Monogram on Transformation

Introduction: Transformation is a process by which we can change the shape, size, orientation and position of an object by applying mathematical operation. In computer graphics transformation plays an important role in order to find and retrieve transformed position of an object with the help of their corresponding transformation matrix.

The coordinate of transformed object can be obtained by multiplying the transformation matrix (new coordinate) with the object matrix (old coordinate). There is a large application of transformation techniques which can be used in our daily life.

Transformation can be accomplished by two methods

1. Geometric Transformation
2. Coordinate Transformation

1. Geometric Transformation:

This is a type of transformation in which an object itself is transformed with respect to stationary coordinate system. It is applied to each point of the object.

2. Coordinate Transformation:

This is a type of transformation in which the object is held stationary while the coordinate system is transformed relative to the object.

Now let us talk about the geometric transformation.

Geometric transformation:

Change in shape, size, orientation and position of an object can be accomplished with geometric transformation. The basic types of geometric transformations are translation, rotation and scaling. The other transformations that are applied to object are reflection and shearing.

Here I would like to focus on rotation. Here rotation should be with respect to origin and with respect to an arbitrary point.

Translation: A translation is applied to an object by repositioning it along a straight line path from one coordinate location to another. We can translate a two dimensional point by adding translation distance t_x and t_y to the original coordinate position (x,y) to move the point to a new position(x',y'). It is given mathematically as

$$x' = x + t_x$$

$$y' = y + t_y$$

The translation distance pair (t_x, t_y) is called a translation vector or shift vector.

Rotation: Angular displacement of any object is called rotation. A two dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane. To illustrate the rotation we specify rotation angle α and the position (x_r, y_r) of the rotation point about which the object is to be rotated. If the object is rotated clockwise then sign value of α is chosen as +ve and if it is rotated anticlockwise then sign value of α is chosen as –ve. We first calculate the transformation equations for rotation of a point position P when the reference point is at coordinate origin. Let us consider P is at r distance from origin and angle β is the original angular position of the point and α is the rotation angle.

We can express the transformed coordinates in terms of angle α and β as

$$x = r\cos\beta, \qquad y = r\sin\beta$$

$$x' = r\cos(\alpha+\beta) = r\cos\alpha\cos\beta - r\sin\alpha\sin\beta = x\cos\alpha - y\sin\alpha$$

$$y' = r\cos(\alpha+\beta) = r\cos\beta\sin\alpha + r\sin\beta\cos\alpha = x\sin\alpha + y\cos\alpha$$

from this equation we can find rotation matrix and finaly transformed coordinate can be obtained as $P' = R_{\alpha}.P$

Rotation about an arbitrary point:

Earlier we have discussed rotation about origin. Now the case is different, in this case here the rotation is not about origin but it is with respect to some arbitrary point. For this purpose homogeneous coordinate system will provide a method to obtain rotation about any point. This can be done in the following order:

Step 1: Translate the object or body at the origin.(Translation)

Step 2: Rotate by an angle as given (Rotation)

Step 3: Translate back to its original location. (Inverse Translation)

In matrix form it can be shown as

$$R(x_r, y_r, \alpha) = [T] = T_{(xr,yr)} \cdot R_{\alpha} \cdot T_{(-xr,-yr)}$$

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