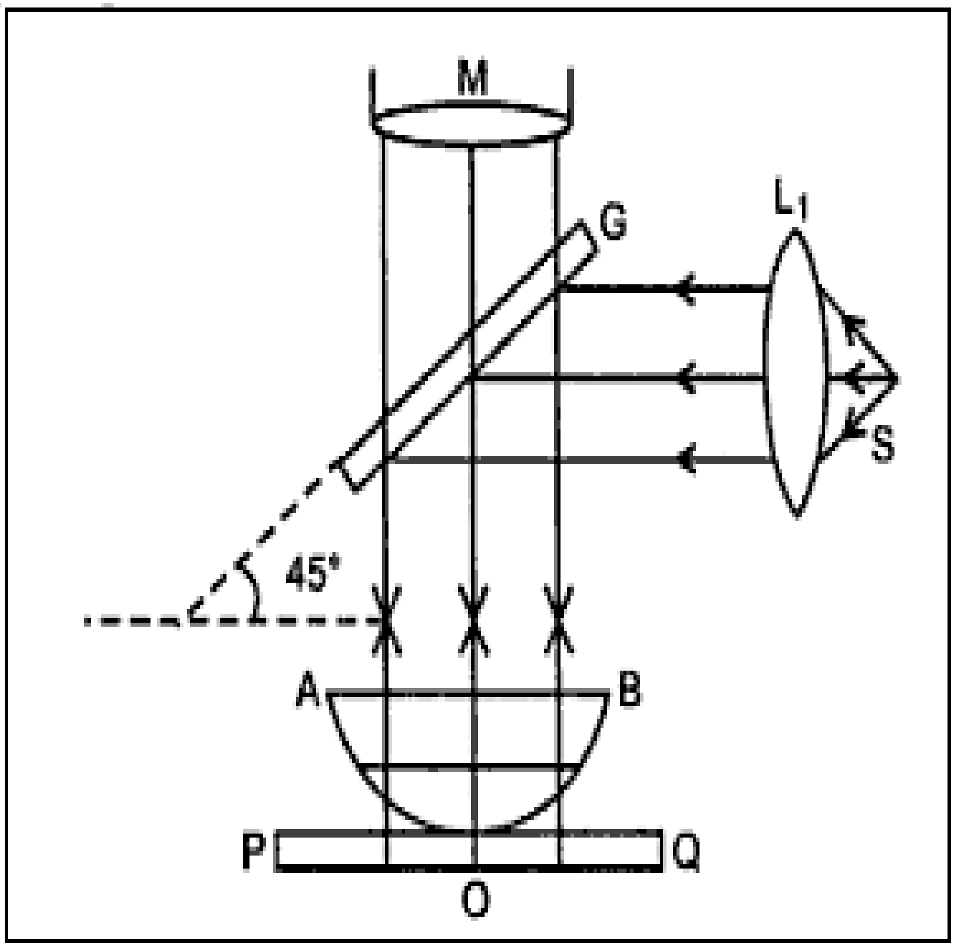
Experimental arrangement

- A Plano-convex lens L of large radius of curvature is placed on a plane glass plate P such that both of them are having a point of contact.

- Light from a monochromatic source is allowed to fall on a glass plate G inclined at an angle 45º to the incident beam.



- The light reflected from the glass plate falls normally on the air film enclosed between Plano convex lens & plane glass plate.



- Light rays reflected upward from the air film, superimpose each other and interference takes place. Due to interference of these rays, alternate bright & dark concentric rings are seen, with the help of microscope.

As the rings are observed in reflected light, the effective path difference is given by $2\mu t \cos r + \frac{\lambda}{2}$, where μ is the refractive index of the film, t is the thickness of the film at point of incidence. For normal incidence r = 0, therefore path difference is $2\mu t + \frac{\lambda}{2}$.

At t = 0, the effective path difference is $\frac{\lambda}{2}$.

Condition for constructive interference-

$$\Delta = n\lambda$$

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$\text{or} \quad 2\mu t = (2n - 1)\frac{\lambda}{2},$$

where n = 1, 2, 3,……

Condition for destructive interference-

$$\Delta = (2n+1)\frac{\lambda}{2}$$

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\text{or} \quad 2\mu t = n\lambda,$$

where n = 0,1,2,3………

Determination of Wavelength of Sodium light using Newton's Ring-

Dark rings-

Let D_n and $D_{(n+p)}$ be the diameters of the n^{th}& $(n+p)^{th}$ dark rings respectively, then

$$D_n^2 = 4n\lambda R \qquad \text{……….(1)}$$

$$D_{n+p}^2 = 4(n+p)\lambda R \qquad \text{………….(2)}$$

where p is any number & R be the radius of curvature of the lens.

Subtracting equation 1 from equation 2,

$$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - \ 4n\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \qquad .....(3)$$

The same result shall be obtained for bright ring.

Show the diameter for bright rings are proportional to square root of odd natural number and for dark rings diameters are proportional to square root of natural number. (2015-16, 2018-19)

Diameter of Bright & Dark Rings

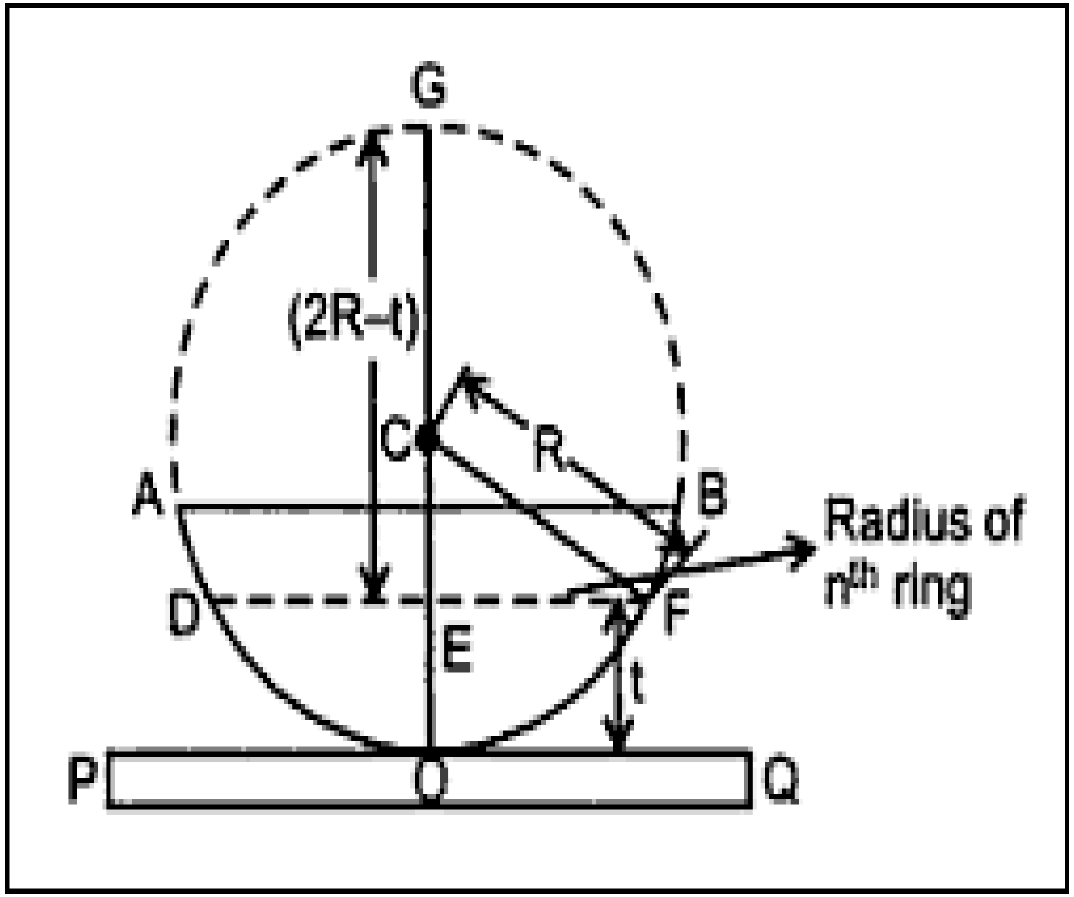
- Let R be the radius of curvature of the lens and r be the radius of a Newton's ring where film thickness is 't'.
- From the property of a circle,

$$DE \times EF = OE \times EG$$

$$r \times r = t\,(2R-t)$$

$$r^2 = 2Rt - t^2 \qquad \ldots\ldots(1)$$

Since t is very small as compared to R,

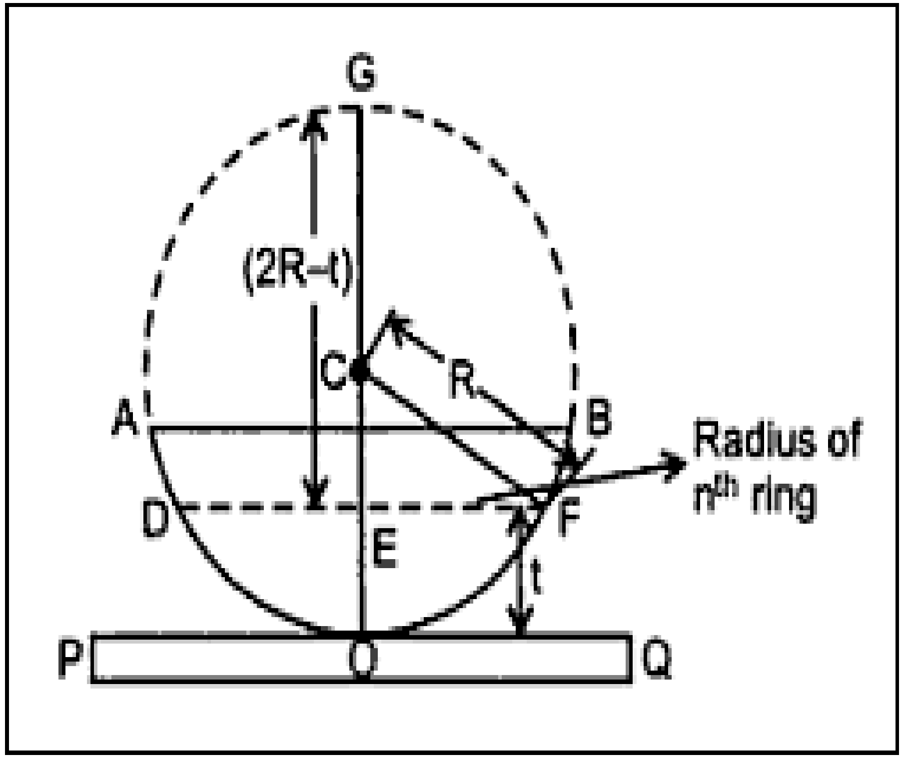


hence

$$r^2 = 2Rt$$

or

$$t = \frac{r^2}{2R} \quad \text{.....(2)}$$



For Bright rings, substituting the value of t in condition for maxima,

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

$$2\mu\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$2\mu\frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

$$r^2 = (2n-1)\frac{\lambda R}{2\mu} \quad \text{........(3)}$$

For n^th ring,

$$r_n^2 = (2n-1)\frac{\lambda R}{2\mu}$$

$$\frac{D_n^2}{4} = (2n-1)\frac{\lambda R}{2\mu} \quad \text{......(4)}$$

where r = $\frac{D}{2}$

$$D_n^2 = 2(2n-1)\frac{\lambda R}{\mu}$$

$$D_n = \sqrt{2(2n-1)\frac{\lambda R}{\mu}} \ldots\ldots\ldots\ldots(5)$$

For air film, μ = 1

$$D_n = \sqrt{2(2n-1)\lambda R}$$

Let $\sqrt{2\lambda R} = K$

Hence D_n= K$\sqrt{2n-1}$

or $D_n \propto \sqrt{2n-1}$

Thus, *the diameter of bright rings are proportional to the square root of the odd natural number*.

For Dark rings, Substituting the value of t in condition in minima,

$$2\mu \frac{r^2}{2R} = n\lambda$$

$$r^2 = \frac{n\lambda R}{\mu}$$

For n^{th} ring.

$$r_n^2 = \frac{n\lambda R}{\mu}$$

$$\frac{D_n^2}{4} = \frac{n\lambda R}{\mu}$$

where $r = \frac{D}{2}$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}}$$

For air film, μ = 1, $D_n = \sqrt{4n\lambda R}$

Let $\sqrt{4\lambda R} = K$

$$D_n = K\sqrt{n}$$

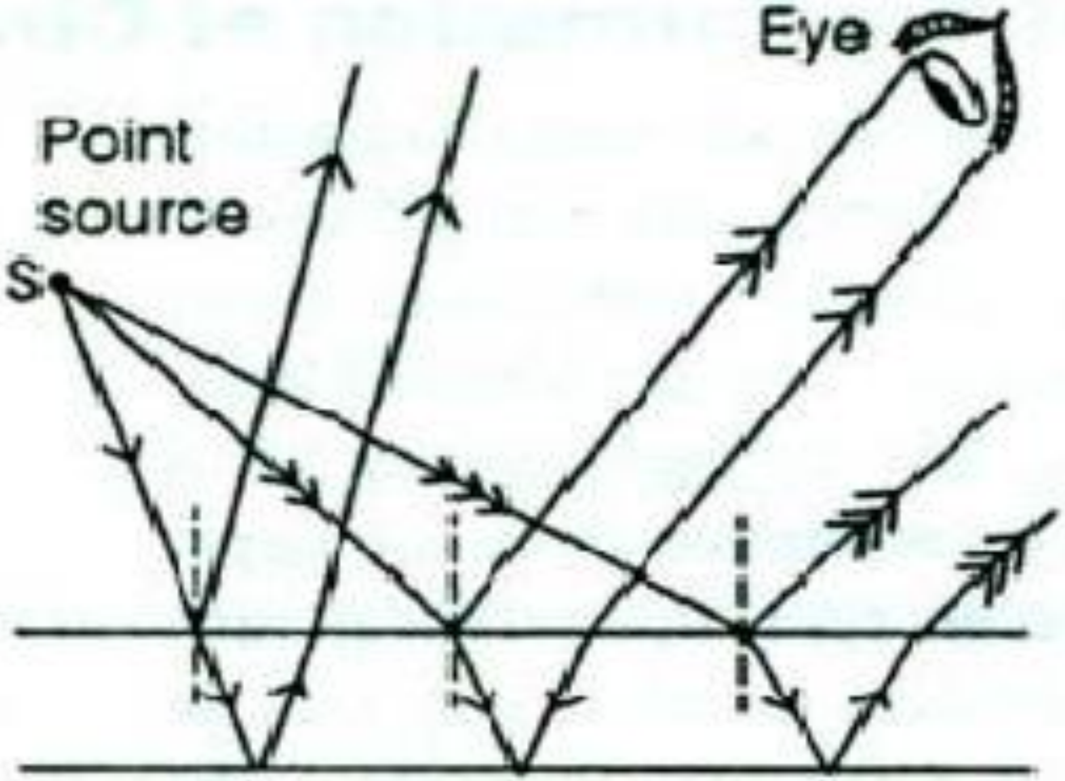
$$D_n \propto \sqrt{n}$$

Thus, the diameters of dark rings are proportional to the square root of the natural number.

Explain the necessity of extended sources. (2018-19)

Necessity of an Extended source

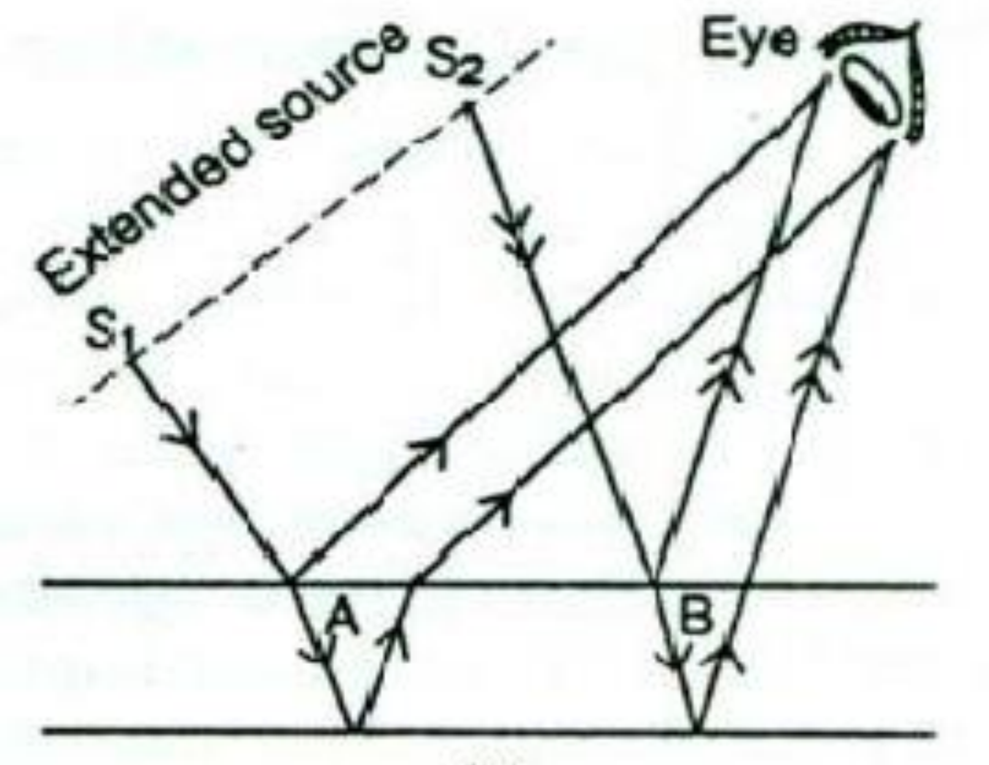
- An extended source is necessary to enable the eye to see whole the film simultaneously.
- Let light from a narrow sources S can be incident on a thin film. For each incident wave we get a pair of parallel interfering wave in fig (a) the interfering pairs of wave due to incident wave 1,2,3 are shown due to the limited size of pupil the ray from only a small portion will be visible.



(a)

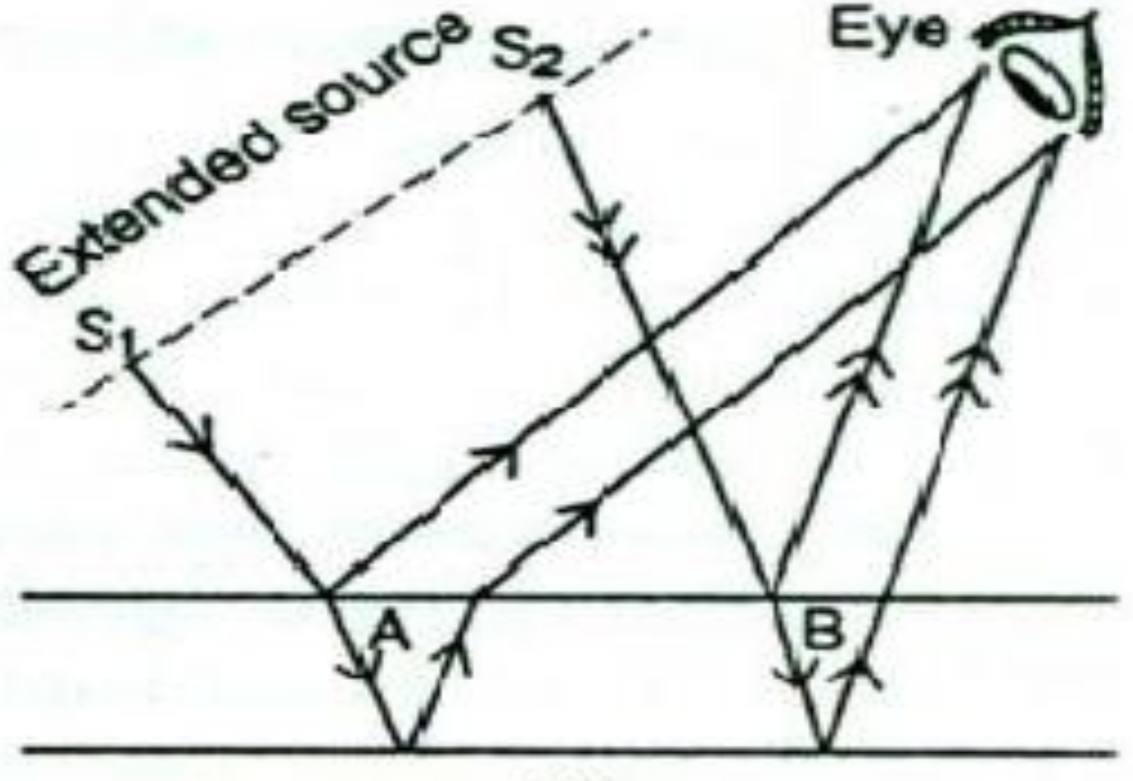
- To observe the different part of film they should move to sideways. Hence with a narrow source it is not possible to observe the whole film simultaneously.

- Now let us consider the case when the film is illuminated by an extended source of light in fig (b), the portion of A film is seen by reflected light originally coming from some point S_1 of the extended source.
- Similarly the portion B is seen by reflected light originally coming from some point S_2 of the same source and so on.



(b)

- Thus in case of extended source of light the rays of different part of are reflected from different part of the film so as to enter the eye placed in a suitable position.
- Thus we may see the entire film simultaneously. So we conclude that utility of an extended source is to enable the eye to view a larger area of the film.



(b)

Unit-III

Wave-optics

Lecture-21

Outline

- Numericals

Calculate the thickness of soap bubble thin film that will result in constructive interference in reflected light. The film is illuminated with light of wavelength 5000Å and refractive index of film is 1.45.

(2020-21)

We know that the condition of constructive interference in the reflected system is

$$2\mu t \cos r = \frac{(2n-1)\lambda}{2}$$

For normal incidence, r = 0 or cos r = 1 and for thinnest film n = 1

Hence, $$t = \frac{\lambda}{4\mu \cos r}$$

$$t = \frac{5000 \times 10^{-10}}{4 \times 1.45 \times 1}$$

$$t = 8.6 \times 10^{-8}\text{m.}$$

Calculate the thickness of the thinnest film ($\mu = 1.4$) in which interference of violet component ($\lambda = 4000$ Å) of incident light can take place by reflection.

We know that the condition of constructive interference in the reflected system is

$$2\mu t \cos r = \frac{(2n-1)\lambda}{2}$$

For normal incidence r = 0 or cos r = 1 and for thinnest film n = 1

Hence $$t = \frac{\lambda}{4\mu \cos r}$$

$$t = \frac{4000 \times 10^{-8}}{4 \times 1.4 \times 1}$$

$$\mathbf{t = 714.3\ \text{Å}}$$

Light of wavelength 6000 Å falls normally on a thin wedge shaped film of refractive index 1.4 forming the fringes that are 2 mm apart. Find the angle of wedge. (Imp)

If θ is the angle of wedge formed by a medium of refractive index μ, then for normal incidence the fringe width for wavelength λ is given by

$$\beta = \frac{\lambda}{2\mu\theta}$$

$\lambda = 6000 \times 10^{-8}$ cm, μ =1.4, β = 2mm = 0.2 cm

$$\theta = \frac{6000 \times 10^{-8}}{2\ x\ 1.4\ x\ 0.2} = 10.71\ \text{x}10^{-5}\ \text{radian}$$

$$\theta = 10.71\ \text{x}10^{-5}\text{x}\ \frac{180}{\pi}\ \text{degree}$$

$$\theta = 0.0061^{0}$$

In Newton's ring experiment the diameter of 4^{th} and 12^{th} dark ring are 0.4 cm and 0.7 cm respectively. Deduce the diameter of 20^{th} dark ring. **(Imp)**

If $D^2{}_{n+p}$ and $D^2{}_n$ be the diameters of (n+p)th and nth dark ring respectively, then

$$D^2{}_{n+p} - D^2{}_n = 4p\lambda R \qquad \text{--------------------(1)}$$

In the given problem n = 4, n+p =12, D_4= 0.4 cm and D_{12}= 0.7 cm

$$D^2{}_{12} - D^2{}_4 = 4 \times 8 \times \lambda \times R \qquad \text{------------------(2)}$$

Suppose the diameter of 20th dark ring is D_{20},

$$D^2{}_{20} - D^2{}_4 = 4 \times 16 \times \lambda \times R \qquad \text{------------------------------------(3)}$$

Dividing equation (2) by (3), we get

$$\frac{D^2{}_{12} - D^2{}_4}{D^2{}_{20} - D^2{}_4} = \frac{4 \times 8 \times \lambda \times R}{4 \times 16 \times \lambda \times R} = \frac{1}{2}$$

$$2 \times (D^2{}_{12} - D^2{}_4) = (D^2{}_{20} - D^2{}_4)$$

$$D^2{}_{20} = 2D^2{}_{12} - D^2{}_4$$

$$D^2{}_{20} = 2(0.700)(0.700) - (0.400)(0.400)$$

$$D^2{}_{20} = 0.98 - 0.16 = 0.82$$

$$D_{20} = \sqrt{0.82} = 0.906\ cm$$

So diameter of 20 th ring = 0.906 cm

A square piece of cellophane film with index of refraction 1.5 has a wedge shaped section so that its thickness at two opposite side is t_1 and t_2. If the number of fringes appearing with wavelength λ = 6000 Å is 10 calculate the difference (t_1 - t_2).

If the order of the fringe appearing at one end o the film be n , then the order of the fringe appearing at other end will be (n +10) for dark fringe.

$$2\mu t_1 cosr = n\lambda \quad \dots\dots\dots\dots\dots\dots (1)$$

$$2\mu t_2 cosr = (n+10)\lambda \dots\dots (2)$$

Subtracting equation (1) from Equation (2) we get

$$2\mu(t_2 - t_1\ )cosr = 10\lambda \dots\dots. (3)$$

If the fringe is seen normal and the angle of wedge very small, then r =0 so that

$$cosr = 1 \dots\dots\dots\dots\dots. (4)$$

Substituting the value of cos r from equation 4 to equation 3 we get

$$2\mu(t_2 - t_1) = 10\lambda$$

$$(t_2 - t_1) = \frac{10\lambda}{2\mu} = \frac{5\lambda}{2\mu}$$

Here

$\mu = 1.5$ and $\lambda = 6000 \times 10^{-3}\ cm$

$$\boldsymbol{(t_2 - t_1) = \frac{5 \times 6000 \times 10^{-3}}{1.5} = 2 \times 10^{-4}\ cm}$$

Unit-III

Wave-optics

Lecture-22

Outline

- Diffraction
- Types of diffraction

(i) Fraunhofer diffraction

(ii) Fresnel diffraction

Diffraction

The bending of light around the sharp corner of opaque obstacle and spreading of light within the geometrical shadow of opaque obstacles is called diffraction of light.

Condition of diffraction:

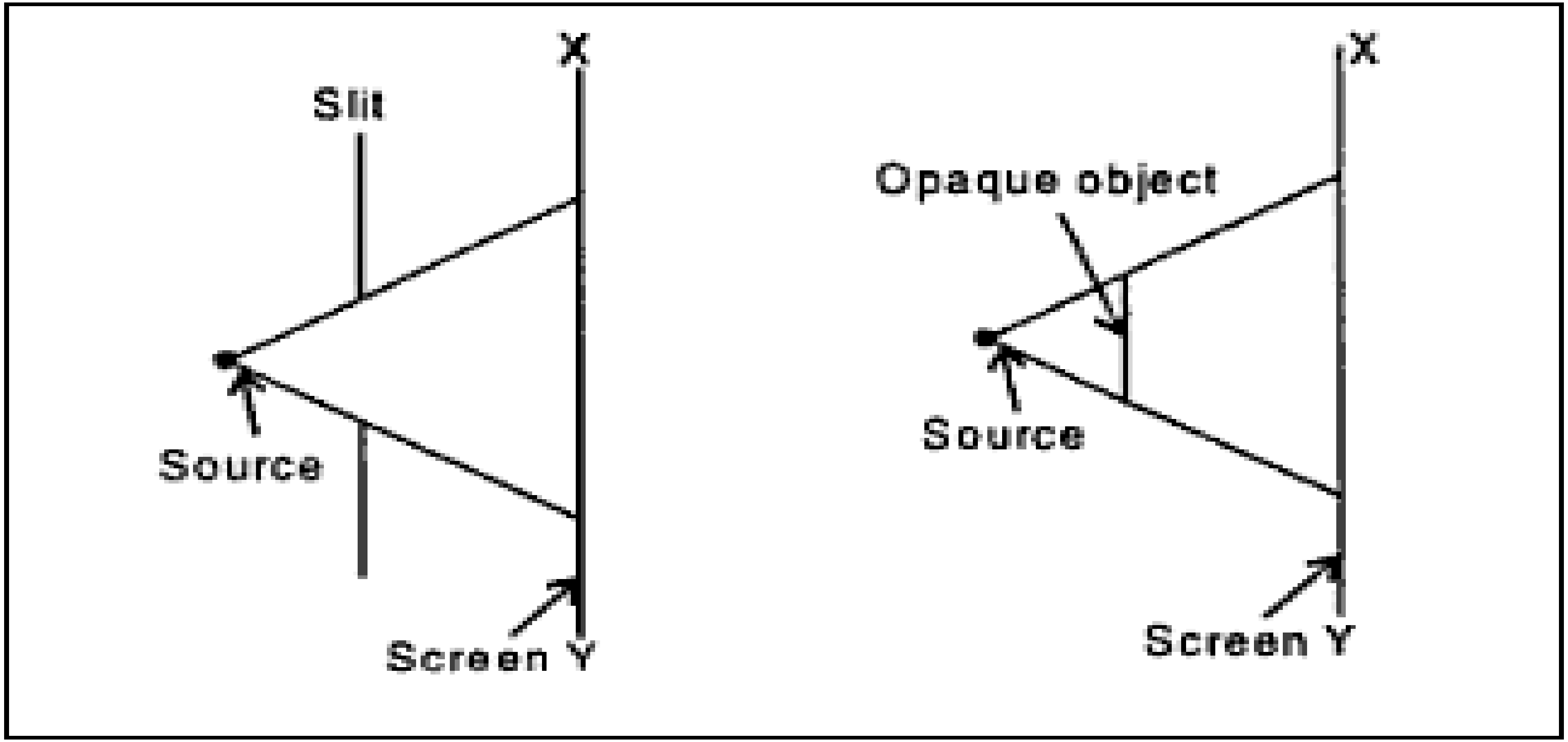
The particle size is equal to wavelength of light.

Types of diffraction:

There are two types of diffraction.

(i) Fraunhofer diffraction

(ii) Fresnel diffraction


X
Slit
Source
Screen Y
X
Opaque object
Source
Screen Y

Difference between Fresnel and Fraunhofer diffraction

S.No	**Fresnel**	**Fraunhofer**
1.	Source and the screen are at finite distance from the diffracting aperture.	Source and the screen are at infinite distance from the diffracting aperture.
2.	Incident wave fronts are spherical or cylindrical.	Wave fronts **incident** on the diffracting obstacle are plane.
3.	For obtaining Fresnal diffraction, zone plates are used.	For this, single, double slits or gratings are used.

4.	Convex lens is not needed to converge the spherical wave fronts.	Plane diffracting wave fronts are converged by means of a convex lens to produce diffraction pattern.
5.	The centre of diffraction pattern may be bright or dark depending upon the number of Fresnel zones.	The centre of the diffraction pattern is always bright.

Discuss the phenomena of Fraunhofer's diffraction at a slit and show that relative intensities of the successive maxima are nearly

$$\mathbf{1 : (4/9\pi^2) : (4/25\pi^2) : \; . \quad . \quad . \quad . \quad .}$$

(2015-16, 2018-19)

or

Obtain an expression for the intensity distribution due to Fraunhofer diffraction at a single slit. (2017-18, 2019-20)

or

Obtain intensity expression for single slit Fraunhofer diffraction pattern.

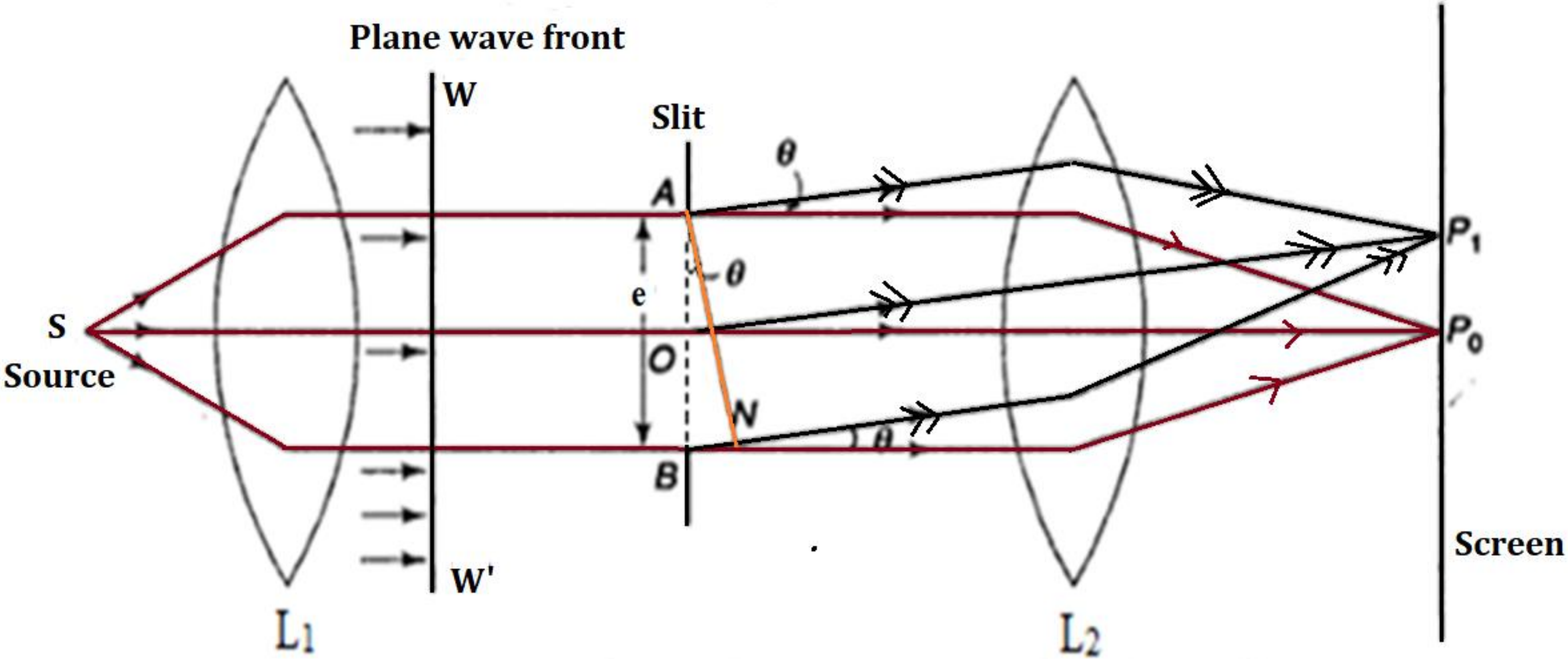
(2015-16)

Discuss the phenomena of Fraunhofer's diffraction at a slit and show that the intensity of first subsidiary maxima is about 4.5% of principal maxima.

(2019-20)

Fraunhofer diffraction at a single slit

Let a parallel beam of monochromatic light of wavelength λ, produced by a point source S be incident upon a converging lens (L_1) and emerging light from it, falls upon a slit AB of width '*e*' where it gets diffracted. If a converging lens (L_2) is placed in the path of the diffracted beam, a real image of the diffraction pattern is formed on the screen in the focal plane of the lens.



Plane wave front
W
Slit
θ
A
θ
e
S
Source
O
N
θ
B
W'
L1
L2
P1
P0
Screen

Path difference is given by,

BN = AB $sin\theta$

BN = $esin\theta$…………(1)

$$\text{Phase difference} = \frac{2\pi}{\lambda}(\mathrm{e}\sin\theta) \qquad \ldots\ldots\ldots\ldots \quad (2)$$

Now, Consider the width AB of the slit is divided into n parts. Each part forms an elementary source. So phase difference between successive wave fronts is

$$\frac{1}{n}(total\ phase) = \frac{1}{n}\left[\frac{2\pi}{\lambda}(\mathrm{e}\sin\theta)\right] = \delta[= \frac{2\alpha}{n}\ (say)]$$

According to the theory of composition of *n* simple harmonic motions of equal amplitude (a) and common phase difference between successive vibrations, the resultant amplitude at P_1 is given by,

$$R = \frac{a\ \sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} = \frac{a \sin \alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

$$where\ \alpha = \frac{\pi}{\lambda}(\mathrm{e} \sin \theta)\ \left\{\because \frac{n\delta}{2} = \frac{\pi}{\lambda}(\mathrm{e} \sin \theta)\right\}$$

$$R = \frac{a \ \sin\alpha}{\frac{\alpha}{n}} \ , \left(for\ small\ \frac{\alpha}{n}, \sin\left(\frac{\alpha}{n}\right) = \frac{\alpha}{n} \right)$$

$$R = \frac{na\sin\alpha}{\alpha}$$

When $n \to \infty, a \to 0\ but\ product\ na\ remains\ finite.\ na \to A\ (say)$

$$R = \frac{\mathrm{A}\sin\alpha}{\alpha} \qquad \ldots\ (3)$$

Resultant intensity,

$$I = R^2 = \frac{A^2 sin^2\alpha}{\alpha^2} \qquad \ldots\ (4)$$

Positions of Maxima and Minima

Principal maximum or central maxima: The resultant amplitude given by eq^n(3) can be written as

$$R = \frac{A}{\propto}\left[\propto - \frac{\propto^3}{3!} + \frac{\propto^5}{5!} - \frac{\propto^7}{7!} + \cdots\right]$$

$$= A\left[1 - \frac{\propto^2}{3!} + \frac{\propto^4}{5!} - \frac{\propto^6}{7!} + \cdots\right]$$

If $\propto = 0$, the value of R will be maximum.

$$\therefore \quad \propto = \frac{\pi}{\lambda}\left(e \sin\theta\right) = 0$$

$$\therefore \ \boldsymbol{\sin\theta = 0 \ or \ \theta = 0} \qquad \ldots\ldots\ldots \ (5)$$

Thus the maximum value of resultant intensity at P_0. This maxima is called Principal maxima.

Position of minima and secondary maxima: Differentiate $eq^n(4)$ with respect to α and equate to zero, that is,

$$\frac{dI}{d\propto} = \frac{d}{d\propto}\left[A^2\left(\frac{\sin\propto}{\propto}\right)^2\right] = 0$$

$$\therefore \quad A^2 . \frac{2\sin\ \propto}{\propto} . \frac{(\propto\ \cos\ \propto - \sin\ \propto)}{\propto^2} = 0$$

So that either $\sin\alpha = 0$

or $(\alpha \cos\alpha - \sin\alpha) = 0$

Position of minima:

The condition sin α = 0 gives the position of minima.

So for minimum intensity, $\sin\alpha = 0$

$$\propto\, = \pm n\pi \;or\; \pm\pi, \pm 2\pi, \pm 3\pi \,\ldots\ldots \pm n\pi$$

$$\frac{\pi}{\lambda}\left(e\sin\theta\right) = \pm n\pi$$

$$\boldsymbol{e\sin\theta = \pm n\lambda}\ldots\ldots\ldots(6)$$

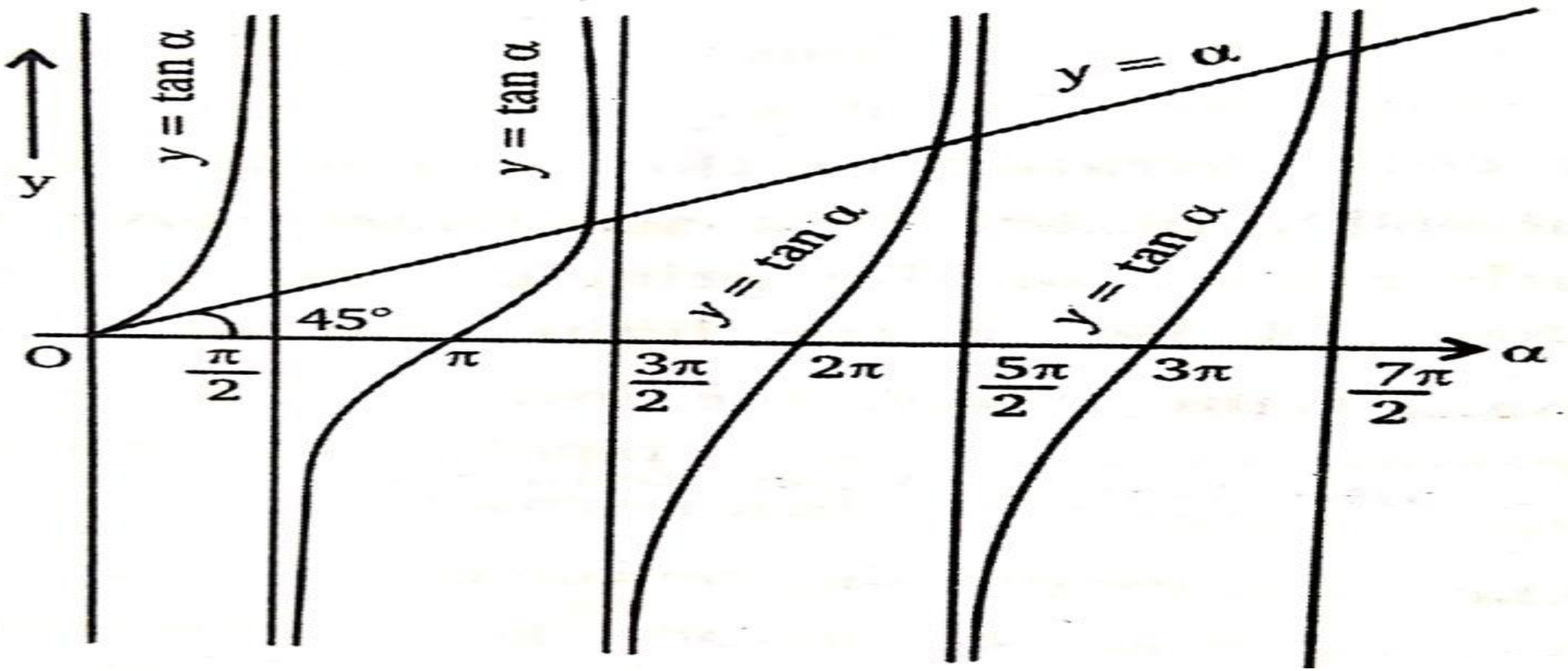
Position of secondary maxima:

The position of secondary maxima are given by

$$(\alpha \cos \alpha - \sin \alpha) = 0$$

$$\alpha = \tan \alpha \qquad \text{............}(7)$$

On plotting graph, $y = \alpha$ and $y = \tan \alpha$



In graph, the points of intersection are the points of maxima.

$$\propto = \mathbf{0}, \pm\frac{\mathbf{3\pi}}{\mathbf{2}}, \pm\frac{\mathbf{5\pi}}{\mathbf{2}}, \pm\frac{\mathbf{7\pi}}{\mathbf{2}} \dots.$$

Here, at α = 0 we get the central maxima. The direction of secondary maxima is given by,

$$\alpha = \frac{\pi}{\lambda}(\mathrm{e}\sin\theta) = \pm(\mathbf{2n} + \mathbf{1})\frac{\boldsymbol{\pi}}{\mathbf{2}}$$

$$\mathrm{e}\boldsymbol{\sin\theta} = \pm(\mathbf{2n} + \mathbf{1})\frac{\boldsymbol{\lambda}}{\mathbf{2}} \qquad \text{....(7)}$$

where$n$ = 1, 2, 3, 4 …….

Here n = 0 is not taken because for n = 0, we get the position of **Principal maxima**.

1) For $n = 0$, $I = A^2 = I_0$ (principal maxima)

2) For n = 1,

$$I_1 = A^2 \left[\frac{\sin(3\pi/2)}{3\pi/2}\right]^2$$

$$= \boldsymbol{A^2 \frac{4}{9\pi^2}}$$

$$= \frac{I_0}{22} \qquad (Ist\ secondary\ maxima)$$

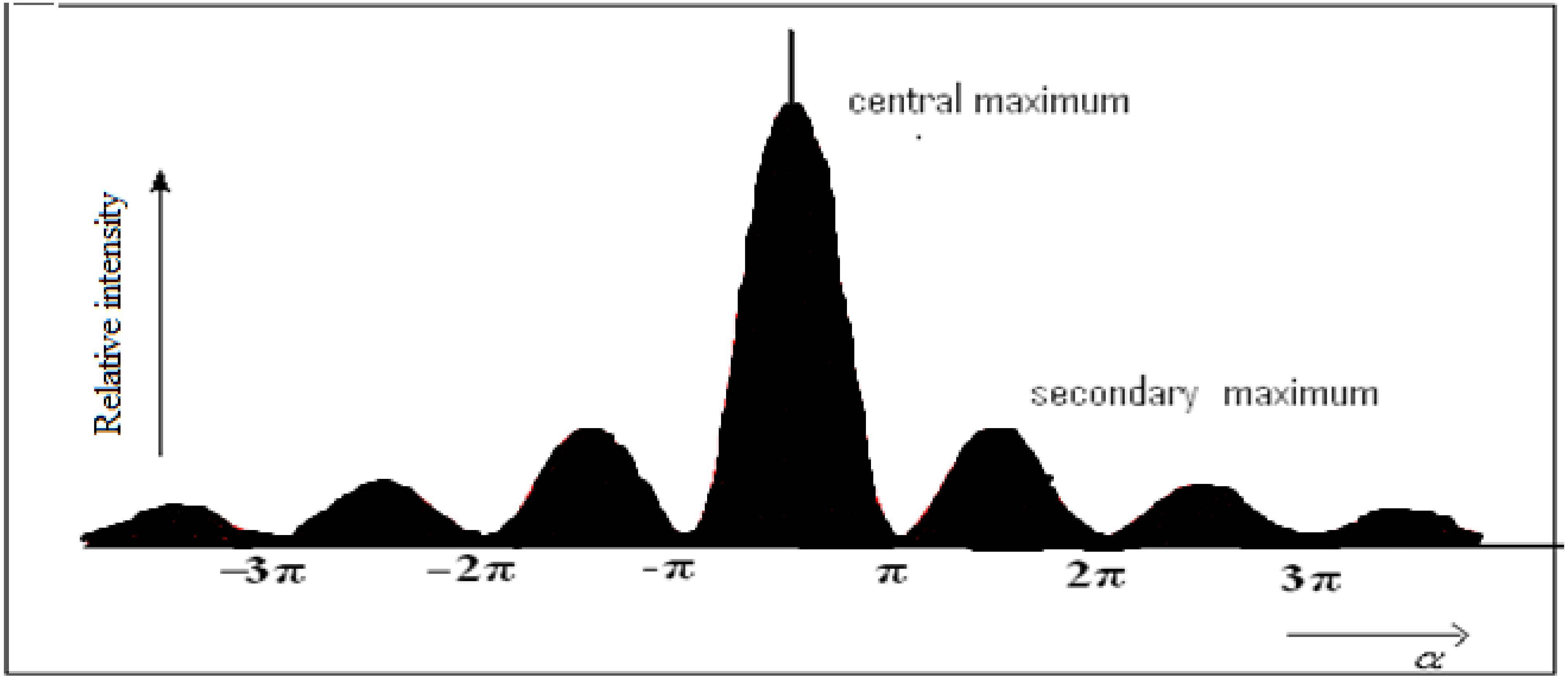
Thus *intensity of first secondary maxima is about (1/22)^th^ i.e. 4.5 % of the intensity of the central maximaor the intensity of first subsidiary maxima is about 4.5% of principal maxim*

3) For n = 2,

$$I_2 = A^2 \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = \boldsymbol{A^2 \frac{4}{25\pi^2}} = \frac{I_0}{61} \qquad (IInd\ secondary\ maxima)$$

Thus *intensity of second secondary maxima is about (1/61)th i.e.* ***1.61*** *% of the intensity of central maxima.*

Thus, $\boldsymbol{I_0 : I_1 : I_2 : I_3}$ **.....=** $\boldsymbol{1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \ldots \ldots}$



Intensity distribution due to single slit diffraction

Unit-III

Wave-optics

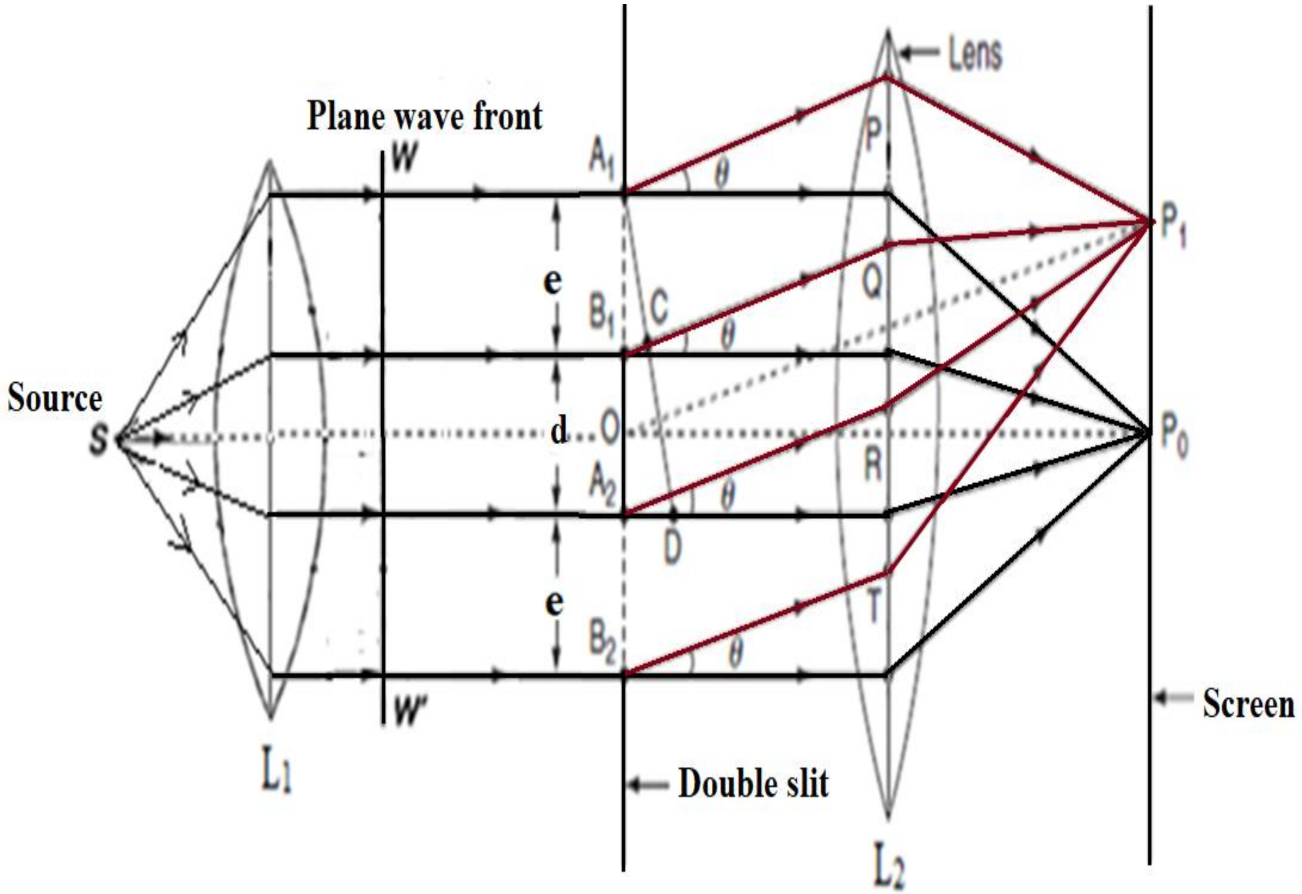
Lecture-23

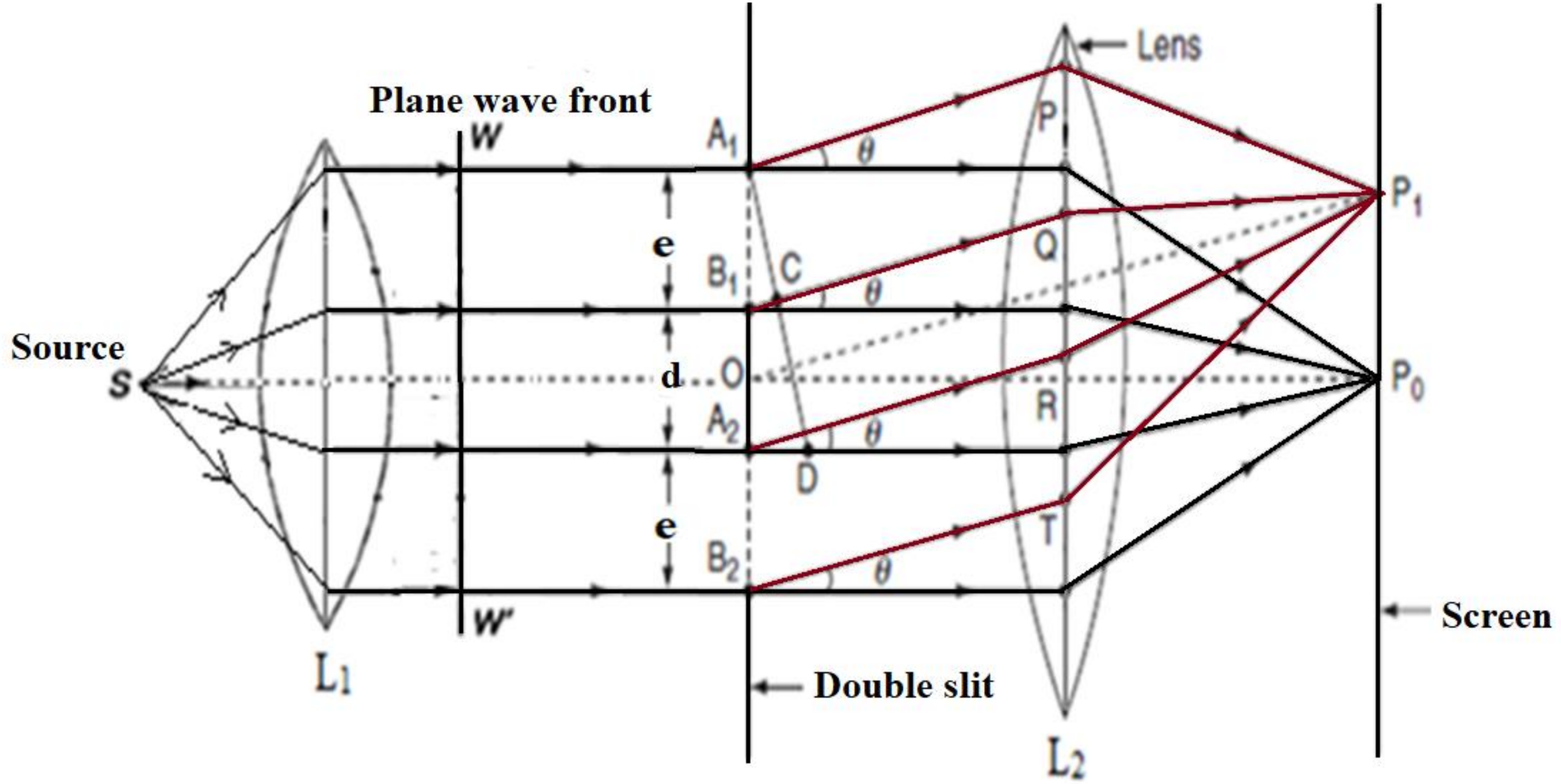
Outline

- Fraunhofer diffraction at double slits
- Diffraction grating

Fraunhofer diffraction at a double slits

- Let a monochromatic plane wave front of wave length ‘λ’ is incident normally on both the slits.
- The double slits have been represented as A_1B_1 and A_2B_2. Let the width of both the slits be equal and it is ‘*e*’ and they are separated by length ‘*d* ’.




Plane wave front
W
W'
Lens
Source
S
L1
A1
B1
C
O
A2
D
B2
e
d
e
θ
P
Q
R
T
P1
P0
Double slit
L2
Screen

P_0 corresponds to the position of the central bright maximum. The intensity distribution on the screen is the combined effect of interference of diffracted secondary waves from the slits.

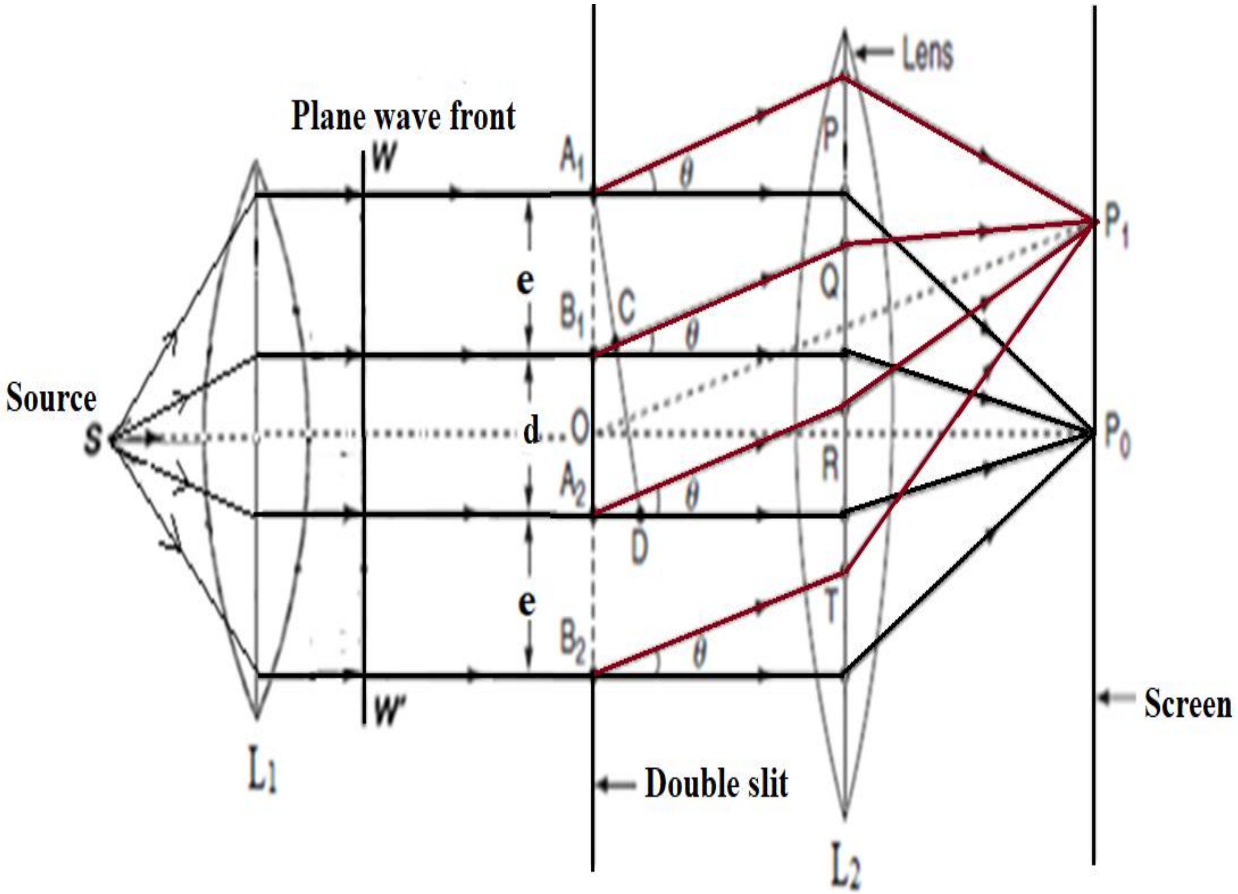
From the triangle A_1B_1C,

$$sin\ \theta = \frac{B_1C}{A_1\ B_1} = \frac{B_1C}{e}$$

$$B_1C = e \sin\theta$$

phase difference,

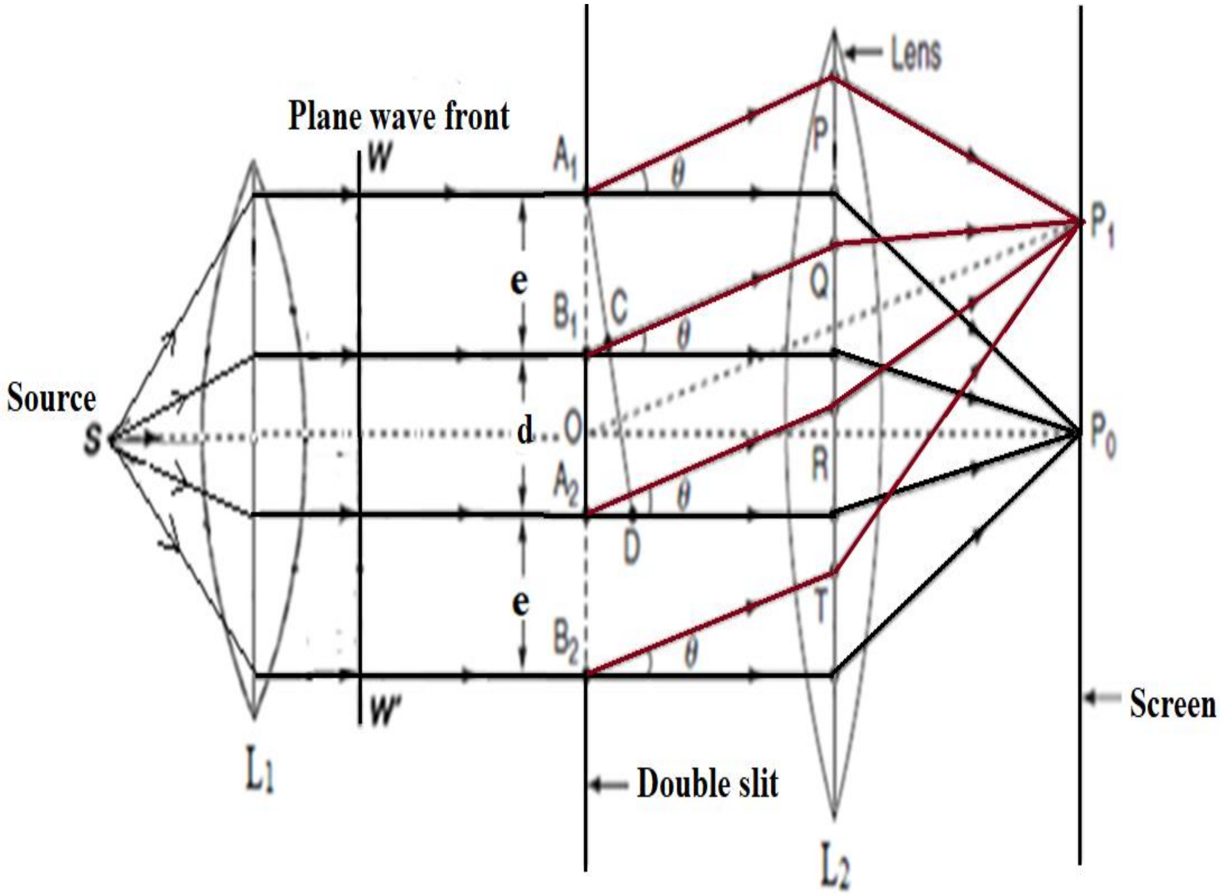
$$2 \propto = \frac{2\pi}{\lambda}\ (e\ sin\theta)$$



$\propto = \frac{\pi}{\lambda}(e\ sin\theta)$... (1)

The diffracted wave amplitudes

[A sinα / α] from the two slits, combine to produce interference. The path difference between the rays coming from corresponding points in the slits A_1B_1 and A_2B_2 can be found by drawing a normal from A_1 to A_2R. A_2D is the path difference between the waves from corresponding points of the slits.



In the $\Delta\, A_1A_2D$,

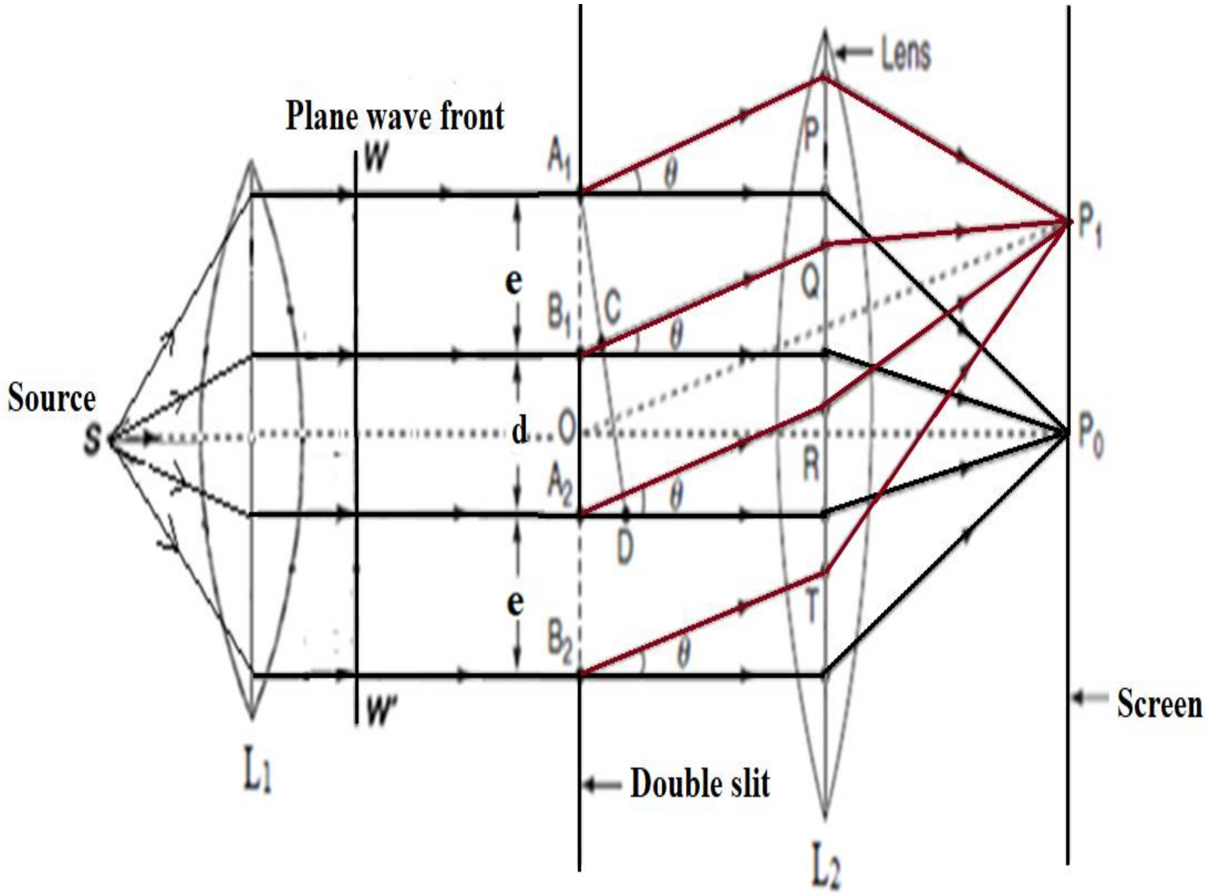
$\sin\theta = A_2D / A_1A_2$

path difference, $A_2D = A_1A_2 \sin\theta$

$A_2D = (e + d)\sin\theta$

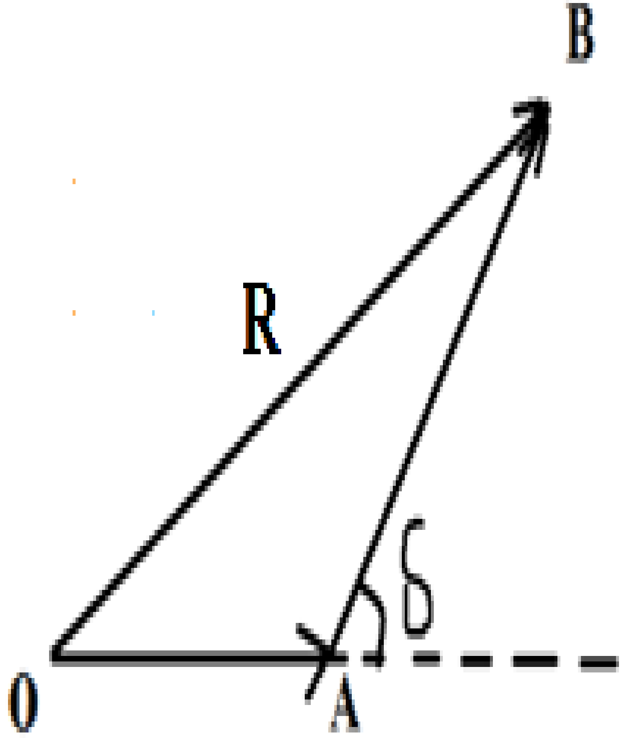
The corresponding phase difference,

$$\delta = 2\beta = \frac{2\pi}{\lambda}(e+d)\sin\theta \quad . \; . \; . \; (2)$$



Applying the theory of interference on the wave amplitudes [(A sin α) / α] at the two slits gives the resultant wave amplitude (*R*).

$$\text{OA} = \frac{A\sin\alpha}{\alpha}, \ \text{AB} = \frac{A\sin\alpha}{\alpha}$$



From figure we have,

$$(OB)^2 = (OA)^2 + (AB)^2 + 2(OA).(AB)\cos\delta$$

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right).\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

$$R^2 = \left(\frac{A \sin \alpha}{\alpha}\right)^2 (2 + 2\cos\delta)$$

$$R^2 = \frac{A^2 sin^2 \alpha}{\alpha^2}\ 4\ cos^2\left(\frac{\delta}{2}\right)$$

$$R^2 = \frac{4A^2 sin^2 \alpha}{\alpha^2}\ cos^2\left(\frac{\delta}{2}\right)$$

[where $\beta = \frac{\delta}{2} = \frac{\pi}{\lambda}\ (e + d)\sin\theta$]

Then the intensity at P_1 is

$$I = R^2 = 4A^2 \frac{sin^2 \propto}{\propto^2} cos^2\beta$$

$$= 4I_0 \frac{sin^2 \propto}{\propto^2} cos^2\beta \qquad \ldots\ldots(3)$$

[since $I_0 = A^2$]

Equation (3) represents the intensity distribution on the screen. In equation (3) the term $\cos^2\beta$ corresponds to interference and $[\sin^2\alpha / \alpha^2]$ corresponds to diffraction.

Interference maxima and minima: If the path difference

$$A_2D = (e + d)\, sin\theta = \pm\, n\, \lambda \qquad \text{………..} \; (4)$$

where n = 1, 2, 3… then 'θ' gives the directions of the maxima due to interference of light waves coming from the two slits.

On the other hand, if the path difference is odd multiples of $\lambda/2$ i.e. then θ gives the directions of minima due to interference of the secondary waves from the two slits.

$$A_2D = (e + d)\sin\theta = \pm(2n - 1)\frac{\lambda}{2} \quad \text{………}(5)$$

Diffraction maxima and minima:

For diffraction minima,

$$e \sin \theta = \pm m\lambda$$

where n = 1, 2, 3… then θgives the directions of diffraction minima. The ± sign indicates minima on both sides with respect to central maximum.

For diffraction maxima, $$\mathrm{e}\ \sin \theta = \pm(2m - 1)\frac{\lambda}{2}$$

The ± sign indicates maxima on both sides with respect to central maximum.

Missing orders in double slit:

The direction of interference maxima are given as,

$(e + d) \sin \theta = n\lambda$ ……. (6) where n = 1, 2, 3, …..

The directions of diffraction minima are given as,

$$e \sin\theta = m\lambda \quad \text{.......(7)} \qquad \text{where } m = 1, 2, 3, \ldots$$

Based on the relative values of e and *d*, certain orders of interference maxima are missing in the resultant pattern.

(i) If $e = d$, then, $2\, e \sin\theta = n\lambda$ and $e \sin\theta = m\lambda$, on dividing (6) by (7), we get

$$\therefore \frac{n}{m} = 2 \quad \text{or} \quad n = 2m$$

If $m = 1, 2, 3 \ldots$ then $n = 2, 4, 6\ldots$ *i.e.*, the interference orders 2, 4, 6 … are missed in the diffraction pattern.

(ii) If $2e = d$, then $3e \sin\theta = n\lambda$ and $e \sin\theta = m\lambda$

$$\therefore \frac{n}{m} = 3 \quad \text{or} \quad n = 3m$$

if $m = 1, 2, 3\ldots$ Then $n = 3, 6, 9\ldots$ *i.e.* the interference orders 3, 6, 9… are missed in the diffraction pattern

(iii) if $e + d = e$ *i.e.* $d = 0$, then two slits are joined. So, the diffraction pattern is due to a single slit of width $2e$.

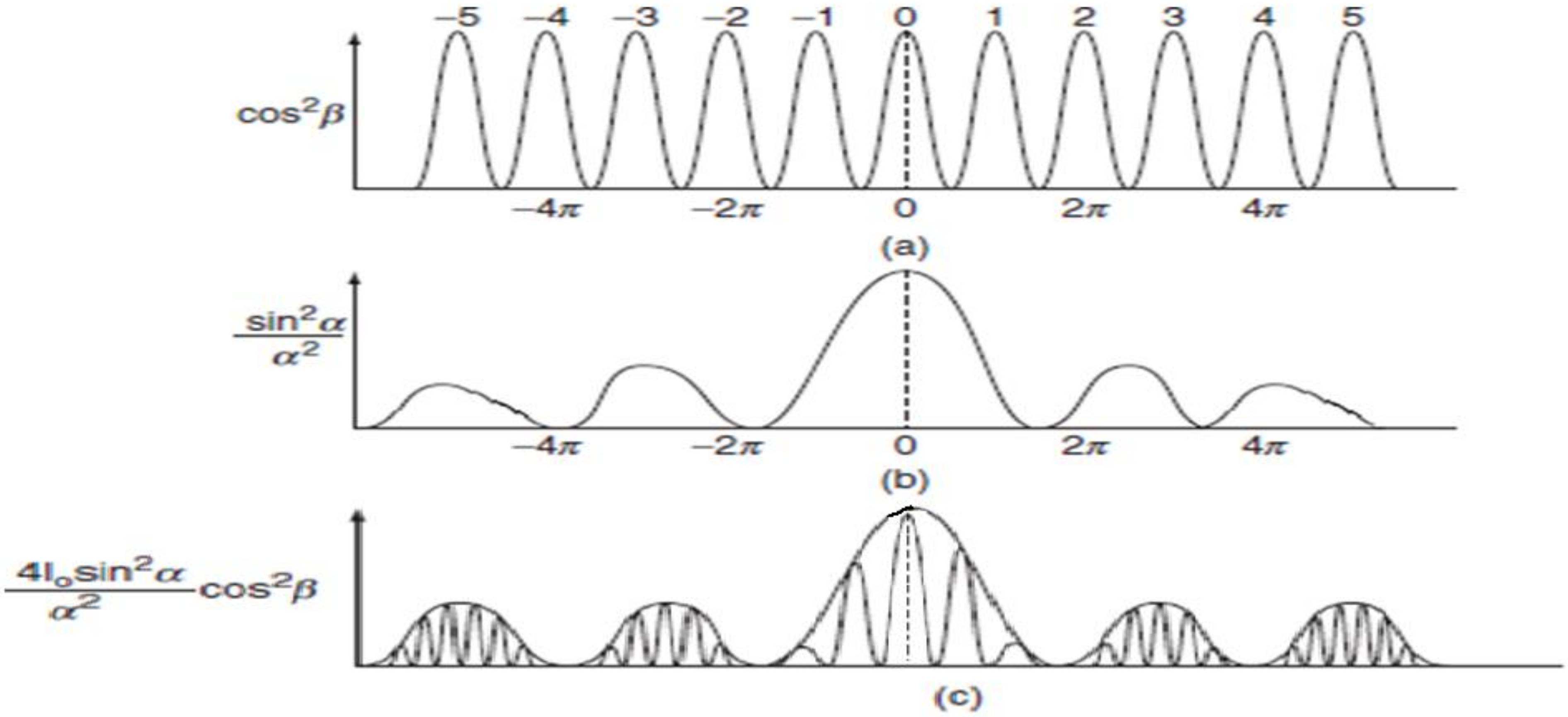
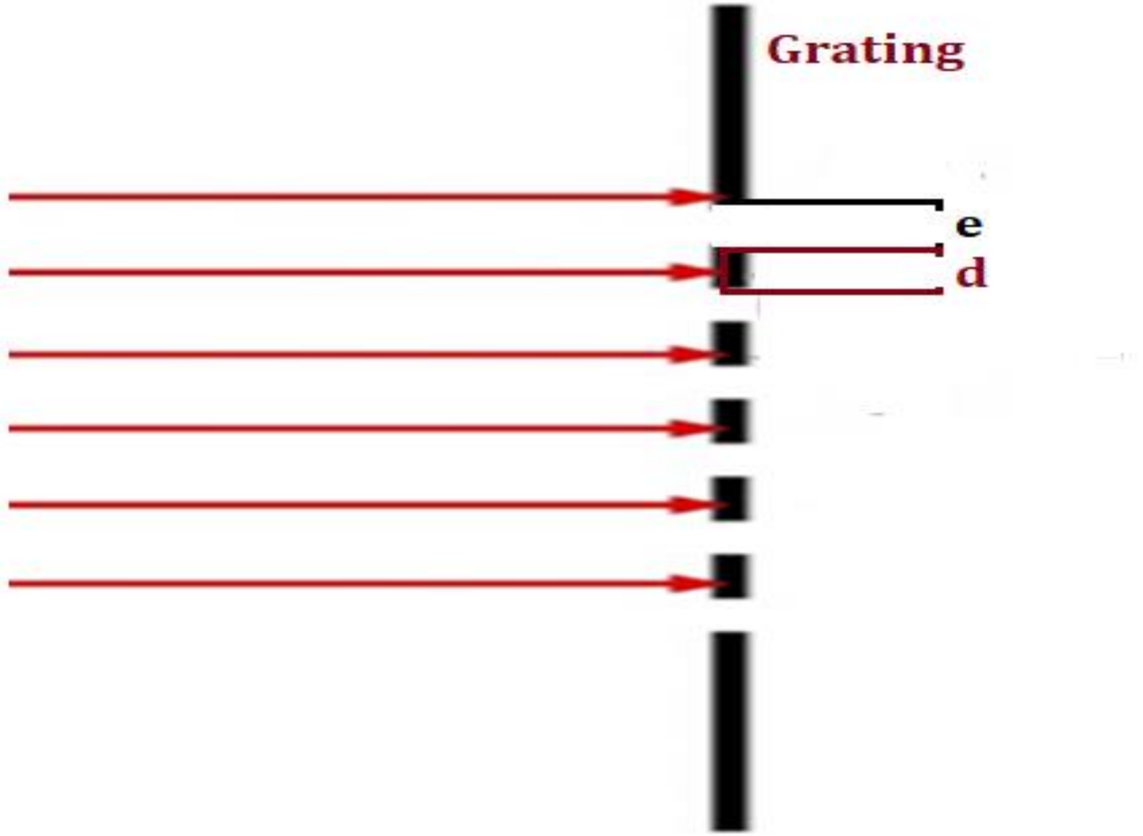


Figure: Intensity distribution due to double slit diffraction.

What do you understand by grating? (2016-17)

A plane diffraction grating is an arrangement consists of a large number of close, parallel, straight, transparent and equidistant slits of same width ***e***, with neighboring slits being separated by an opaque region of width ***d***.



Give the construction and theory of plane transmission grating? Explain the formation of spectra by it. (2015-16)

or

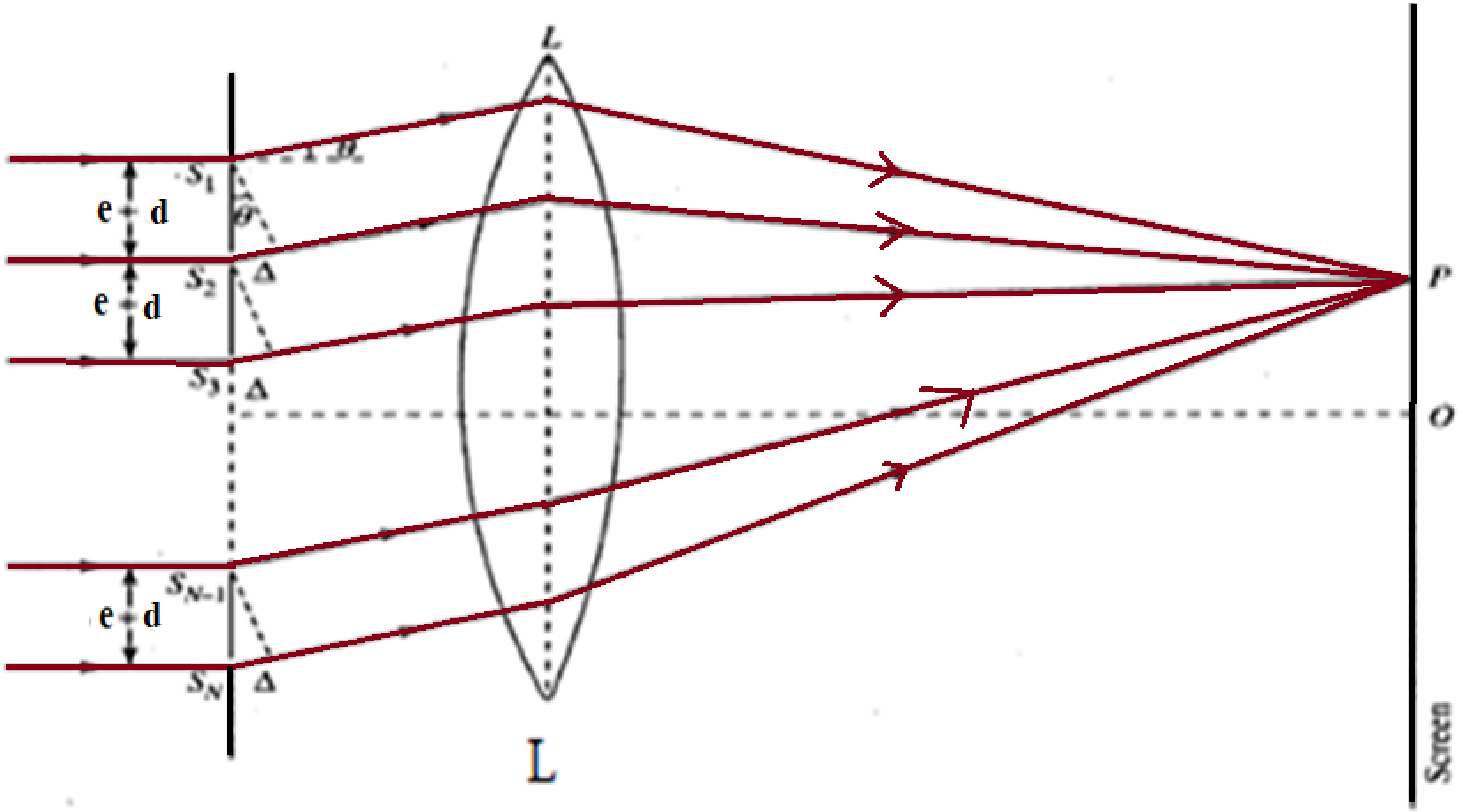
Give the construction and theory of plane transmission grating. (2017-18)

or

What is diffraction grating? Discuss the phenomena of diffraction due to plane diffraction grating. (2015-16)

Diffraction Grating

Let a monochromatic light incident on a plane diffraction grating consists of large number of N parallel slits, each of width *e* and separation *d*. Here **(*e*+*d*)** is called grating element.



The waves diffracted from each slit is equivalent to a single wave of amplitude $R = \frac{A \sin \alpha}{\alpha}$

The path difference between the consecutive waves is same and equal to $(e+d)$ sinθ.

$$\text{Phase difference } (2\beta) = \frac{2\pi}{\lambda}\,(e+d)\sin\theta$$

where $\beta = \frac{\pi}{\lambda}\,(e+d)\sin\theta$

Thus, the resultant amplitude at P is the resultant amplitude of N waves, each of amplitude R and common phase difference, 2β. Hence, the resultant amplitude at P is given by

$$R' = \frac{R\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}$$

$$\left[Here, R = \frac{A\,sin\,\alpha}{\alpha}\ (due\ to\ single\ slit), \delta = 2\beta, \qquad n = N\right]$$

$$R' = \frac{R\sin N\beta}{\sin\beta} = \frac{A\sin\alpha}{\alpha}\cdot\frac{\sin N\beta}{\sin\beta}$$

Resultant intensity at P is

$$I' = R'^2 = \frac{A^2 sin^2\alpha}{\alpha^2}\cdot\frac{sin^2 N\beta}{sin^2\beta} \qquad ....(1)$$

The factor $\frac{A^2 sin^2\alpha}{\alpha^2}$ gives the intensity pattern due to a single slit while the factor $(\frac{sin^2 N\beta}{sin^2\beta})$ gives the distribution of intensity due to interference from all the N points.

Principal maxima: From eq ^{n}(1), intensity will be maximum when

$$\sin \beta = 0$$

$$\beta = \pm n\pi \qquad \text{where, n= 0, 1, 2 ....}$$

But sin$N\beta$ is also equal to zero. Hence, $\frac{\sin N\beta}{\sin \beta}$ becomes indeterminate. Its limiting value can be evaluated by L'Hospital rule:

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \to \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin \beta)}$$

$$= \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = N$$

Therefore, the intensity at $\beta = \pm n\pi$ is given by,

$$I_{max} = A^2 \left(\frac{sin \propto}{\propto}\right)^2 N^2 \qquad \text{...........(2)}$$

$$\therefore \; \boldsymbol{I_{max} \propto N^2}$$

The direction of principal maxima are given by,

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda} (\mathrm{e} + d) \sin\theta = \pm n\pi$$

$$(\mathrm{e} + \boldsymbol{d}) \sin \boldsymbol{\theta} = \pm \boldsymbol{n\lambda} \qquad \text{…………….(3)} \qquad n = 0, 1, 2, \dots$$

For $\boldsymbol{n} = \mathbf{0}$, $\boldsymbol{\theta} = \mathbf{0}$, this gives the direction of zero order **principal maxima**. The values of n = 1, 2, 3,………gives the direction of first, second, third……order principal maxima.

Secondary minima: For minimum intensity, $\sin N\beta = 0$

$$N\beta = \pm m\pi$$

$$\frac{\pi}{\lambda}\,\mathrm{N}(\mathrm{e} + d) \sin \theta = \pm m\pi$$

$$\mathrm{N}(\mathrm{e} + d) \sin \theta = \pm m\lambda \qquad \text{……….(4)}$$

where m can take all integral values except 0, N, $2N$, $3N$,……..nN.

Secondary maxima: For secondary maxima,

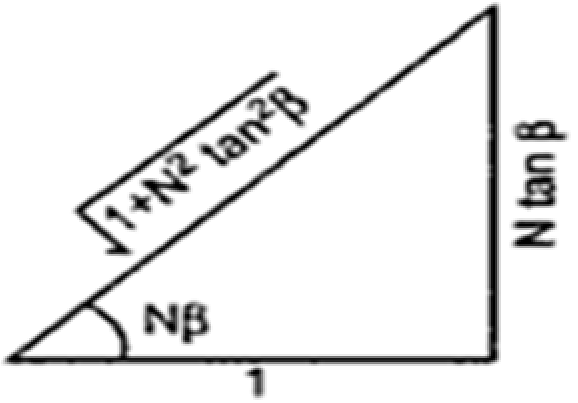
$$\frac{dI}{d\beta} = \frac{d}{d\beta}\left(\frac{A^2 sin^2 \alpha}{\alpha^2} \cdot \frac{sin^2 N\beta}{sin^2 \beta}\right) = 0$$

$$\frac{A^2 sin^2 \propto}{\alpha^2} \cdot 2\left[\frac{sinN\beta}{sin\beta}\right] \cdot \left[\frac{NcosN\beta sin\beta - sinN\beta cos\beta}{sin^2 \beta}\right] = 0$$

$$NcosN\beta sin\beta - sinN\beta cos\beta = 0$$

$$\boxed{\tan N\beta = N \tan \beta} \qquad \text{.......(5)}$$

To find the intensity of secondary maxima, we make the use if the triangle shown below:



We have, $\sin N\beta = \frac{N\tan\beta}{\sqrt{(1+N^2 tan^2\beta)}}$

$$\frac{sin^2 N\beta}{sin^2\beta} = \frac{(N^2 tan^2\beta)/(1+N^2 tan^2\beta)}{sin^2\beta}$$

$$= \frac{(N^2 tan^2\beta)}{(1+N^2 tan^2\beta)sin^2\beta} = \frac{N^2}{1+(N^2-1)sin^2\beta}$$

Putting this value of $(sin^2 N\beta)/sin^2\beta$ in eq^n (1)

$$I' = R'^2 = \frac{A^2 sin^2\alpha}{\alpha^2}\cdot\frac{N^2}{1+(N^2-1)sin^2\beta} \qquad \ldots(6)$$

Dividing eq ^{n} (6) by eq ^{n} (2), we get

$$\frac{Intensity\ of\ secondary\ maxima}{Intensity\ of\ primary\ maxima} = \frac{I'}{I_{max}} = \frac{1}{1 + (N^2 - 1)sin^2\beta}$$

Hence, the greater the value of N, the weaker are secondary maxima.

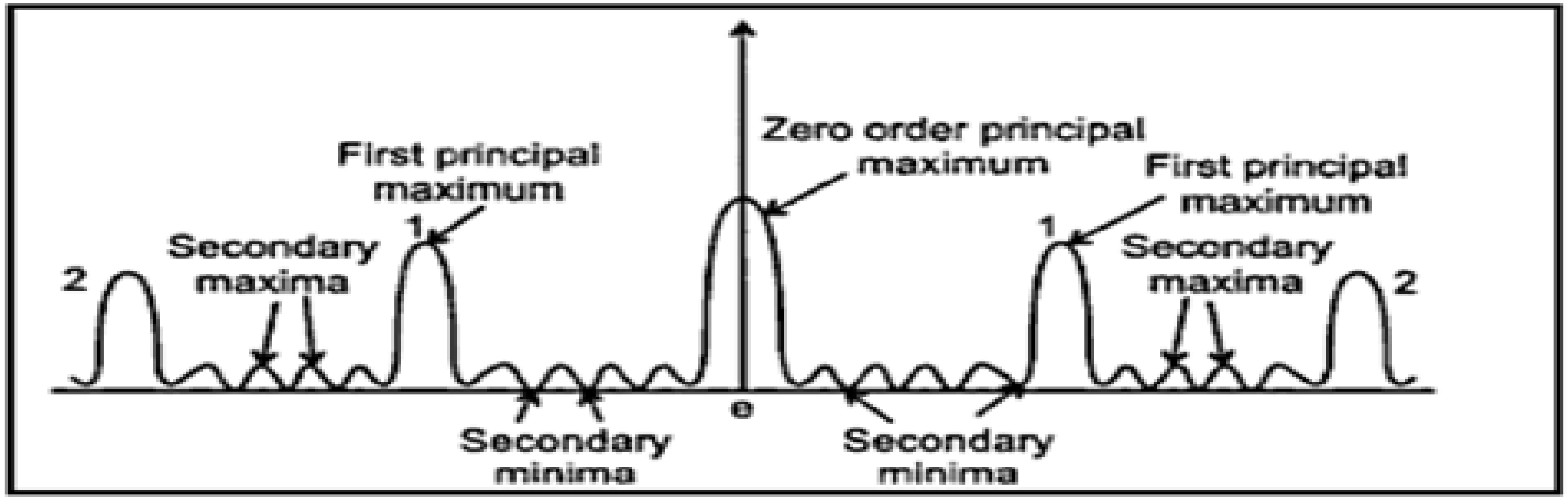


Figure: Intensity distribution due to N slit diffraction

Unit-III

Wave-optics

Lecture-24

Outline

- Formation of spectra by diffraction grating
- Missing order

Explain the formation of spectra by diffraction grating.

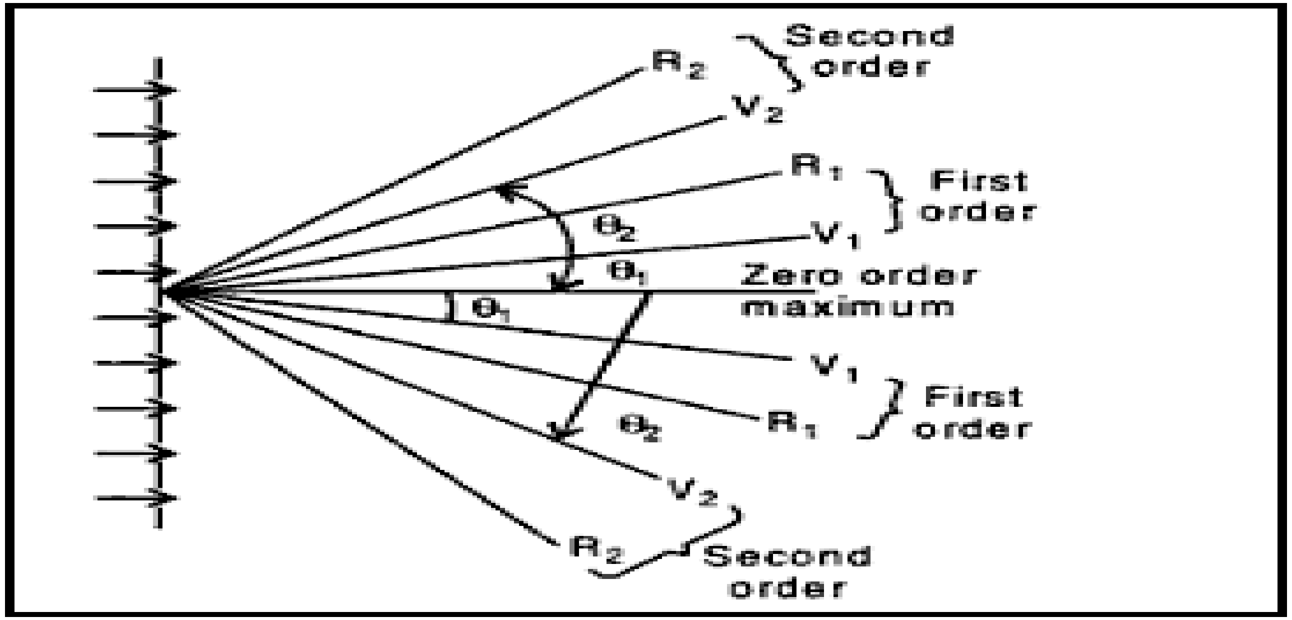
(2015-16, 2017-2018)

Formation of spectra by diffraction grating

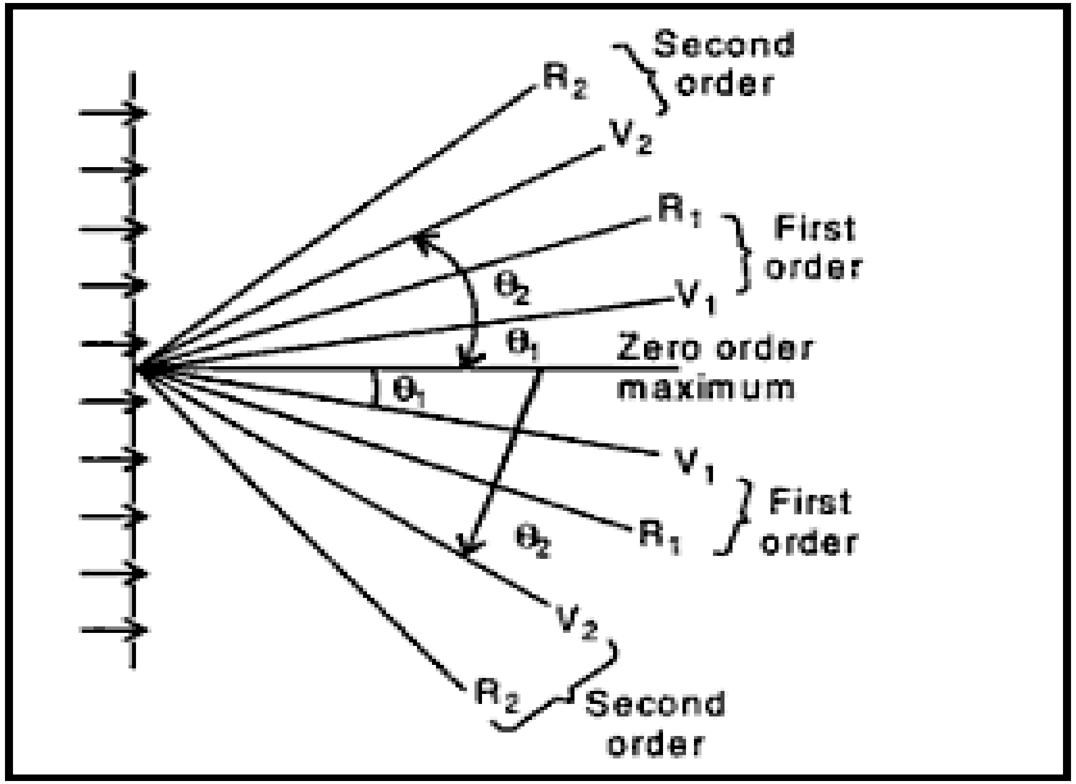
The direction of the n^{th} principal maxima is given by

$(e+d)\sin\theta_n = n\lambda$

where θ_n is the angle of diffraction and (e+d) is the grating element. From this equation, it can be concluded that



1. The angle of diffraction θ_n is different for different order principal maxima for a given wavelength.
2. If we use white light then the light of different wavelength is diffracted in a different direction for a particular order. Longer the wavelength, greater the angle of diffraction.

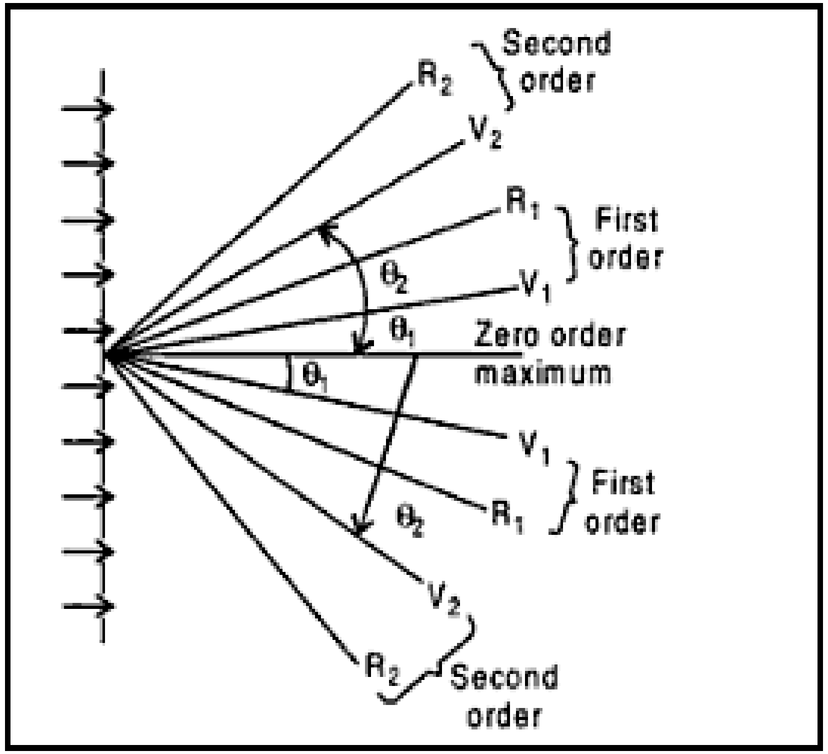


3. In grating spectra, violet colour is in the innermost position and red is the outermost position. As the order of the spectrum increase, the intensity decreases.

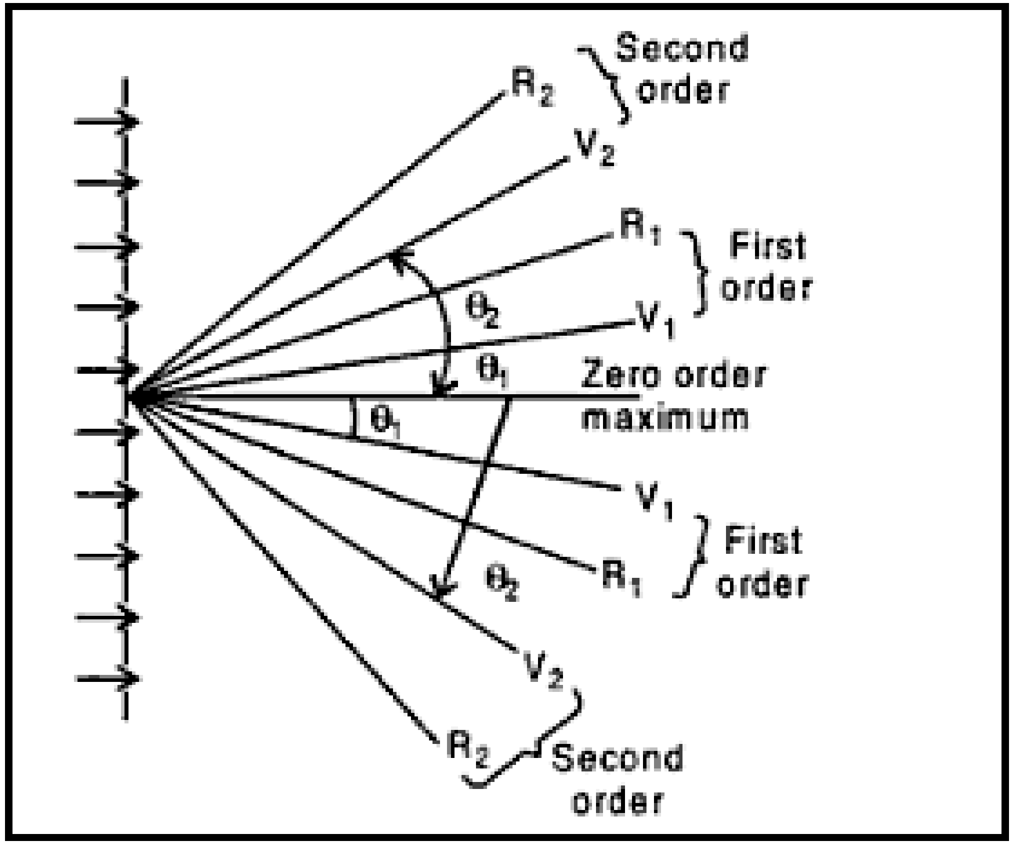
$$n = \frac{(\mathrm{e} + d)\sin\theta}{\lambda}$$

4. The maximum orders available in grating spectra can be obtained from the following condition. For nth principal maxima,

$(e+d)\sin\theta = n\lambda$



- At central maxima (n = 0), all maxima of different wavelengths coincide to form a central image of color similar to incident light.
- For n = 1, all principal maxima of different wavelength form a spectrum of the first order.
- Similarly for n = 2, all principal maxima of different wavelength form a second order spectrum.



What do you understand by missing order spectrum? What particular spectra would be absent if the width of transparencies twice of opacities of grating? **(2015-16, 2016-17).**

Missing order spectra in Diffraction Grating

As we know, the condition for maxima in grating,

$$(e+d)\sin\theta = n\lambda, \qquad n = 0,1,2,3\ldots\ldots \qquad (1)$$

and condition for minima in grating

$$e\sin\theta = m\lambda, \qquad m = 1,2,3\ldots\ldots.. \qquad (2)$$

Dividing eq ^{n} (1) by eq ^{n} (2),

$$\frac{(e+d)}{e} = \frac{n}{m} \quad or \quad n = \frac{(e+d)}{e}m$$

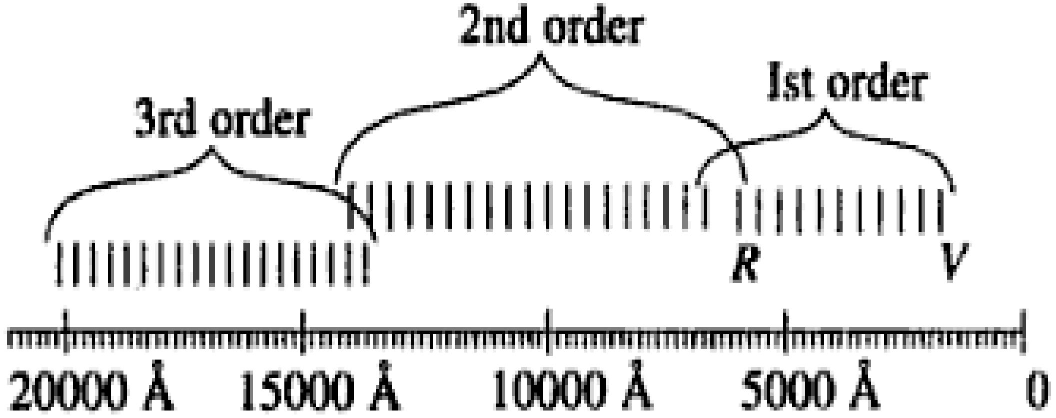
This is the required condition of missing order spectra in the diffraction pattern.

i) When $d = e$, then $n = 2m$

Therefore, when m = 1,2,3,…….missing orders are 2, 4, 6,……..

ii) When $d = 2e$, then $n = 3m$

Therefore, when m = 1,2,3,…….missing orders are 3, 6, 9,……..



What particular spectra would be absent if the width of the transparencies and opacities of the grating are equal. (2016-17)

According to missing order condition in diffraction grating, we know

$$n = \left(\frac{e + d}{e}\right) m$$

Here, e = d

$$n = 2m$$

where, m =1,2,3

Missing order m = 2,4,6………

What do you mean by resolving power on an optical instrument?

(2018-19, 2020-21)

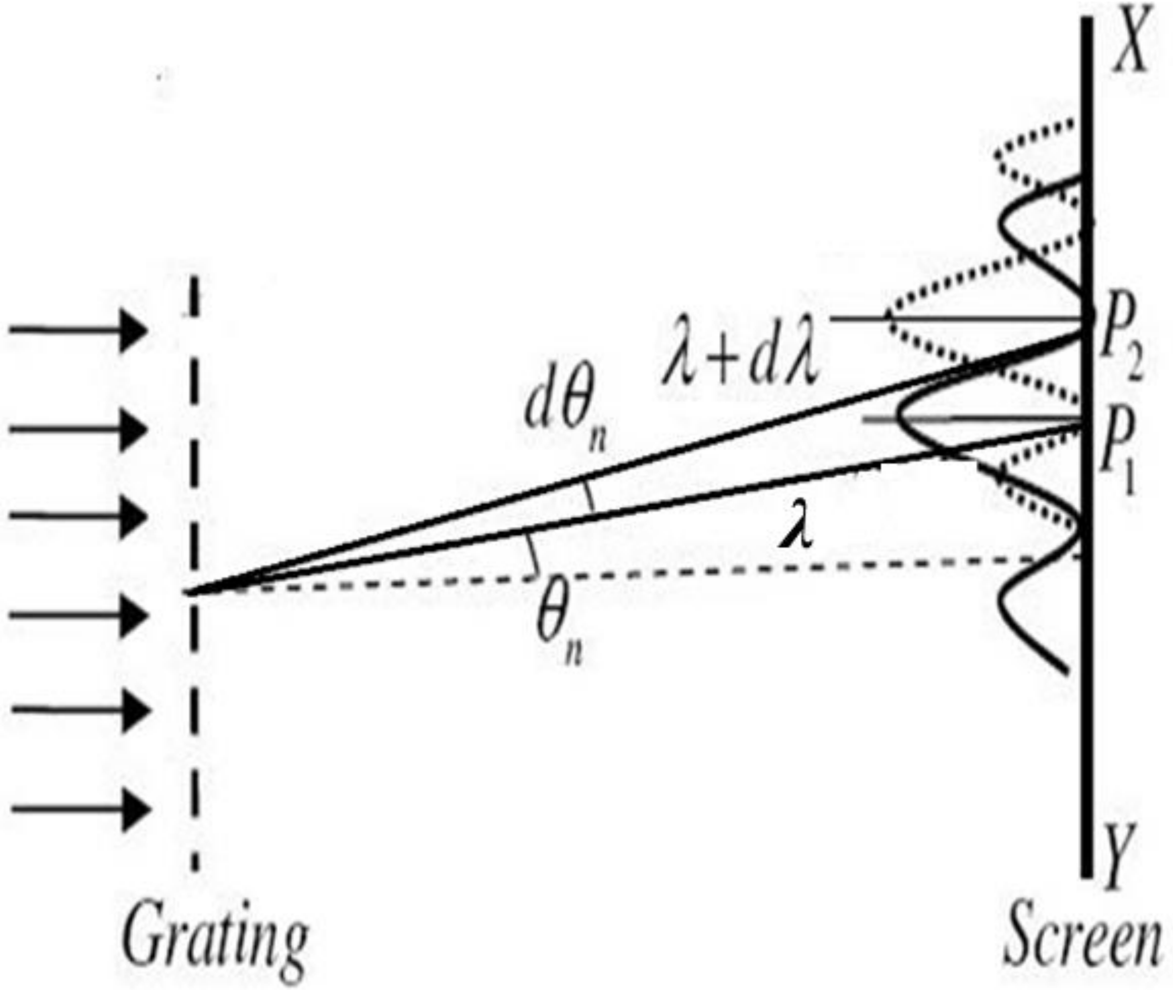
What is resolving power of grating? **(2015-16)**

Resolving Power

The capacity of an optical instrument to show two close objects separately is called resolution and the ability of an optical instrument to just resolve the images of two close point objects is called its **resolving power**.

Resolving power of a grating

It is defined as the ratio of the wavelength of any spectral line to the smallest wavelength difference between neighboring lines for which the spectral can be just resolved at the wavelength λ.
It can be expressed mathematically as $\frac{\lambda}{d\lambda}$.



Let the direction of n^{th} principal maxima for wavelength λ is given by

$$(e + d) \sin \theta_n = n \lambda$$

$$N (e + d) \sin \theta_n = Nn \lambda$$

and the first minima will be in the direction given by

$$N (e + d) \sin (\theta_n + d\theta_n) = m \lambda$$

where m is an integer except 0, N, $2N$ … because at these values condition of maxima will be satisfied.

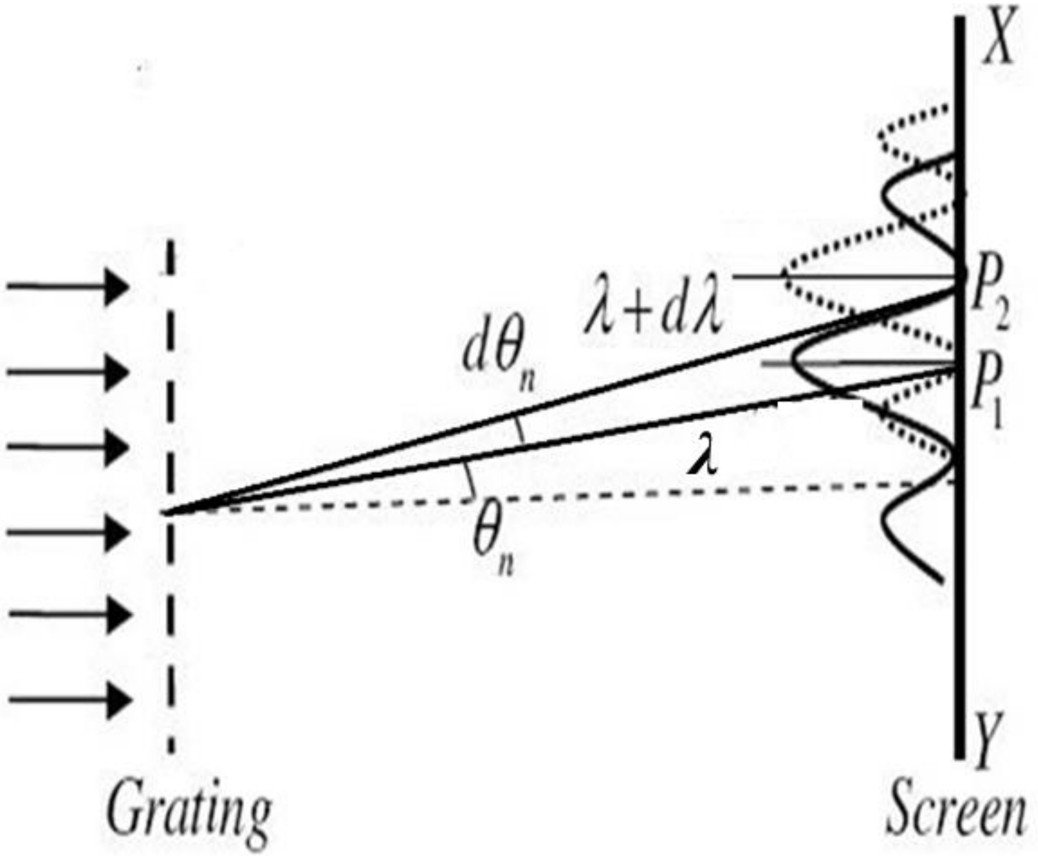
The first minima adjacent to the n^{th} maxima will be in the direction $(\theta_n + d\theta_n)$ only when $m = (nN + 1)$. Thus

$$N(e + d)\sin(\theta_n + d\theta_n) = (nN + 1)\lambda \qquad \text{…. (1)}$$

For just resolution, the principal maxima for the wavelength $\lambda + d\lambda$ must be formed in the direction $(\theta_n + d\theta_n)$, therefore

$$(e + d)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda)$$

$$N(e + d)\sin(\theta_n + d\theta_n) = Nn(\lambda + d\lambda) \qquad \text{…. (2)}$$



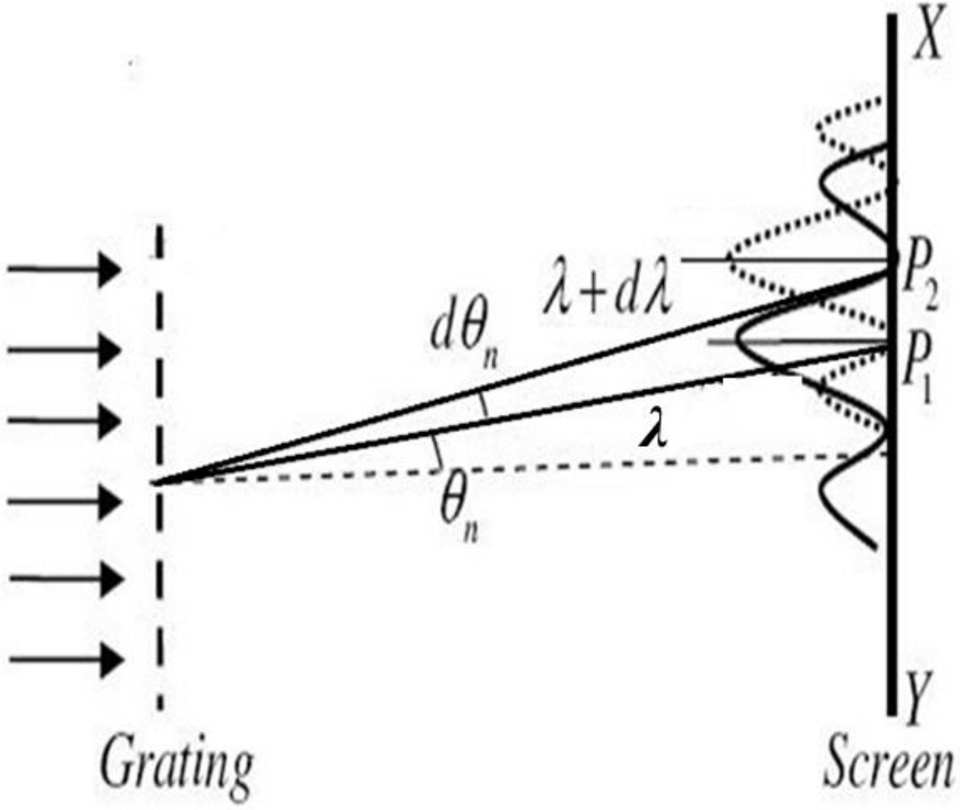
Now equating (1) and (2),

$$(nN + 1)\,\lambda = Nn\,(\lambda + d\lambda)$$

$$\lambda = Nn\,d\lambda$$

Thus resolving power of grating is,

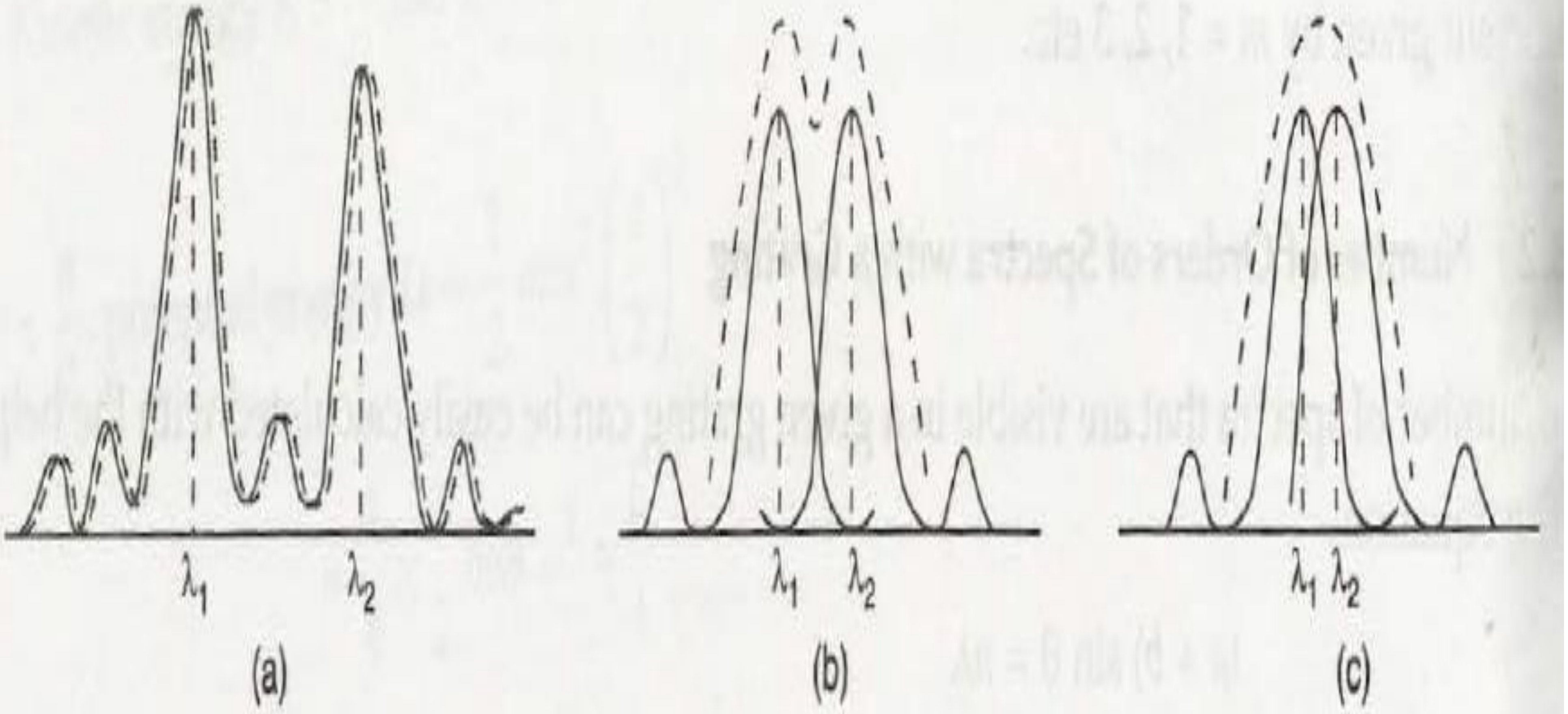
$$\frac{\lambda}{d\lambda} = \mathrm{Nn}$$



What is a Rayleigh criterion of resolution? (2015-16, 2016-17, 2020-21)

The Rayleigh criterion of resolution

According to Rayleigh, the two point sources or two equally intense spectral lines are **just resolved** by an optical instrument when the central maximum of the diffraction pattern due to one source falls exactly on the first minimum of the diffraction pattern of the other and vice-versa.



λ_1
λ_2
(a)
λ_1
λ_2
(b)
λ_1
λ_2
(c)

Define dispersive power of a plane transmission diffraction grating.

(2017-18, 2018-19, 2019-20)

Dispersive power of diffraction grating

The **dispersive power** of a diffraction grating is defined as the **rate of change of the angle of diffraction with the change in the wavelength** of light used.

$$\omega = \frac{d\theta}{d\lambda}$$

For plane diffraction grating, $(e + d)\sin\theta = n\lambda$, ...(1)

Differentiate this eq ^{n} with respect to λ, we get

$$(e + d)\cos\theta \frac{d\theta}{d\lambda} = n$$

$$or \quad \frac{d\theta}{d\lambda} = \frac{n}{(e + d)\cos\theta}$$

$$or \quad \frac{d\theta}{d\lambda} = \frac{n}{(e + d)(1 - \sin^2\theta)^{1/2}} \quad .....(2)$$

From eq ^{n} (1), we have,

$$\sin\theta = \frac{n\lambda}{(e+d)}$$

Substituting the above value of $\sin\theta$ in eq ^{n} (2),

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)[1-(n\lambda/e+d)^2]^{1/2}}$$

$$\frac{d\theta}{d\lambda} = \frac{1}{[(\frac{e+d}{n})^2 - \lambda^2]^{1/2}}$$

The above equation leads to the following conclusion:

1. ω is directly proportional to the order of spectrum i.e. the higher is the order, greater is the dispersive power.

2. ω is inversely proportional to the $(e+d)$ i.e. the dispersive power is greater for a grating having larger number of lines per cm.

3. ω is inversely proportional to the cos θ i.e. larger the value of θ, higher is the dispersive power.

Unit-III

Wave-optics

Lecture-25

Outline

- Numericals

A diffraction grating used at normal incidence gives a yellow line (λ = 6000Å) in a certain spectral order superimposed on a blue line (λ = 4800Å) of next higher order. If the angle of diffraction is sin^{-1}(3/4), calculate the grating element.

(2015-2016)

The direction of principal maxima for normal incidence for wavelength λ_1 is

$$(e + d) \sin\theta = n\lambda_1 \qquad \ldots\ldots(1)$$

Let n^{th} order maximum of λ_1 coincide with $(n+1)^{th}$ order maximum of λ_2, then

$$(e + d) \sin\theta = n\lambda_1 = (n + 1)\,\lambda_2$$

or $$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

From (1), $(e+d)\sin\theta = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$

$$(e+d) = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)\sin\theta}$$

$$= \frac{6000\times10^{-8}\times4800\times10^{-8}}{(6000-4800)\times10^{-8}\times(3/4)}$$

$$= 3.2 \times 10^{-4} cm$$

Light of wavelength 5500 Åfalls normally on slit of width 22.0 $\times 10^{-5}$ cm. Calculate the angular position of two minima on either side of central maxima. **(2015-16)**

For single slit diffraction, the angular position of minima is

$$e \sin \theta = n\lambda$$

$\sin \theta = 2\lambda / e (n = 2)$

$\theta = \sin^{-1} (2\lambda/e)$

$= \sin^{-1} (2 \times 5500 \times 10^{-8}) / (22 \times 10^{-5})$

$\theta = 30°$

A diffraction grating used at normal incidence gives a green line (λ = 5450Å) in a certain spectral order superimposed on a violet line (λ = 4100Å) of next higher order. If the angle of diffraction is 30°, then how many lines per cm are there in grating?

(2015-16)

For grating, $(e+d)\sin\theta = n\lambda$

Let n^{th} maxima of λ_1 coincide with $(n+1)^{\mathrm{th}}$ maxima of λ_2, then we have

$(e+d)\sin\theta = n\,\lambda_1 = (n+1)\ \lambda_2$

$n\,\lambda_1 = (n+1)\ \lambda_2$

$n = \frac{\lambda_2}{\lambda_1-\lambda_2}$, We have, $(e+d)\sin\theta = \frac{\lambda_2\,\lambda_1}{\lambda_1-\lambda_2}$

$\sin\theta = \frac{\lambda_2\,\lambda_1}{(\lambda_1-\lambda_2)(e+d)}$ ……..(1)

$$\lambda_1 = 5400\ A^{\circ} = 5400 \times 10^{-8} cm,\ \lambda_2 = 4050\ A^{\circ} = 4050 \times 10^{-8} cm,$$

$$\lambda_1 - \lambda_2 = 1350\ \times 10^{-8} cm,\ \ \theta = \ 30^{\circ}$$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{4050 \times 10^{-8}}{1350\ \times 10^{-8}} = 3$$

Now from (1), $(e + d) = \frac{5400 \times 10^{-8} \times 4050 \times 10^{-8}}{1350\ \times 10^{-8} \times \sin 30^{\circ}}$ =33.10×10^{-7} cm

Number of lines per cm = $\frac{1}{(e+d)} = 3 \times 10^{5}$

A plane transmission grating has 15000 lines per inch. Find the resolving power of grating and the smallest wavelength difference that can be resolved with a light of 6000Ǻ in the second order. **(2016-17)**

The number of lines per inch on the grating, N = 15000

n = 2, Resolving power = $\frac{\lambda}{d\lambda} = nN$

$$= 2 \times 15000 = 30000$$

The smallest wavelength difference,

$$d\lambda = \frac{\lambda}{nN}$$

$$= \frac{6000 \times 10^{-8}}{30000}$$

$$= 0.20 \times 10^{-8} \text{cm}$$

A plane transmission grating has 16,000 lines to an inch over a length of 5 inches. Find in the wavelength region of 6000 Å, in the second order (i) the resolving power of grating and (ii) the small wavelength difference that can be resolved. **(Imp)**

The number of lines per inch on the grating, N = 16000

Length of grating = 5 inches, So total number of lines = 80000,

n = 2,

(i) Resolving power = $\frac{\lambda}{d\lambda} = nN = 2 \times 80000 = 160000$

(ii) The smallest wavelength difference,

$$d\lambda = \frac{\lambda}{nN} = \frac{6000 \times 10^{-8}}{160000} = 3.75 \times 10^{-10}\,\mathrm{cm}$$

Light of wavelength 5000 Å is incident normally on a slit. The central maximum falls out 30^0 on both sides of the direction of incident light. Calculate the slit width. **(Imp)**

For single slit, the direction of minima is given by,

e $\sin\theta = n\lambda$ where n = 1, 2, 3…..

Therefore, the angular spread of the central maximum on either side of incident light is,

$$\sin\theta = \frac{\lambda}{e}$$

$$e = \frac{\lambda}{\sin\theta} = \frac{5\times10^{-8}}{\sin 30} = 10^{-7}cm$$

Calculate the angle at which the first dark band and the next bright band are formed in the Fraunhofer diffraction pattern of a slit 0.3 mm wide (λ = 5890Å).

For single slit, the direction of minima is given by,

$e \sin\theta = n\lambda$ where n = 1, 2, 3…..

For first dark band, n =1

$$\sin\theta = \frac{\lambda}{e} = \frac{5\times10^{-8}}{0.03} = 0.00196$$

$$\theta = \sin^{-1}(0.00196) = 0.112^{\circ}$$

Thank you